Modeling $1/f$ noise using models with overlapping pulses

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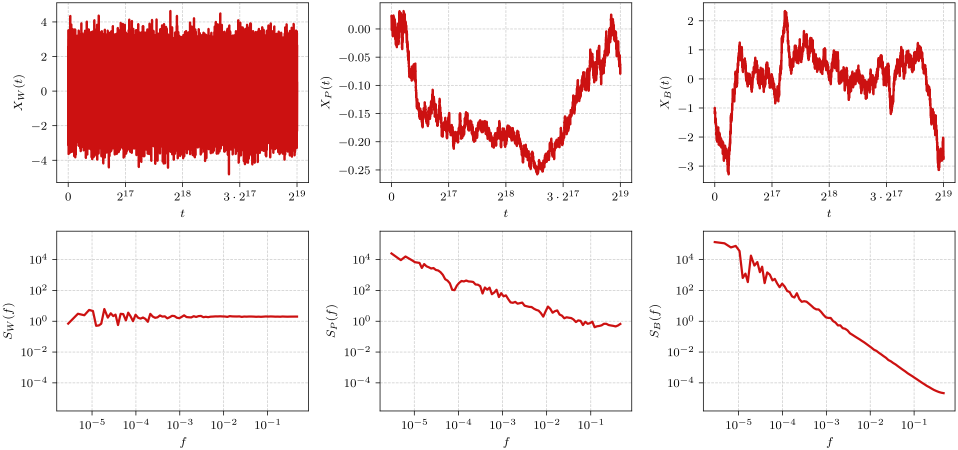
$1/f$ noise?

$$X_W(t) = \xi(t)$$

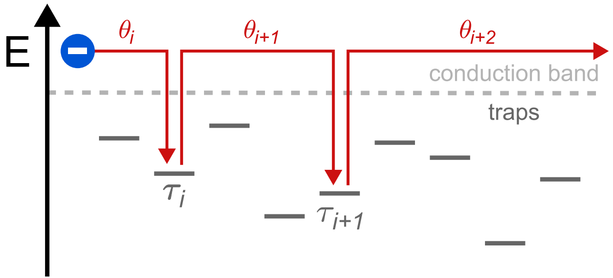
$$X_P(t) = ???$$

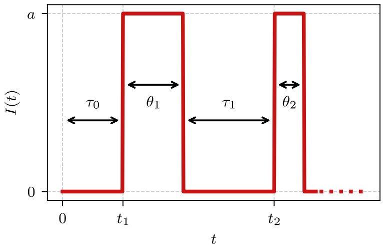
$$X_B(t) = \int_0^t \xi(t)\mathrm{d}t$$



Non-overlapping pulse model

Model generating non-overlapping pulses





τ - escape (detrapping, generation) time, θ - capture (trapping, recombination) time.

Power spectral density

In general, for non-overlapping pulses:

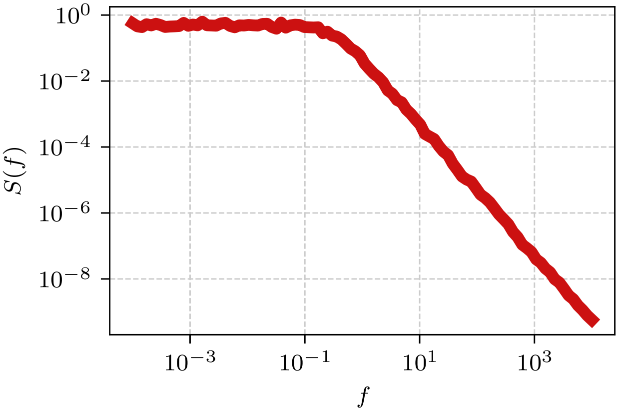
$$S(f) = \lim_{T\to\infty} \left\langle \frac{2}{T} \left| \sum_k e^{-2\pi i f t_k} F_k(f) \right|^2 \right\rangle .$$

For rectangular pulses, and given that τ_i and θ_i are independent:

$$F_k(f) = \frac{\mathrm{i}a}{2\pi f} \left(e^{-2\pi \mathrm{i} f \theta_k} - 1 \right),$$

$$\boxed{S(f) = \frac{a^2 \bar{\nu}}{\pi^2 f^2} \operatorname{Re} \left[\frac{(1-\chi_\theta(f))(1-\chi_\tau(f))}{1-\chi_\theta(f)\chi_\tau(f)} \right]} .$$

Lorentzian spectral density



Homogeneous material:

$$\theta_i \sim \text{Exp}\left(\gamma_\theta\right).$$

Identical traps:

$$\tau_i \sim \text{Exp}\left(\gamma_\tau\right).$$

Spectral density:

$$S(f) = \frac{4a^2\bar{\nu}}{(\gamma_\tau + \gamma_\theta)^2 + 4\pi^2 f^2}.$$

Obtaining $1/f$ noise

With heavy-tailed distributions

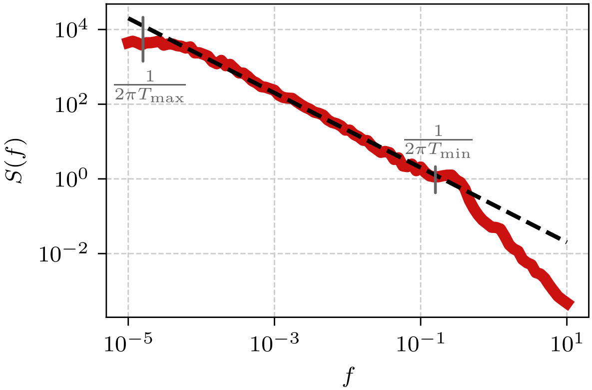
$$\tau_i \sim \text{Pareto}\,(\alpha, T_{\text{min}}, T_{\text{max}})\,,$$

$$\theta_i \sim \text{Pareto}\,(\alpha, T_{\text{min}}, T_{\text{max}})\,,$$

we obtain

$$S(f) \propto \frac{1}{f^{\alpha}}, \quad \text{for } 0 < \alpha < 1,$$

$$S(f) \propto \frac{1}{f^{2-\alpha}}, \quad \text{for } 1 < \alpha < 2.$$



Heterogeneous detrapping process...

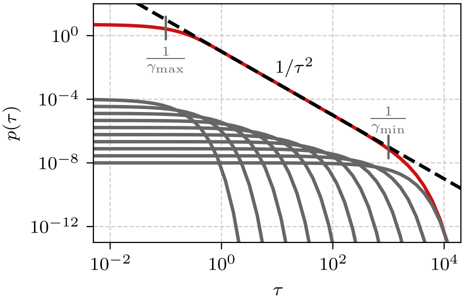
If trapping centers are unique:

$$\gamma_i \sim \mathcal{U}\left(\gamma_{\text{min}}, \gamma_{\text{max}}\right),$$

$$\tau_i \sim \text{Exp}\left(\gamma_i\right),$$

then the detrapping time distribution:

$$p(\tau) \propto \int_{\gamma_{\text{min}}}^{\gamma_{\text{max}}} \gamma_i \exp\left(-\gamma_i \tau\right) \mathrm{d}\,\gamma_i \propto \frac{1}{\tau^2}.$$



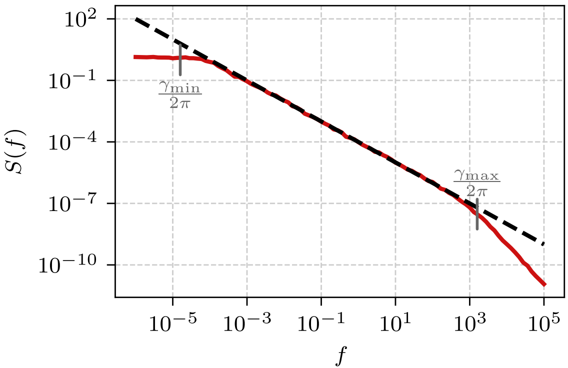
...in homogeneous materials leads to $1/f$ noise

Homogeneous material:

$$\theta_i \sim \mathrm{Exp}\left(\gamma_\theta\right).$$

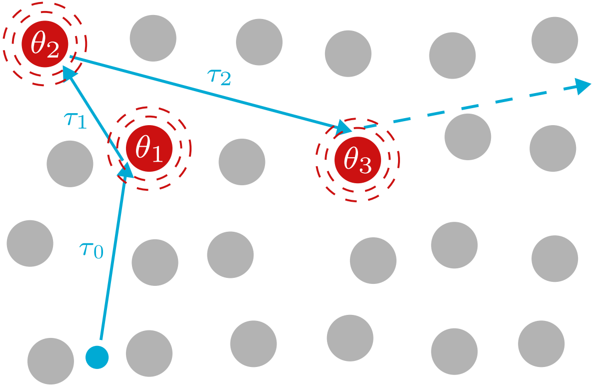
For long pulses, $\gamma_\theta \ll \gamma_{\max}$:

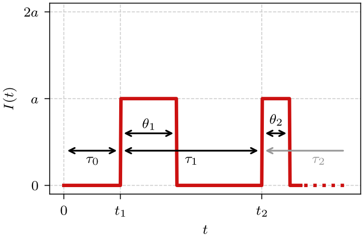
$$S(f) \approx \frac{a^2 \bar{\nu}}{\gamma_{\max}} \cdot \frac{1}{f}.$$



Overlapping pulse model

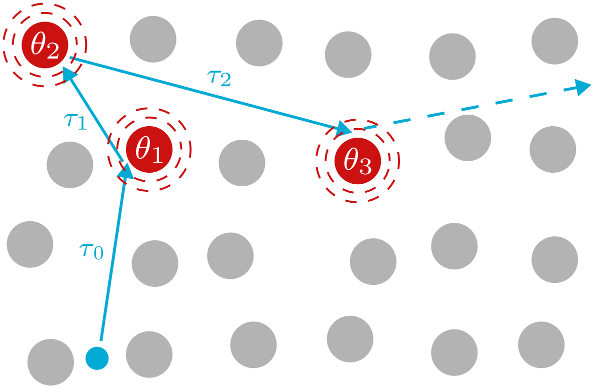
Model generating overlapping pulses

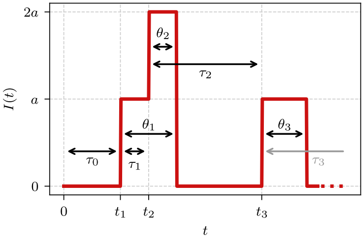




τ - free-flight (interaction) time, θ - relaxation time.

Model generating overlapping pulses





τ - free-flight (interaction) time, θ - relaxation time.

Power spectral density

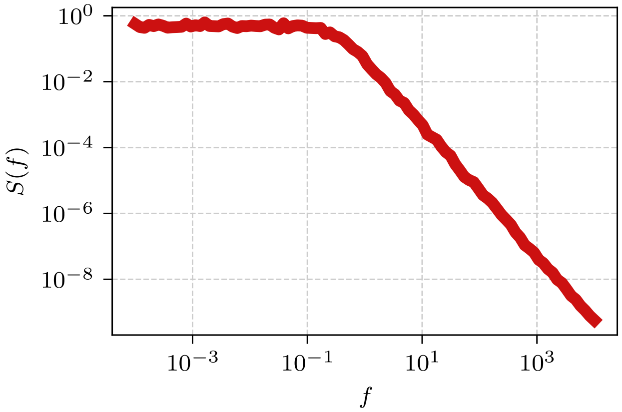
For rectangular non-overlapping pulses (previously) we have obtained

$$S(f) = \frac{a^2 \overline{\nu}}{\pi^2 f^2} \operatorname{Re}\left[\frac{(1-\chi_\theta(f))(1-\chi_\tau(f))}{1-\chi_\theta(f)\chi_\tau(f)}\right].$$

For rectangular overlapping pulses we obtain:

$$\boxed{S(f) = \frac{a^2 \overline{\nu}}{\pi^2 f^2} \operatorname{Re}\left[\frac{1-\chi_\theta(f)}{1-\chi_\tau(f)} \cdot \{1-\chi_{-\theta}(f)\chi_\tau(f)\}\right].}$$

Lorentzian spectral density



Homogeneous material:

$$\tau_i \sim \mathrm{Exp}\left(\gamma_\tau\right).$$

Identical excitation centers:

$$\theta_i \sim \mathrm{Exp}\left(\gamma_\theta\right).$$

Power spectral density:

$$S(f) = \frac{4a^2\bar{\nu}}{\gamma_\theta^2 + 4\pi^2 f^2}.$$

Obtaining $1/f$ noise

Homogeneous material:

$$\tau_i \sim \mathrm{Exp}\left(\gamma_\tau\right).$$

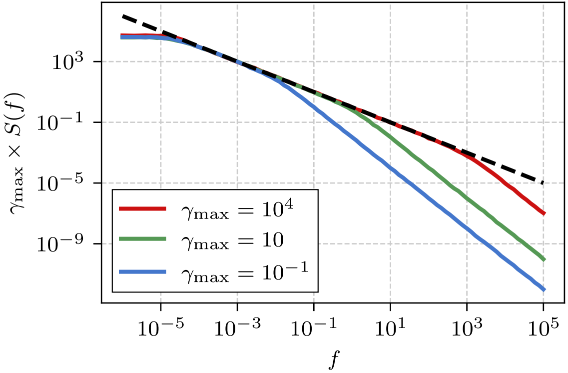
Unique excitation centers:

$$\gamma_i \sim \mathcal{U}\left(\gamma_{\mathrm{min}}, \gamma_{\mathrm{max}}\right),$$

$$\theta_i \sim \mathrm{Exp}\left(\gamma_i\right).$$

Power spectral density:

$$S(f) \approx \frac{a^2 \bar{\nu}}{\gamma_{\mathrm{max}}} \cdot \frac{1}{f}.$$



$1/f$ noise is obtained for long and short pulses (here $\gamma_\tau = 1$).

This talk was based on

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$1/f$ noise from the sequence of nonoverlapping rectangular pulses

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We analyze the power spectral density of a signal composed of nonoverlapping rectangular pulses. First, we derive a general formula for the power spectral density of a signal constructed from the sequence of nonoverlapping pulses. Then we perform a detailed analysis of the rectangular pulse case. We show that pure $1/f$ noise can be observed until extremely low frequencies when the characteristic pulse (or gap) duration is long in comparison to the characteristic gap (or pulse) duration, and gap (or pulse) durations are power-law distributed. The obtained results hold for the ergodic and weakly nonergodic processes.

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I. INTRODUCTION

Flicker noise, also $1/f$ noise or pink noise, is a phenomenon well-known for almost a century since it was first observed by Johnson in a vacuum tube experiment [1,2]. Since then power-law scaling in the power spectral density of $1/f^{\beta}$ form (with $0.5 \lesssim \beta \lesssim 1.5$) has been reported in different experiments and empirical data sets across varied fields of research [3–7], especially in solids [8–10]. One of the peculiarities of $1/f$ noise is that it is observed for low frequencies and no cutoff frequency has been observed in many cases, e.g., 300 years' worth of weather data [11] or a three-week experiment with semiconductors [12], no cutoff frequency has been observed [13]. In other cases, the cutoff frequency can be observed [14–16], but $1/f$ noise is still observed over a broad range of frequencies.

Given observations in various research fields, one would expect that a general explanation of $1/f$ noise is due. However, even almost a century after discovery, there is no generally accepted model of $1/f$ noise. There are numer-

stochastic processes can lead to spurious long-range memory processes [29–32]. These are completely different approaches as the true long-range memory models rely on nonlocal operators, while the models exhibiting spurious long-range memory rely on locally nonlinear potentials, which often result in nonergodic or nonstationary behavior.

Here we will consider a different model, one which is not affected by the nonlinear transformations of amplitude and thus reproduces $1/f$ noise not due to fluctuations in amplitude but due to temporal dynamics. The approach we take here is most similar to renewal theory models [33] and random telegraph noise models, as we model a system which abruptly switches between two states ("on" and "off"). Thus the signal generated has the characteristic look of a telegraph signal or pulse sequence [34]. In Ref. [35], Halford suggested that $1/f$ noise could be modeled by a sequence of well-behaved perturbations with power-law distributed durations. Heiden [36] considered a sequence of pulses, with the coupling between pulse amplitude, duration, and the gap duration, and

$1/f$ noise from the trapping–detrapping process of individual charge carriers

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Abstract

We consider a signal generated by a single charge carrier drifting through the homogeneous condensed matter. We assume that the trapping centers are distributed uniformly across the material so that the trapping process is a homogeneous Poisson process. We assume that the detrapping rate of an individual trapping center is random and uniformly distributed. We show that under these assumptions, and if the trapping rate is low in comparison to the maximum detrapping rate, $1/f$ noise in the form of Hooge's relation is obtained. Hooge's parameter is shown to be a ratio between the characteristic trapping rate and the maximum detrapping rate.

1 Introduction

Many materials, devices, and systems exhibit different kinds of fluctuations or noise [1–3]. Most widely known and well understood are the white noise and the Brownian noise. White noise is characterized by absence of temporal correlations, and flat power spectral density of $S(f) \sim 1/f^0$ form. Examples of the white noise include thermal and shot noise. Thermal noise is known to arise from the random motion of the charge carriers. It occurs at any finite temperature regardless of whether the current flows. Shot noise, on the other hand, is a result of the discrete nature of the charge carriers and the Poisson statistics of waiting times before each individual detection of the charge carrier. The Brownian noise is a temporal integral of the white noise, and thus exhibits no correlations between the increments of the signal, it is characterized by a power spectral density of $S(f) \sim 1/f^2$ form.

The nature of the $1/f$ noise (also referred to as flicker noise or pink noise), characterized by power spectral density of $S(f) \sim 1/f$ form, remains open to discussion despite almost 100 years since the first reports [4,5]. This kind of noise is of particular interest as it is observed across various physical [1,2,6–20], and non–physical [21–27] systems. As far as the $1/f$ noise cannot be obtained by the simple procedure of integration, differentiation, or simple transformation of some common signals, and the general mechanism generating such signals has not yet

Thank you!

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