

Poll-delayed imitation in the noisy voter model

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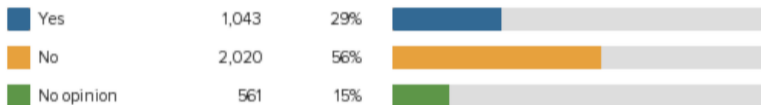


Do public polls help us discover truth?

Should schools in America teach Arabic Numerals as part of their curriculum?

> All respondents in my account

> Weighted according to U.S. Census figures for gender and age, 18 and older

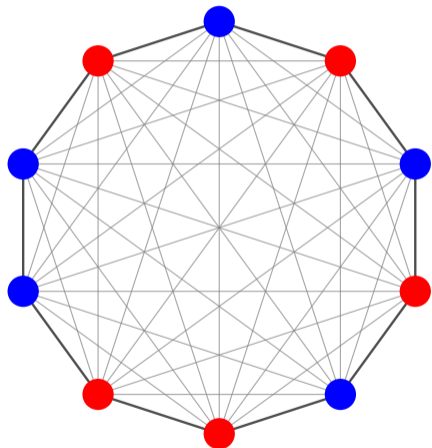


Margin +/- 3% 3,624 responses from 05/07/2019 to 05/11/2019

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Source: twitter.com

In (voter) models (some) things are simpler



We can assume

- N identical agents
- binary opinions
- peer-pressure
- arbitrary social network

Then

- truthful polls do help.

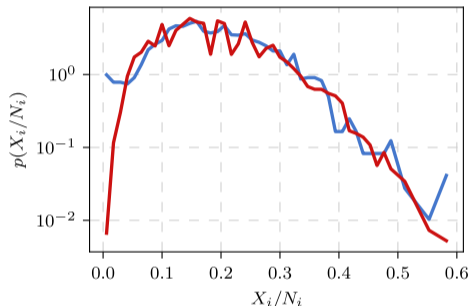
“Non-extensive” noisy voter model

- X agents in state “1”
- ε independent transition rate
- unit peer-induced transition rate

$$\lambda(X \rightarrow X+1) = \lambda^+ = (N-X) \cdot [\varepsilon + X],$$
$$\lambda(X \rightarrow X-1) = \lambda^- = X \cdot [\varepsilon + (N-X)].$$

Converges to distribution:

$$X \sim \text{BetaBin}(\varepsilon, \varepsilon, N).$$



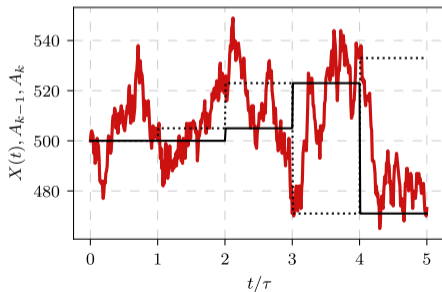
Simulated PDF (red curve) against empirical data (blue curve).

Replicating election data: [Kononovicius (Complexity, 2017)], [Braha & de Aguiar (Plos One, 2017)]

Introducing poll-delayed imitation

Noisy voter model:

$$\begin{aligned}\lambda^+(t) &= (N - X(t)) \cdot [\varepsilon + X(t)], \\ \lambda^-(t) &= X(t) \cdot [\varepsilon + (N - X(t))].\end{aligned}$$



Poll-delayed NVM:

$$\begin{aligned}\lambda^+(t) &= (N - X(t)) \cdot \left[\varepsilon + X \left(\left\lfloor \frac{t}{\tau} - 1 \right\rfloor \tau \right) \right], \\ \lambda^-(t) &= X(t) \cdot \left[\varepsilon + \left(N - X \left(\left\lfloor \frac{t}{\tau} - 1 \right\rfloor \tau \right) \right) \right].\end{aligned}$$

With $A_k = X(k\tau)$, rates:

$$\begin{aligned}\lambda^+(t) &= (N - X(t)) \cdot [\varepsilon + A_{k-1}], \\ \lambda^-(t) &= X(t) \cdot [\varepsilon + (N - A_{k-1})].\end{aligned}$$

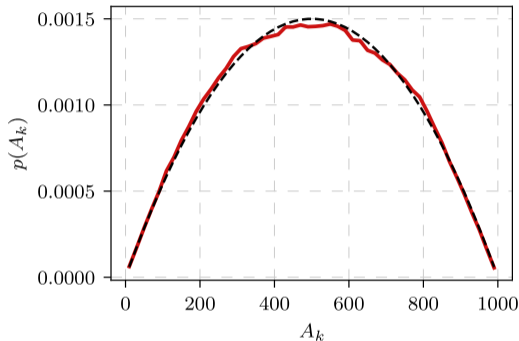
Small τ limit

$\tau \ll N^{-2}$ limit is boring, because

- A_k updates are frequent ($= \tau$),
- X updates are rare ($\gtrsim \tau$).

Effectively:

$$A_{k-1} = X \left(\left\lfloor \frac{t}{\tau} - 1 \right\rfloor \tau \right) \approx X(t).$$



Simulated PDF (red curve) and BetaBin($\varepsilon, \varepsilon, N$) (black dashed curve). Parameters: $\varepsilon = 2$, $\tau = 10^{-7}$, $N = 10^3$.

Macroscopic simulation method

Observe that for $k\tau \leq t < (k+1)\tau$:

$$\varepsilon_+^{(k)} = \varepsilon + A_{k-1} = \text{const},$$

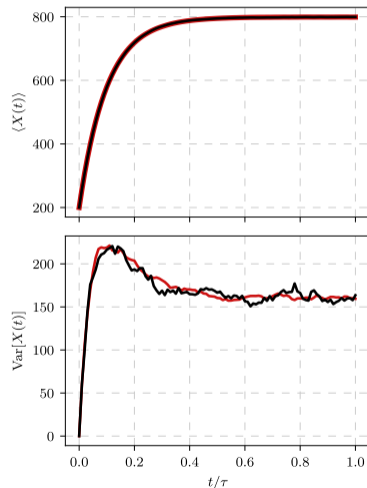
$$\varepsilon_-^{(k)} = \varepsilon + (N - A_{k-1}) = \text{const}.$$

Thus:

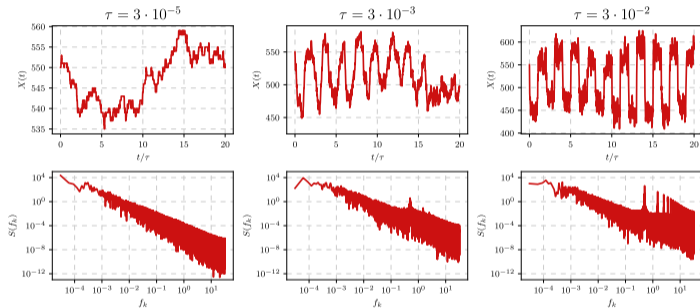
$$\lambda_+(t) = [N - X(t)] \cdot \varepsilon_+^{(k)},$$

$$\lambda_-(t) = X(t) \cdot \varepsilon_-^{(k)}.$$

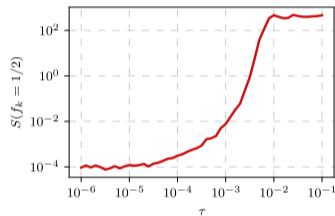
(\Rightarrow) Ensemble mean and variance within a single period: macro (red) and Gillespie (black). Relevant parameters: $\tau = 3 \cdot 10^{-2}$, $A_{-1} = 800$, $A_0 = X(0) = 200$.



A peek at time series



Sample time series (top) and spectra (bottom).



Power at $f_k = \frac{1}{2}$.

Large τ limit

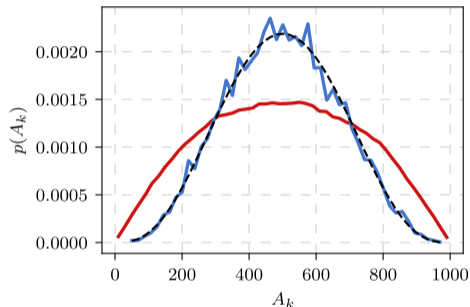
In general,

$$p_{i+1}(x) = \sum_{v=0}^N \sum_{u=0}^N p_{\mathcal{T}}(x|v, u) p_i(v) p_{i-1}(u).$$

For $\tau \gg \frac{1}{N+2\varepsilon}$, we have convergence in a single period. Then

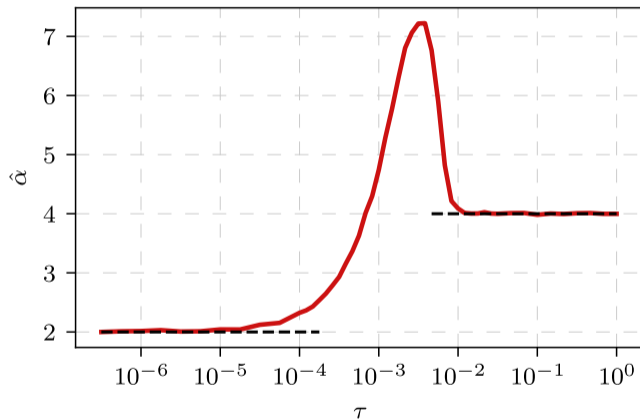
$$p_{i+1}(x) = \sum_{u=0}^N p_{\mathcal{T}}(x|u) p_{i-1}(u)$$

and can be used to derive moments.

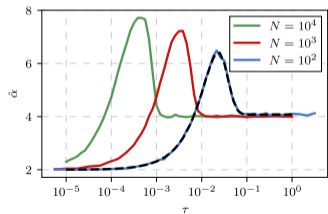
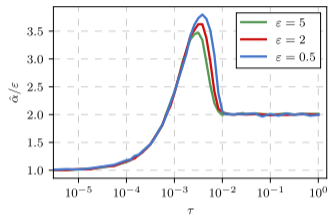


(red) $\tau = 10^{-7}$, (blue) $\tau = 3 \cdot 10^{-2}$,
(dashed) BetaBin ($2\varepsilon, 2\varepsilon, N$)

Intermediate τ



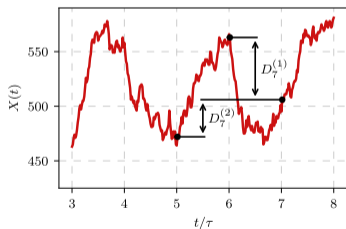
Simulated $\hat{\alpha}$ of BetaBin($\hat{\alpha}$, $\hat{\alpha}$, N).



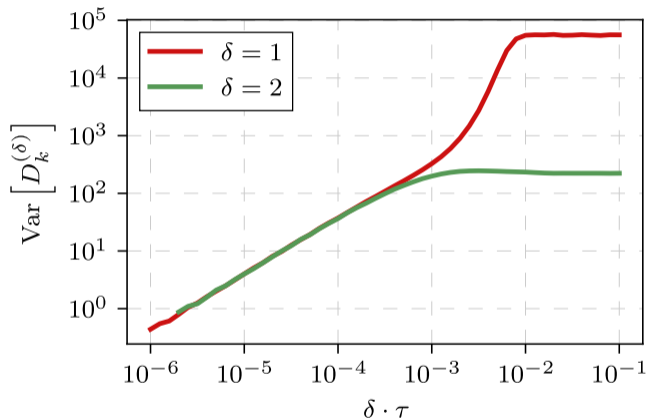
Reason for anomalous damping behavior?

Examine swings between δ -consecutive polls:

$$D_k^{(\delta)} = A_k - A_{k-\delta}.$$



Sample series and δ -swings.



Variance of δ -swing distribution.

Thank you!

Delayed interactions in the noisy voter model through the periodic polling mechanism

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Abstract

Effects of delayed interactions on the stationary distribution of the noisy voter model. Delayed interactions occur through the periodic polling mechanism and replace the nearest-agent interactions. In our analysis, we require that the polling period aligns with the polling outcomes. As expected, when the polling period is relatively short, the stationary distribution is effectively identical to the original model. As the polling period increases, a new phase emerges, but the model with delayed interactions still converges to stationary distribution. The observed scaling behavior is non-trivial. As the polling period increases, the distribution intensifies, yet there is a critical intermediate polling period for which fluctuation intensity.

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Power-law distribution from superposition of normal distributions

March 12, 2024 Aleksejus Kononovicus #interactive-models #statistics

Last time we have seen that we can recover power-law distribution from a superposition of exponential distributions. This idea served a basis for our [1] paper. When presenting some of these results at a conference I was asked question if exponential distribution is necessary, can't one use normal distribution instead?

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