Intrinsic and spurious long-range memory in financial markets and ABMs through the lense of first passage times

Aleksejus Kononovicius, Vygintas Gontis

Institute of Theoretical Physics and Astronomy, Vilnius University

aleksejus.kononovicius@tfai.vu.lt
http://kononovicius.lt/, http://rf.mokslasplius.lt/

Long-range memory

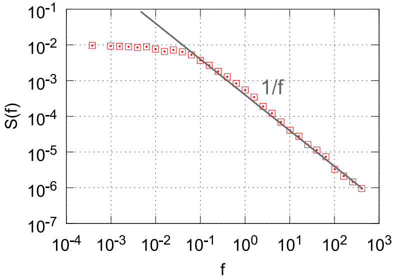
We study long-range memory

via power spectral density. If power spectral density of the time series takes power-law form:

$$S(f) \sim 1/f^{\beta}, \quad 0.5 < \beta < 2,$$

then we say that the time series exhibits long-range memory.

In the ideal case $\beta \approx 1$.



2014-2020 Operational Programme for the European Union Funds Investments in Lithuania

Intrinsic or emergent memory?

Memory is explicitly included

in:

- ARCH family models,
- fractional Brownian motion framework,
- agent-based models featuring intelligent agents with memory.

Memory may emerge from:

- Markovian point processes,
- stochastic differential equations,
- agent-based models featuring zero-intelligence agents.

2014-2020 Operational Programme for the European Union Funds Investments in Lithuania

Fractional Gaussian noise / fractional Brownian motion

Though exact fGn and fBm generation methods vary,

one can obtain spectral density of the power-law form, $S(f) \sim 1/f^{\beta}$, with β determined by the Hurst exponent, H.

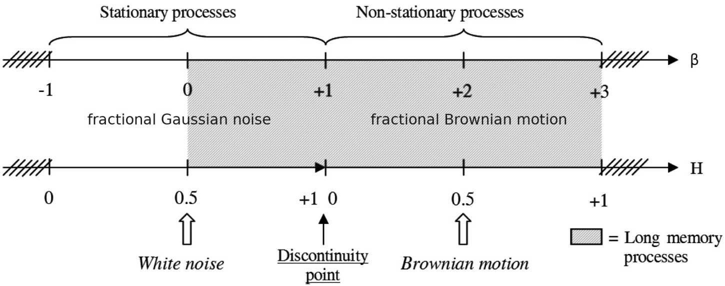


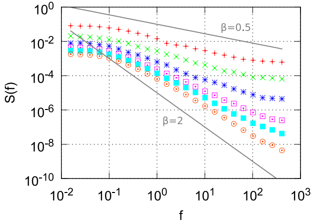
Figure from [Esposti et al., Chaos 18 (2008)]

Simple emergent memory models

Points process

for inter-event times:

$$\tau_{k+1} = \tau_k + \gamma \tau_k^{2\mu-1} + \tau_k^{\mu} \zeta_k$$



Stochastic differential eq.

for intensity:

$$\mathrm{d}x = \left(\eta - \frac{\lambda}{2}\right) x^{2\eta-1}\mathrm{d}t + \sigma x^{\eta}\mathrm{d}W$$

Agent-based model

defined via herding switching rates:

$$\lambda_{c\to f} = \frac{h}{\tau} N_c \left(\varepsilon_{cf} + N_f\right),$$
$$\lambda_{f\to c} = \frac{h}{\tau} N_f \left(\varepsilon_{fc} + N_c\right).$$

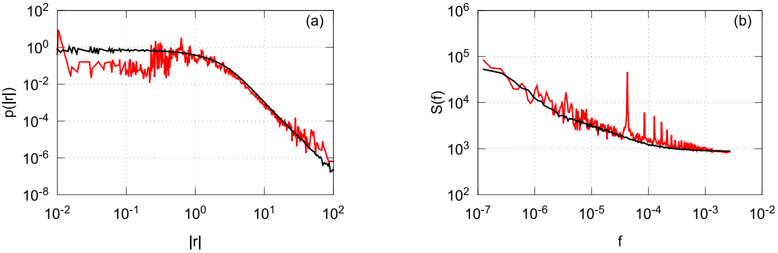
Point process - [Kaulakys and Meškauskas, Phys. Rev. E 58 (1998)]

SDE - [Kaulakys and Ruseckas, Phys. Rev. E 70 (2004)]

ABM - [Kononovicius and Gontis, Physica A 391 (2012)]

2014-2020 Operational Programme for the European Union Funds Investments in Lithuania

Sophisticated emergent memory models
match stylized facts exactly

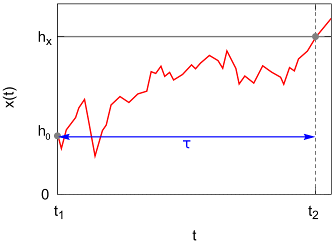


Absolute return PDF and PSD of the ABM (black curve) and empirical data (red curve).

[Gontis and Kononovicius, PLoS ONE 9 (2014)]

2014-2020 Operational Programme for the European Union Funds Investments in Lithuania

First passage time framework



It is well known that for

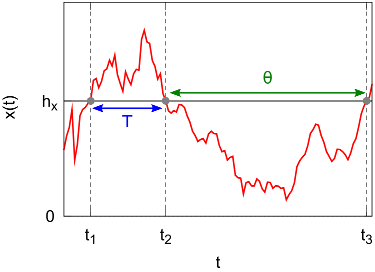
- 1-D Markov processes – $P_{h_0,h_x}(\tau) \sim \tau^{-3/2}$,
- fBm time series – $P_{h_0,h_x}(\tau) \sim \tau^{-(2-H)}$.

where h_0 is the starting point, h_x is the threshold.

2014-2020 Operational Programme for the European Union Funds Investments in Lithuania

[Redner (CUP, 2001)], [Ding and Yang, Phys. Rev. E 52 (1995)]

Bursting statistics



We use related concepts

which we refer to as burst, T, and inter-burst, θ, times.

Key difference from FPT

is that the starting point is in the vicinity of the threshold:

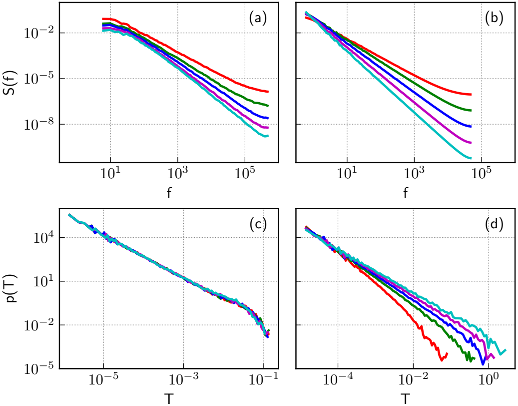
$$h_0 = h_x + \epsilon,$$

here $\epsilon \downarrow 0$ for T, while $\epsilon \uparrow 0$ for θ.

[Gontis et al., ACS 15 (2012)]

2014-2020 Operational Programme for the European Union Funds Investments in Lithuania

Bursting statistics of ABM and fBm time series

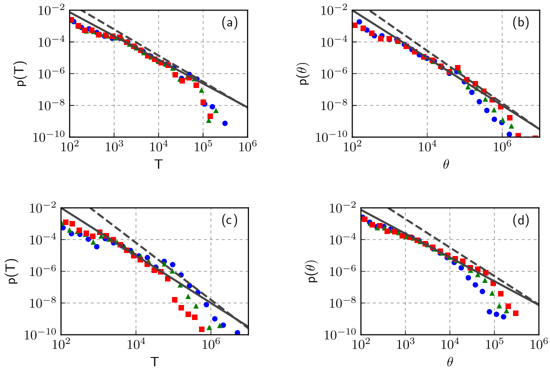


Time series from ABM and fBm, PSDs of which are similar, compare (a) and (b). As we can see their burst duration PDFs differ, compare (c) and (d).

2014-2020 Operational Programme for the European Union Funds Investments in Lithuania

[Gontis and Kononovicius, Entropy 19 (2017)]

Bursting statistics of the Forex time series



One minute trading activity (top) and volatility (bottom) burst (left) and inter-burst (right) PDFs. Symbols represent empirical data (EUR/USD) for different thresholds. Solid line represents 3/2 law, while dashed line represents what would be expected from $2 - H$ law.

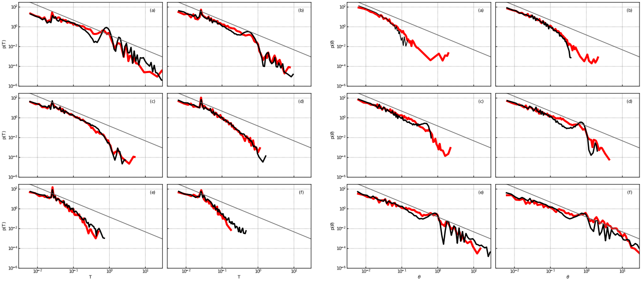
2014-2020 Operational Programme for the European Union Funds Investments in Lithuania

[Gontis and Kononovicius, Physica A 483 (2017)]

Markovian model compared to empirical data

For the extreme thresholds

3/2 law disappears, yet this disappearance is well accounted by the sophisticated ABM.



Sophisticated ABM (black) and empirical (red) burst (left) and inter-burst (right) duration PDFs in volatility time series.

2014-2020 Operational Programme for the European Union Funds Investments in Lithuania

[Gontis and Kononovicius, arXiv:1712.05121 [q-fin.ST]]

Thank You!

aleksejus.kononovicius@tfai.vu.lt
http://kononovicius.lt/, http://rf.mokslasplius.lt/