Order book model with herd behavior and long–range memory

Aleksejus Kononovicius, Julius Ruseckas

Institute of Theoretical Physics and Astronomy, Vilnius University

aleksejus.kononovicius@tfai.vu.lt
http://kononovicius.lt/, http://rf.moksplasplius.lt/

This talk based on an article

Order book model with herd behavior exhibiting long–range memory

Aleksejus Kononovicius, Julius Ruseckas

Institute of Theoretical Physics and Astronomy, Vilnius University

Abstract

In this work, we propose an order book model with herd behavior. The proposed model is built upon two distinct approaches: a recent empirical study of the detailed order book records by Kanazawa *et al.* [Phys. Rev. Lett. 120, 138301] and financial herd behavior model. Combining these approaches allows us to create a more plausible financial market model, which is also capable to replicate the long-range memory phenomenon of the absolute return and the trading activity. We compare the statistical properties of the model against the empirical statistical properties of the Bitcoin exchange rates and New York stock exchange tickers. We also show that the fracture in the spectral density of the high–frequency absolute return time series might be related to the mechanism of convergence towards the equilibrium price.

The article will appear in Physica A (DOI: 10.1016/j.physa.2019.03.059)

2014-2020 Operational Programme for the European Union Funds Investments in Lithuania

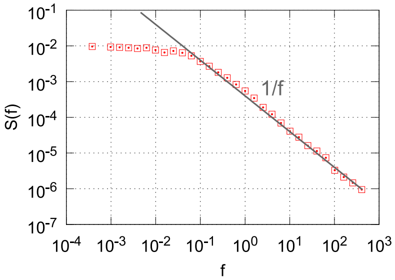
Long–range memory phenomenon

If spectrum of a time series takes power-law form:

$$S(f) \sim 1/f^{\beta}, \quad 0.5 < \beta < 2,$$

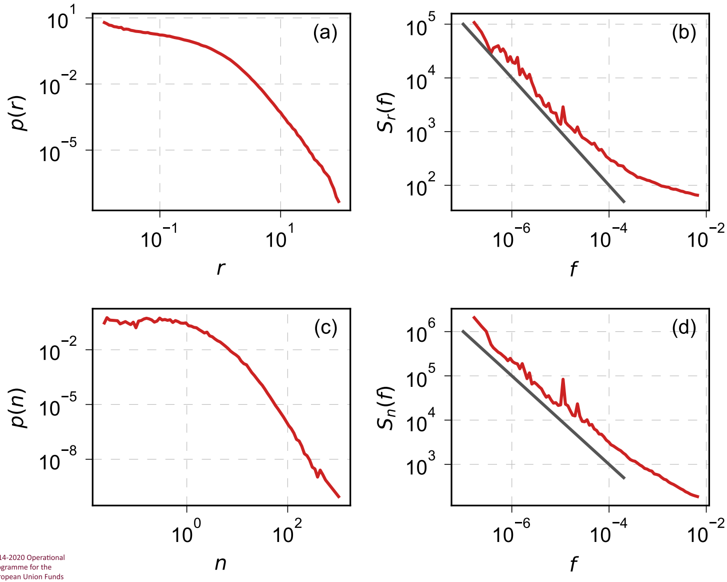
then we say that the time series exhibits long–range memory.

In the ideal case $\beta \approx 1$ as $f \to 0$.



2014-2020 Operational Programme for the European Union Funds Investments in Lithuania

Long–range memory in the Bitcoin time series



2014-2020 Operational Programme for the European Union Funds Investments in Lithuania

Simple stochastic model of long–range memory

It is known that solutions, $y(t)$, of the SDE in Ito sense,

$$\mathrm{d}y = \left(\eta - \frac{\lambda}{2}\right) y^{2\eta-1}\mathrm{d}t + y^{\eta}\mathrm{d}W,$$

with $\eta > 1$, have power–law spectrum:

$$S(f) \sim f^{-\beta}, \quad \beta = 1 + \frac{\lambda - 3}{2(\eta - 1)},$$

and also that the stationary distribution of y is also a power–law (exponent λ).

e.g.: [Ruseckas and Kaulakys, PRE 81: 031105 (2010)]

financial market application: [Gontis *et al.*, Physica A 389: 100–106 (2010)]

2014-2020 Operational Programme for the European Union Funds Investments in Lithuania

Imitative behavior model

- Suppose we have N agents, which choose between two states.
- Let X be the number of agents choosing the first state.
- Let each agent change his opinion either independently (σ_i) or due to imitation (h).

Then system–wide transition rates given by:

$$\lambda(X \rightarrow X+1) = (N-X)(\sigma_1 + hX),$$
$$\lambda(X \rightarrow X-1) = X(\sigma_2 + h[N-X]).$$

It is straightforward to show that $\frac{X}{N} \sim \mathcal{B}e\left(\frac{\sigma_1}{h}, \frac{\sigma_2}{h}\right)$ when $N \rightarrow \infty$.

originally proposed in [Kirman, Q. J. of Economics 108: 137–156 (1993)]

equivalent to Voter model [Clifford and Sudbury, Biometrika 60: 581–588 (1973)]

2014-2020 Operational Programme for the European Union Funds Investments in Lithuania

Imitative behavior and the long–range memory

- Suppose that two states correspond to chartist and fundamentalist trading strategies.
- Suppose that chartists trade based on the mood:

$$D_c = r_0 X_c \xi, \quad \xi = \frac{X_o - X_p}{X_c}.$$

- Suppose that fundamentalists trade based on the fundamental information.

$$D_f = X_f \ln \frac{P_f}{P}.$$

- From Walras equilibrium condition we can define return:

$$D_c + D_f = 0, \quad \Rightarrow \quad r = r_0 \frac{X_c}{X_f} \Delta\xi = r_0 \frac{X}{N - X} \Delta\xi.$$

2014-2020 Operational Programme for the European Union Funds Investments in Lithuania

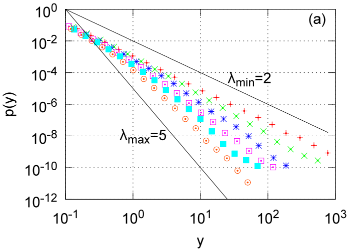
[Kononovicius and Gontis, Physica A 391: 1309–1314 (2012)]

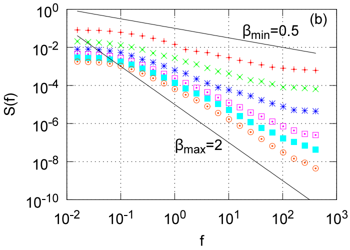
Properties of $y = \frac{X}{N-X}$

Corresponding SDE for $y = \frac{X}{N-X}$ is:

$$\mathrm{d}y = \left(\sigma_1 + \frac{2h - \sigma_2}{\tau(y)} y\right)(1+y)\mathrm{d}t + \sqrt{2h\frac{y}{\tau(y)}}(1+y)\mathrm{d}W,$$

with feedback scenario $\tau(y) = y^{-\alpha}$.

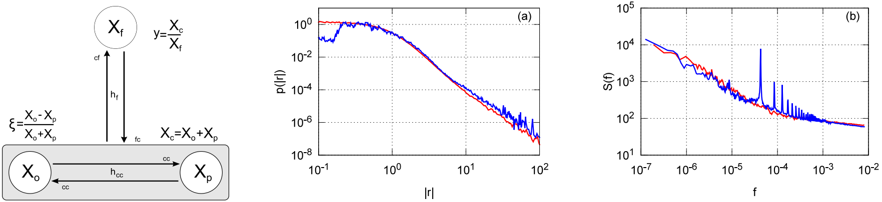




2014-2020 Operational Programme for the European Union Funds Investments in Lithuania

Imitative behavior matching to the empirical data

If we describe ξ dynamics using the same imitative behavior model and include "exogenous" noise, we were able to replicate the empirical data from various stock markets.



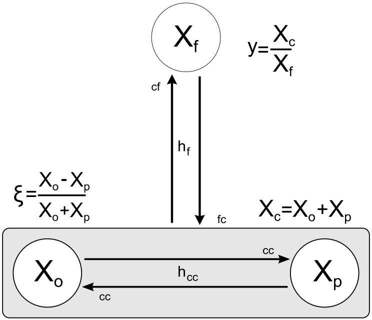
Model with y and ξ dynamics and "exogenous" noise (red curve) against NYSE 1 minute return (blue curve).

[Gontis and Kononovicius, PLoS ONE 9: e102201 (2014)]

What is the nature of the “exogenous” noise?

Similarly as in ARCH family models, we have assumed that y and ξ dynamics drive the unobserved volatility of the market, while some exogenous randomness affects what the observed return:

$$r_t \sim \mathcal{N}(0, \sigma), \quad \sigma = f(y, \xi).$$

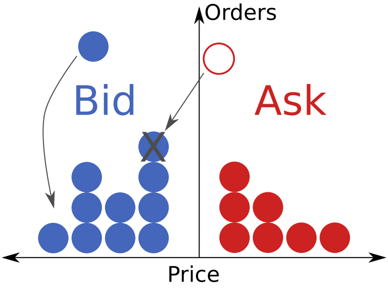


- The nature of the exogenous noise is dubious.
- The model implies certain things about trading activity which do not relate to the observed reality.

2014-2020 Operational Programme for the European Union Funds Investments in Lithuania

Order book dynamics?!

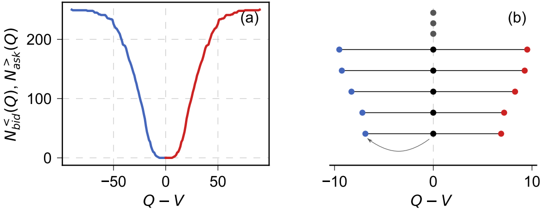
- There is no market marker – prices are "negotiated"
- **Bid** = Buy
- **Ask** = Sell
- **Limit orders** – to buy/sell at specified price (time?)
- **Market order** – to buy/sell now (price?)



2014-2020 Operational Programme for the European Union Funds Investments in Lithuania

Behavior of agents in order–book terms

- Chartists place limit orders to the both sides of the order book at quotes (=desired prices) $Q_i = V \pm S_i$.
- V is updated instantaneously after each transaction.
- Chartists submit market orders based on the current market mood.
- Fundamentalists submit market orders (only) based on the market fundamentals.



2014-2020 Operational Programme for the European Union Funds Investments in Lithuania

[Kanazawa *et al.*, PRL 120: 138301 (2018)]

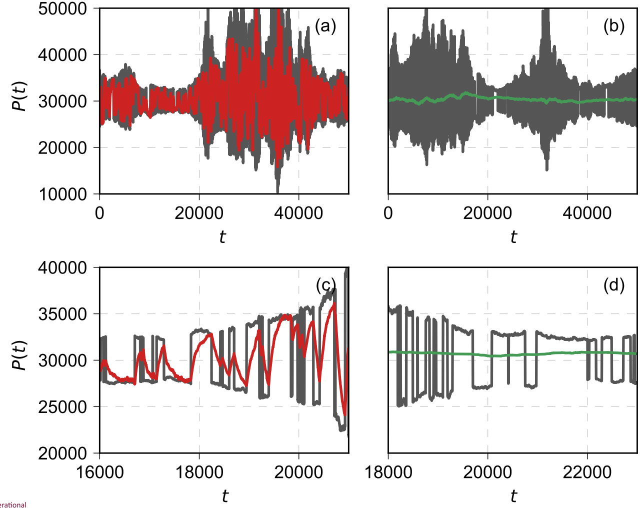
Possible events within our model

and their reasonable time scales

- Chartists adjust their valuation, cancel and deposit limit orders (instantaneous).
- Chartists submit market orders, $p_{bid} = \frac{1+\xi(t)}{2}$ (seconds).
- Fundamentalists submit market orders, bid if $P_f > P(t)$, ask if $P_f < P(t)$ (seconds).
- Chartists flip their mood, $\xi(t + \Delta t) = -\xi(t)$ (tenths of minutes).
- Agents switch between chartist and fundamentalist trading strategies (multiple hours).

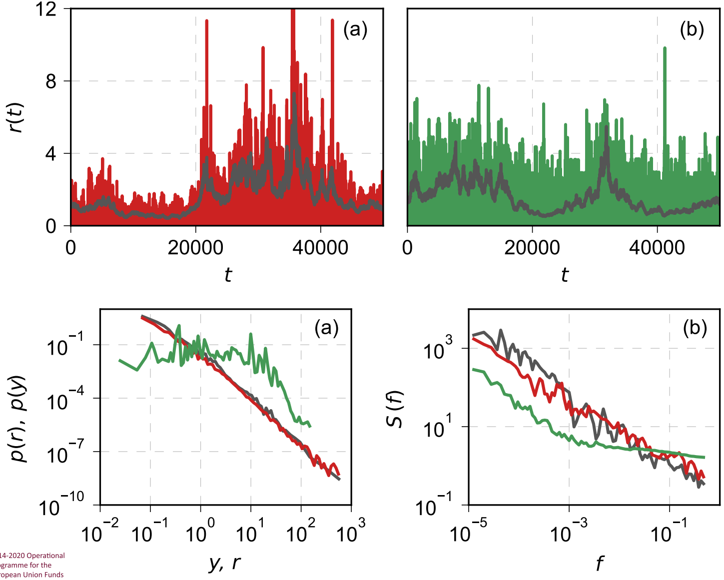
2014-2020 Operational Programme for the European Union Funds Investments in Lithuania

Trade submission must be faster
than switching dynamics



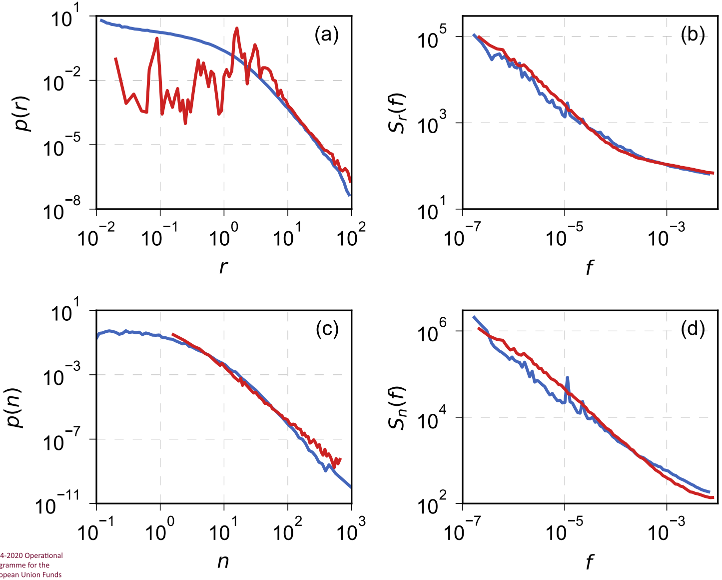
2014-2020 Operational Programme for the European Union Funds Investments in Lithuania

Then y and return are correlated



2014-2020 Operational Programme for the European Union Funds Investments in Lithuania

Matching the Bitcoin time series



Conclusions

- We have combined stochastic (phenomenological), herd behavior (abstract) and order book (realistic) models.
- The new model replicates return and trading activity statistics.
- It touches upon the topic of efficient markets and price convergence. It seems that the markets catch up with "equilibrium" in a trading day or two.
- Opens up a path for Bayesian inference – the model provides a physical meaning for the size of order book and its sides.

Based on [A. Kononovicius, J. Ruseckas, accepted to PhysA, doi: 10.1016/j.physa.2019.03.059]

Similar Bayesian approaches are already in development [T. Lux, J Eco Dyn Con 91: 391–408 (2018)].

2014-2020 Operational Programme for the European Union Funds Investments in Lithuania

Thank you!

email: aleksejus.kononovicius@tfai.vu.lt
www: http://kononovicius.lt/, http://rf.mokslasplius.lt/