Compartmental voter model

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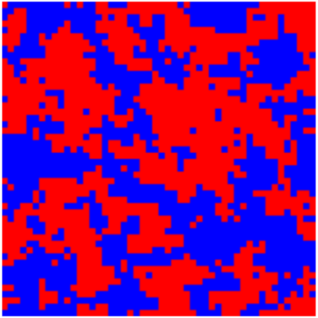
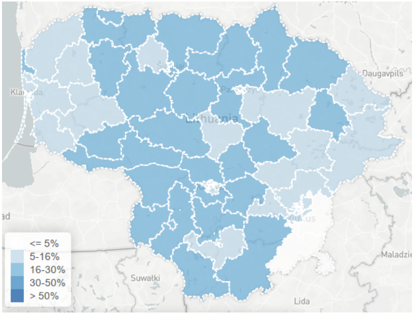
The core idea





- The models: single compartment, multiple points in time.
- The data: multiple compartments, single point in time.

Image sources: (left) http://rf.mokslasplius.lt/voter-model/, (right) http://rinkimurezultatai.lt

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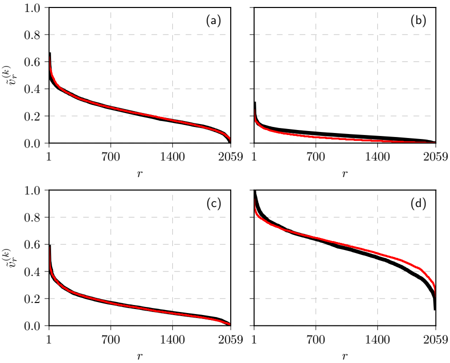
Simplest solution: independent compartments



Rank–size distribution of the vote shares for 4 main parties in the Lithuanian Seimas 1992 elections: model (red) vs data (black).

RSDs are used for observations over compartments, PDFs are used for observations over time.

Figure: Kononovicius, Complexity 2017: 7354642 (2017)

Other: (Sano *et al.*, 2017), (Braha & de Aguiar, 2017), (Fenner *et al.*, 2017)

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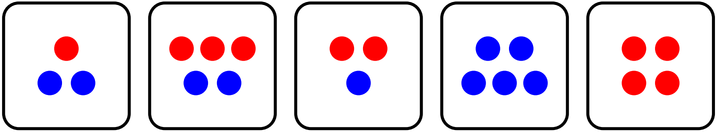
Static setup

- Consider N agents of T types.
- Let agent types be fixed.
- Let agents move between M compartments of capacity C.

N=20, T=2, M=5, C=5



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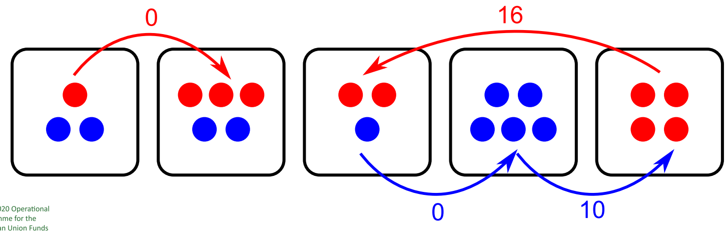
Movement dynamics

Consider movement rates (from i to j for type k):

$$\lambda_{(k)}^{i\to j} = \begin{cases} X_i^{(k)} \left(\varepsilon^{(k)} + X_j^{(k)}\right) & \text{if } i \neq j \text{ and } N_j < C, \\ 0 & \text{otherwise,} \end{cases}$$

here $X_i^{(k)}$ is the number of agents of type k in compartment i, while N_j is the total number of agents in compartment j.

N=20, T=2, M=5, C=5, ε=2



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Infinite capacity ($C = N$)

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Stationary distribution

Given $C = N$ one can write closed form expression for the total entry and exit rates:

$$\lambda_{(k)}^{i+} = \sum_{j=1}^{M} \lambda_{(k)}^{j \to i} = \left[N^{(k)} - X_i^{(k)} \right] \left(\varepsilon^{(k)} + X_i^{(k)} \right),$$

$$\lambda_{(k)}^{i-} = \sum_{j=1}^{M} \lambda_{(k)}^{i \to j} = X_i^{(k)} \left([M-1] \, \varepsilon^{(k)} + \left[N^{(k)} - X_i^{(k)} \right] \right).$$

Which means that $X_i^{(k)}$ is distributed according to Beta-binomial distribution in this particular case.

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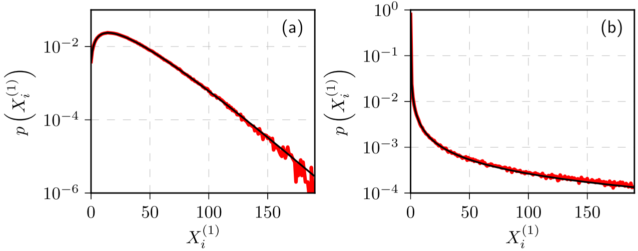
Numerical verification of stationary distribution



Model (red curves): $N = 3000$, $T = 1$, $M = 100$ and $C = N$ (all cases), $\varepsilon^{(1)} = 2$ (a) and 0.03 (b). Beta-binomial fit (black curves): $N = 3000$, $\alpha = \varepsilon^{(1)}$ and $\beta = (M-1)\,\varepsilon^{(1)}$ (all cases).

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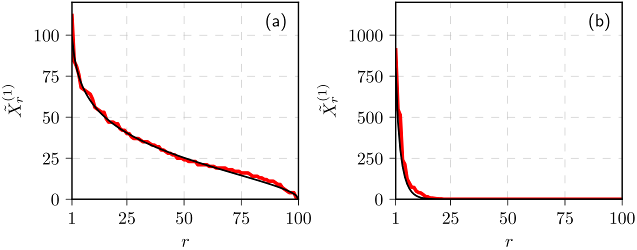
Numerical inquiry into rank–size distribution



Same simulation as in the previous slide: $N = 3000$, $T = 1$, $M = 100$ and $C = N$ (all cases), $\varepsilon^{(1)} = 2$ (a) and 0.03 (b).

Beta–binomial distribution provides a rather good fit as if compartments would be truly independent.

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Finite capacity $C < N$

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Obtaining stationary distribution

General idea

- In general we don't know close forms of the total entry and exit rates.
- Notice that the compartmental model is actually a multivariate finite–state Markov chain.
- In theory **we can reduce any finite–state Markov chain to one–dimensional Markov chain** by relabeling states.

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Obtaining stationary distribution II

Technical implementation

- Given $N = 10$, $T = 2$, $M = 2$, $C = 6$ and ε.
- The state vector is $\left\{ X_1^{(1)}, X_1^{(2)} \right\}$.
- List all possible states: $\{0, 4\}, \ldots, \{5, 1\}$.
- Relabel states: $\{0, 4\} \rightarrow 0, \{0, 5\} \rightarrow 1, \ldots$
- Write down the transition matrix and find its eigensystem.
- Aggregate over eligible states to get the desired result: $P(X_1^{(1)} = 0) = P(\{0, 4\}) + P(\{0, 5\}) + P(\{0, 6\})$.

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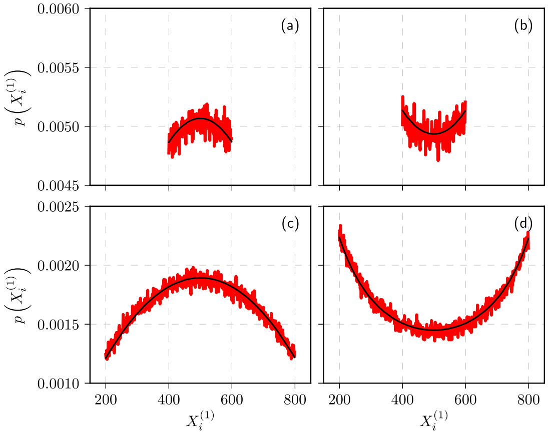
Univariate MC: $T = 1$, $M = 2$



Model (red curves): $N = 1000$ (all cases), $C = 600$ ((a) and (b)) and 800 ((c) and (d)), $\varepsilon^{(1)} = 2$ ((a) and (c)) and 0.03 ((b) and (d)). Truncated Beta-binomial distribution (black curves): $N = 1000$, $\alpha = \varepsilon$, $\beta = (M-1)\,\varepsilon$ (all cases).

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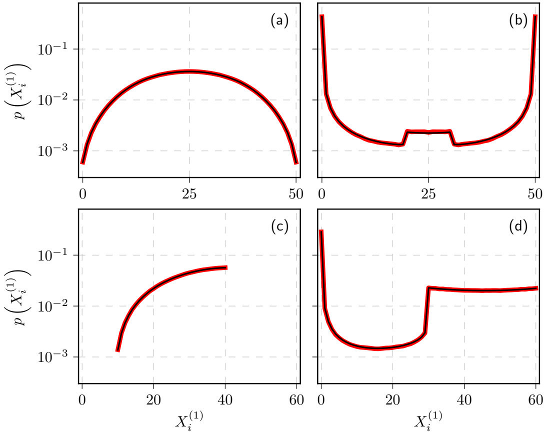
Simplest multivariate MCs



Model (red curves): $N = 100$, $M = 2$ and $T = 2$ ((a) and (b)), $N = 90$, $M = 3$ and $T = 1$ ((c) and (d)), $C = 40$ (c), 60 ((a) and (d)) and 80 (b), $\varepsilon = 2$ ((a) and (c)) and 0.03 ((b) and (d)). Fits (black curves) were obtained from the eigensystem.

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Empirical examples

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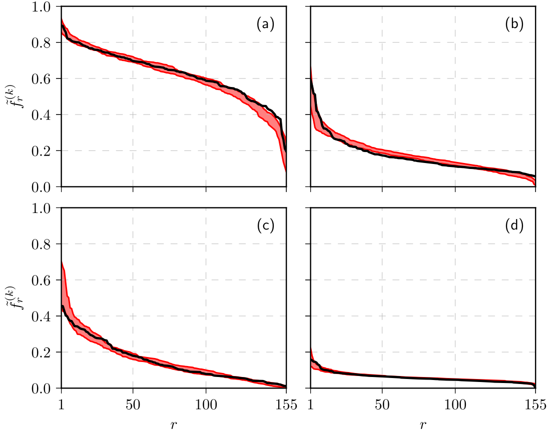
Ethnic groups in London (UK census 2011)



Considered groups (black curves): (a) White, (b) Asian, (c) Black, (d) other. Model (red areas): $N^{(w)} = 48515$, $N^{(a)} = 12865$, $N^{(b)} = 11470$ and $N^{(o)} = 4495$ ($N = 77345$), $\varepsilon^{(w)} = 2.5$, $\varepsilon^{(a)} = 4$, $\varepsilon^{(b)} = 1.5$, $\varepsilon^{(o)} = 15$, $M = 155$, $C = 600$.

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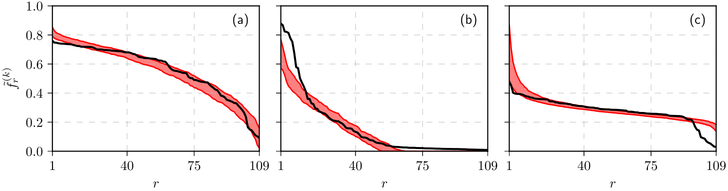
Religious groups in Leicester (UK census 2011)



Considered groups (black curves): (a) Christians, (b) no religion, (c) other. Model (red areas): $N^{(c)} = 30411$, $N^{(n)} = 8829$ and $N^{(o)} = 15151$ ($N = 54391$), $\varepsilon^{(c)} = 2.5$, $\varepsilon^{(n)} = 0.01$, $\varepsilon^{(o)} = 50$, $M = 109$, $C = 600$.

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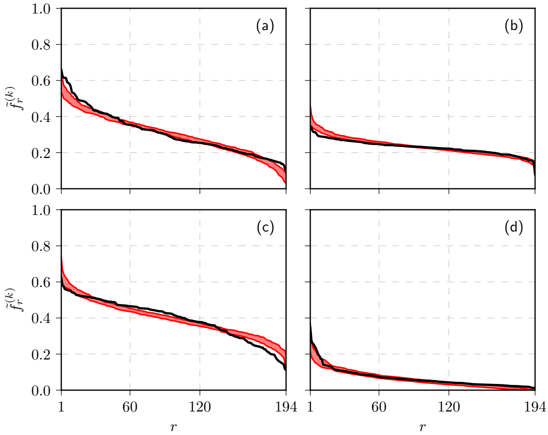
Working class in Sheffield (UK census 2011)



Considered groups (black curves): (a) higher, (b) intermediate and (c) lower occupations, (d) unemployed. Model (red areas): $N^{(1)} = 29876$, $N^{(2)} = 22310$, $N^{(3)} = 38218$ and $N^{(u)} = 6596$ ($N = 97000$), $\varepsilon^{(1)} = 3$, $\varepsilon^{(2)} = 50$, $\varepsilon^{(3)} = 12$, $\varepsilon^{(u)} = 2$, $M = 194$, $C = 600$.

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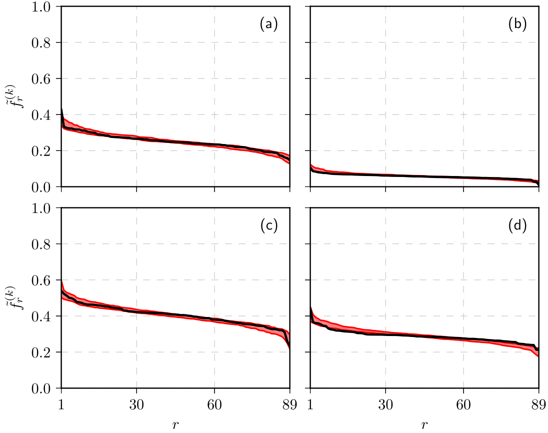
Vote shares in Vilnius (LT Seimas election 1992)



Considered groups (black curves): (a) Sajudzio koalicija, (b) Lietuvos krikscioniu demokartu partija, (c) Lietuvos demokratine darbo partija, (d) other. Model (red areas): $N^{(s)} = 11125$, $N^{(l)} = 2581$, $N^{(d)} = 17978$ and $N^{(o)} = 12816$ ($N = 44500$), $\varepsilon^{(s)} = \varepsilon^{(l)} = \varepsilon^{(d)} = 25$, $\varepsilon^{(o)} = 75$, $M = 89$, $C = 600$.

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This talk was based on a preprint

Compartmental voter model

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Abstract

Numerous models in opinion dynamics focus on the temporal dynamics within a single spatial unit (e.g., country). While the opinions are often observed across multiple spatial units (e.g., polling stations) at a single point in time (e.g., elections). Aggregates of these observations, while quite useful in many applications, neglect the underlying spatial heterogeneity in opinions. To address this issue we build a simple compartmental agent–based model in which all agents have fixed opinions, but are able to change their compartments. We demonstrate that this model is able to generate compartmental rank–size distributions consistent with the empirical data.

1 Introduction

Most well–known models of opinion dynamics seem to imply that a steady state, either consensus or polarization, is inevitable [1–7]. However, local and spatial heterogeneity and ongoing exchange of opinions and cultural traits is a characterizing feature of social systems. Variety modifications of the well–known models were proposed to account for these features, such as inflexibility [8, 9] or spontaneous flipping [10, 11]. Some of the models were modified to account for the theories from the social sciences [12, 13]. Effects of these modifications are still being actively reconsidered in context of network theory, non–linearity, complex contagion and applications towards financial markets [14–24]. Nevertheless even these modified models assume that opinion dynamics occur and

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It is available online as arXiv: 1906.01842 [physics.soc-ph].

Thank you!

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