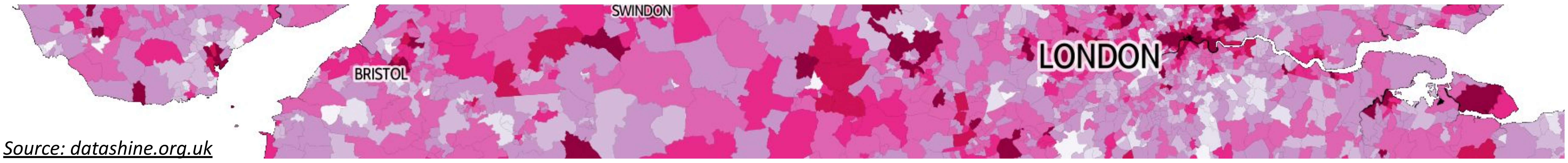
SCALING OF EMPIRICAL COMPARTMENTAL DISTRIBUTIONS

Justas Kvedaravicius[1], Aleksejus Kononovicius[2]
[1]Faculty of Physics, Vilnius University, Lithuania,
[2]Institute of Theoretical Physics and Astronomy, Faculty of Physics, Vilnius University, Lithuania
justas.kvedaravicius@ff.stud.vu.lt



Source: datashine.org.uk

Introduction

Sociophysics is a physical approach to modeling social phenomena. Yet the approach has a lack of comparison between voter model's theoretical results and empirical (observed) data [1, 2]. To enable comparison some authors explicitly include space (geography) in modelling opinion dynamics [1, 3]. To understand the implications of space to opinion dynamics, it's worth studying how empirical data can entail structures of opinion or socio-demographic indicators. **This research analyzes changes in empirical compartmental distributions. Further comparison is done against spin-lattice systems, namely Ising and noisy voter models.**

UK 2011 census data and randomized spatial model

Empirical data from United Kingdom 2011 census have been chosen as it is available at scales of varying size. It details demographic indicator (age, sex, economic status) values for local geographic units (e.g. small villages, communities) as well as national regions (e.g. metropolitan areas, countries). We refer to such distinct territorial units as empirical compartments. Notably distributions of empirical compartments vary for chosen demographic indicators at different scales.

Firstly, a randomized spatial model is proposed to assess the nature of observed distributions of empirical compartments. Model takes empirical compartments from the finest scale and combines them randomly to replicate compartments of a coarser scale. Some diversity measures (e.g. standard deviation) are chosen for which relative changes in their values are calculated. It is shown that results produced by the proposed model follow the inverse square root law. However, relative changes in the diversity measures for the empirical data differ from the theory thus suggesting existence of spatial structures.

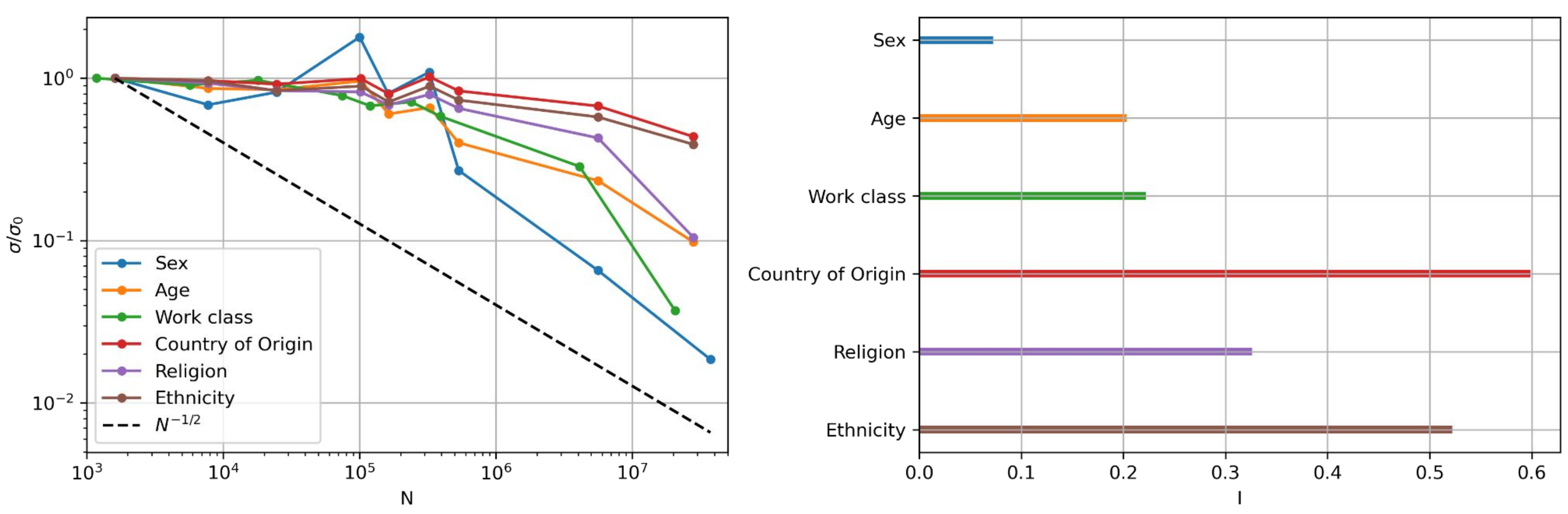


Fig. 1. Relative changes of the standard deviation for chosen demographic indicators (left). Diversity index *I* values were calculated for each of the indicator (right). Index value 0 represents the case of randomized spatial model.

Other considered diversity measures exhibit different power laws. For example, relative changes in Theil index (which is a case of generalized entropy) follow inverse linear law.

Quantitative diversity index

Building on the qualitative description of spatial structures a new measure is introduced for quantitative description. A numerical diversity index (*I*) is developed to describe certain configurations of spin-lattice models. These states are namely ferromagnetic (*I* = 1), anti-ferromagnetic (*I* = -1) or random configuration (*I* = 0). The latter is an instance of already proposed random model and is an intermediate between the other two.

Noisy voter and Ising (Metropolis) models

Diversity index values were calculated for spin models at thermal equilibrium. Metropolis dynamics were employed for simulation of Ising model (at first). Proposed index *I* excelled at capturing spatial structure for Metropolis dynamics at critical temperature. The lattice is known to exhibit aligned domains of various size at the critical point. However, it failed to capture polarization at low temperature values, where one major domain of spins is present.

Noisy voter model showed logarithmic dependence of index in terms of noise probability. The noise was introduced through a parameter *p* which represented probability to stochastically change spin value. Results of this model were similar to the Metropolis dynamics above critical point temperature where Ising model is at anti-ferromagnetic state.

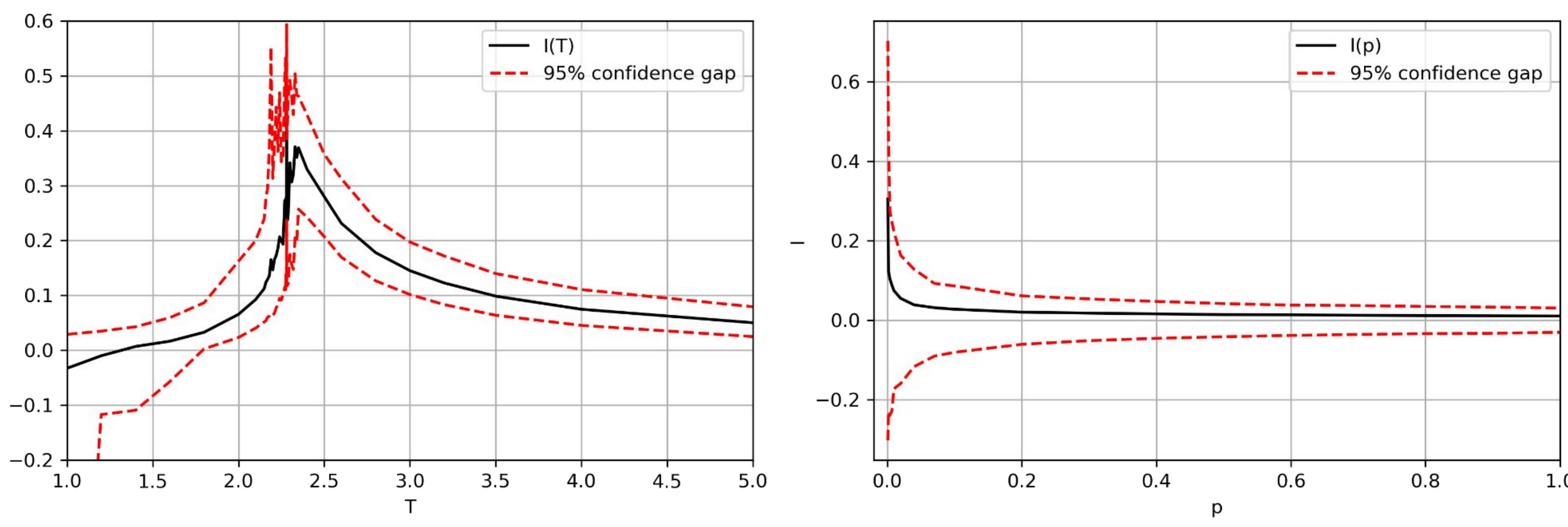


Fig. 2. Diversity index values for Ising model with Metropolis dynamics (left) and noisy voter model (right). Solid line shows index median value while dashed red line is 95% confidence gap. Ising model critical temperature point is approximately at 2.269.

Ising model with Kawasaki dynamics

Furthermore, Kawasaki dynamics were used for diversity index calculations on Ising model. This interpretation allows spins to exchange places and conserves overall magnetization of the lattice. Biggest diversity index values for such lattice were at phase coexistence state. Different spins are separated into small inter-connected regions at said phase and similarly to Metropolis case index values are at their highest.

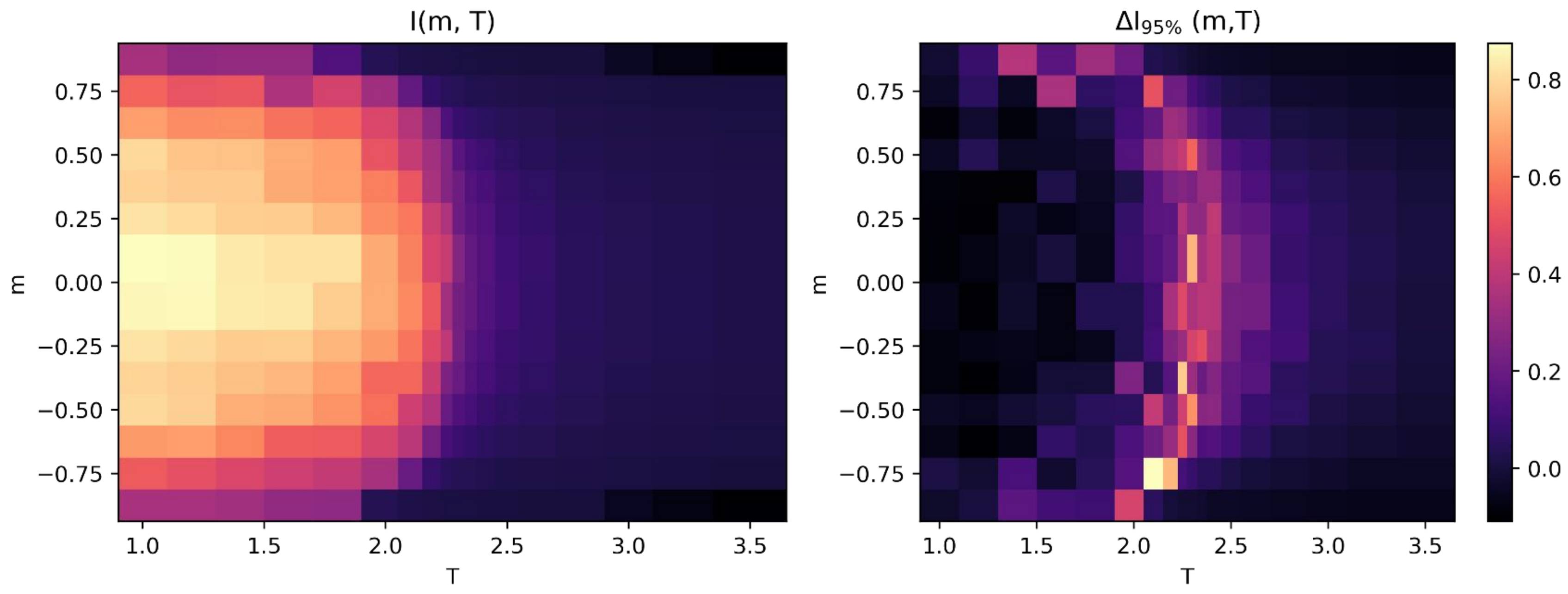


Fig. 3. Diversity index median values for Ising model with Kawasaki dynamics (left) and total value for 95% confidence gap of said index (right). Magnetization and temperature are parameters for Kawasaki dynamics.

[1] J. Fernandez-Gracia, K Suchecki, J. J. Ramasco, M. Miguel, V. Eguiluz, Is the voter model a model for voters?, Physical Review Letters 112, 158701 (2014).
[2] A. F. Peralta, J. Kertész, G. Iñiguez, Opinion dynamics in social networks: From models to data., 2022.
[3] A. Kononovicius, Compartmental voter model, Journal of Statistical Mechanics: Theory and Experiment 2019, 103402 (2019).