Poll-delayed imitation in the noisy voter model

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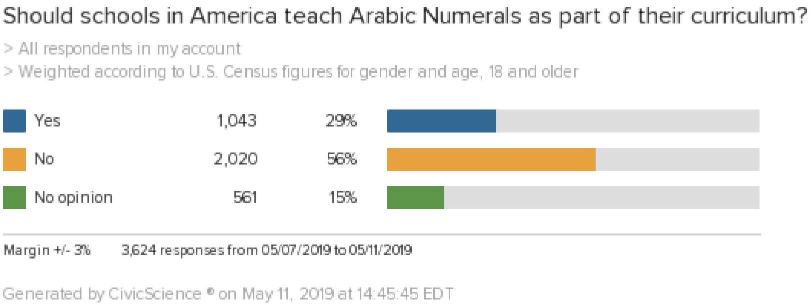
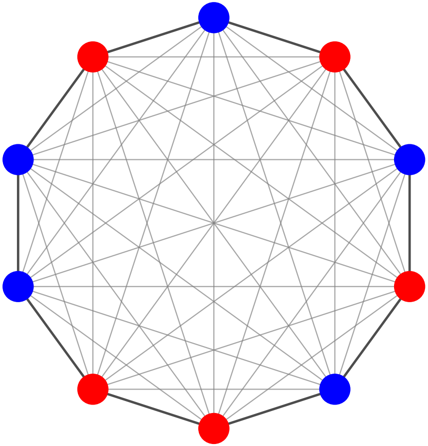
Do public polls help us discover truth?



Source: twitter.com

In (voter) models (some) things are simpler



We can assume

- N identical agents
- binary opinions
- peer-pressure
- arbitrary social network

Then

- truthful polls do help.

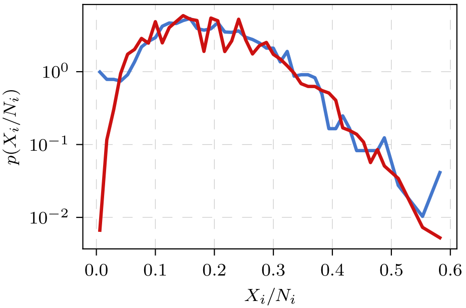
“Non-extensive” noisy voter model

- X agents in state “1”
- ε independent transition rate
- unit peer-induced transition rate

$$\lambda\left(X \to X+1\right) = \lambda^{+} = \left(N - X\right) \cdot \left[\varepsilon + X\right],$$
$$\lambda\left(X \to X-1\right) = \lambda^{-} = X \cdot \left[\varepsilon + \left(N - X\right)\right].$$

Converges to distribution:

$$X \sim \text{BetaBin}\left(\varepsilon, \varepsilon, N\right).$$



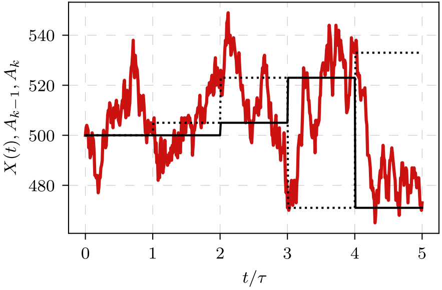
Simulated PDF (red curve) against empirical data (blue curve).

Replicating election data: [Kononovicius (Complexity, 2017)], [Braha & de Aguiar (Plos One, 2017)]

Introducing poll-delayed imitation

Noisy voter model:

$$\lambda^+(t) = (N - X(t)) \cdot [\varepsilon + X(t)]\,,$$
$$\lambda^-(t) = X(t) \cdot [\varepsilon + (N - X(t))]\,.$$



Poll-delayed NVM:

$$\lambda^+(t) = (N - X(t)) \cdot \left[\varepsilon + X\left(\left\lfloor \frac{t}{\tau} - 1 \right\rfloor \tau\right)\right],$$
$$\lambda^-(t) = X(t) \cdot \left[\varepsilon + \left(N - X\left(\left\lfloor \frac{t}{\tau} - 1 \right\rfloor \tau\right)\right)\right].$$

With $A_k = X(k\tau)$, rates:

$$\lambda^+(t) = (N - X(t)) \cdot [\varepsilon + A_{k-1}]\,,$$
$$\lambda^-(t) = X(t) \cdot [\varepsilon + (N - A_{k-1})]\,.$$

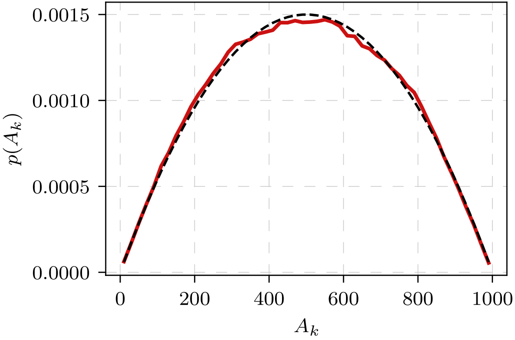
Small τ limit

$\tau \ll N^{-2}$ limit is boring, because

- A_k updates are frequent ($= \tau$),
- X updates are rare ($\gtrsim \tau$).

Effectively:

$$A_{k-1} = X\left(\left\lfloor \frac{t}{\tau} - 1 \right\rfloor \tau\right) \approx X(t).$$



Simulated PDF (red curve) and BetaBin $(\varepsilon, \varepsilon, N)$ (black dashed curve). Parameters: $\varepsilon = 2$, $\tau = 10^{-7}$, $N = 10^3$.

Macroscopic simulation method

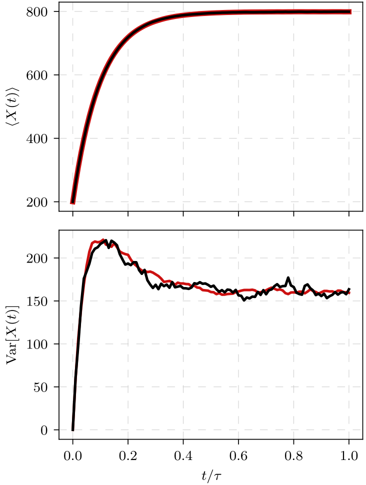
Observe that for $k\tau \le t < (k+1)\,\tau$:

$$\varepsilon_{+}^{(k)} = \varepsilon + A_{k-1} = \text{const},$$
$$\varepsilon_{-}^{(k)} = \varepsilon + (N - A_{k-1}) = \text{const}.$$

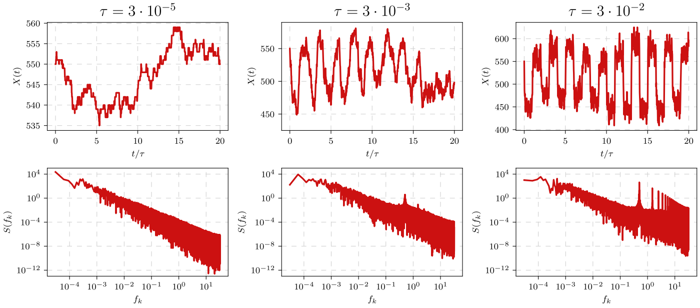
Thus:

$$\lambda_{+}(t) = [N - X(t)] \cdot \varepsilon_{+}^{(k)},$$
$$\lambda_{-}(t) = X(t) \cdot \varepsilon_{-}^{(k)}.$$

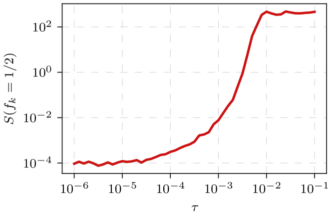
(⟹) Ensemble mean and variance within a single period: macro (red) and Gillespie (black). Relevant parameters: $\tau = 3 \cdot 10^{-2}$, $A_{-1} = 800$, $A_0 = X(0) = 200$.



A peek at time series



Sample time series (top) and spectra (bottom).



Power at $f_k = \frac{1}{2}$.

Large τ limit

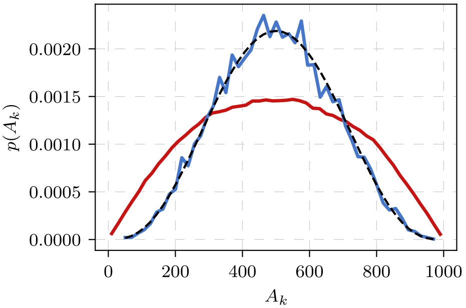
In general,

$$p_{i+1}(x) = \sum_{v=0}^{N} \sum_{u=0}^{N} p_{\mathcal{T}}(x|v,u)\, p_i(v) p_{i-1}(u).$$

For $\tau \gg \frac{1}{N+2\varepsilon}$, we have convergence in a single period. Then

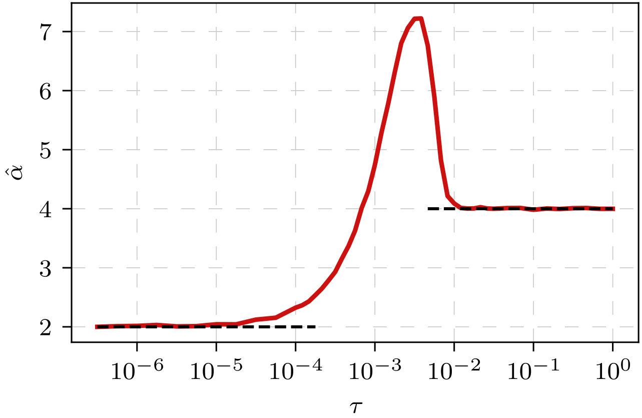
$$p_{i+1}(x) = \sum_{u=0}^{N} p_{\mathcal{T}}(x|u)\, p_{i-1}(u)$$

and can be used to derive moments.

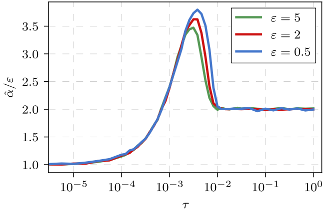


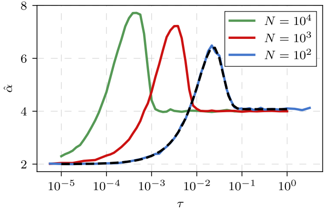
(red) $\tau = 10^{-7}$, (blue) $\tau = 3 \cdot 10^{-2}$, (dashed) BetaBin $(2\varepsilon, 2\varepsilon, N)$

Intermediate τ



Simulated $\hat{\alpha}$ of BetaBin$(\hat{\alpha}, \hat{\alpha}, N)$.

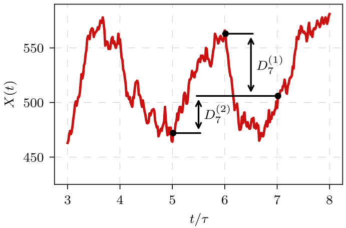




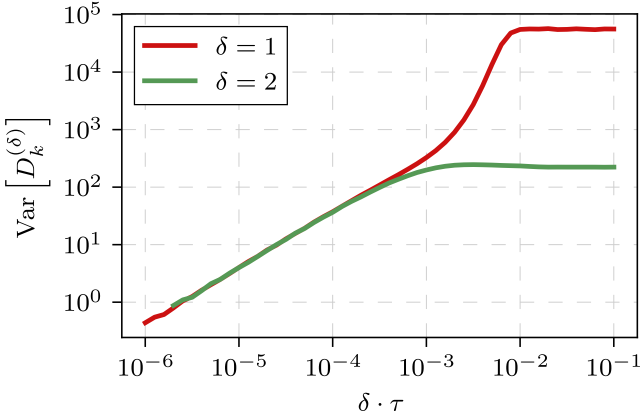
Reason for anomalous damping behavior?

Examine swings between δ-consecutive polls:

$$D_k^{(\delta)} = A_k - A_{k-\delta}.$$

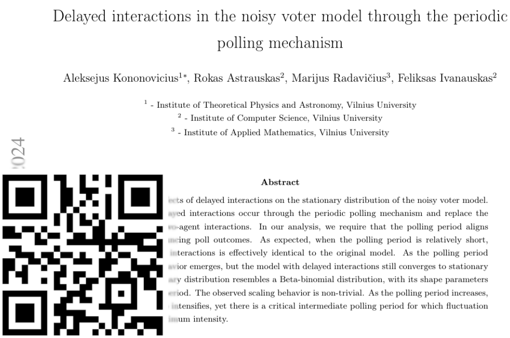


Sample series and δ-swings.



Variance of δ-swing distribution.

Thank you!



Delayed interactions in the noisy voter model through the periodic polling mechanism

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Abstract

...ts of delayed interactions on the stationary distribution of the noisy voter model. ...ed interactions occur through the periodic polling mechanism and replace the ...-agent interactions. In our analysis, we require that the polling period aligns ...ing poll outcomes. As expected, when the polling period is relatively short, ...teractions is effectively identical to the original model. As the polling period ...ior emerges, but the model with delayed interactions still converges to stationary ...y distribution resembles a Beta-binomial distribution, with its shape parameters ...iod. The observed scaling behavior is non-trivial. As the polling period increases, ...tensifies, yet there is a critical intermediate polling period for which fluctuation ...um intensity.