Modeling of Lithuanian parliamentary elections using ABM

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Collaborators

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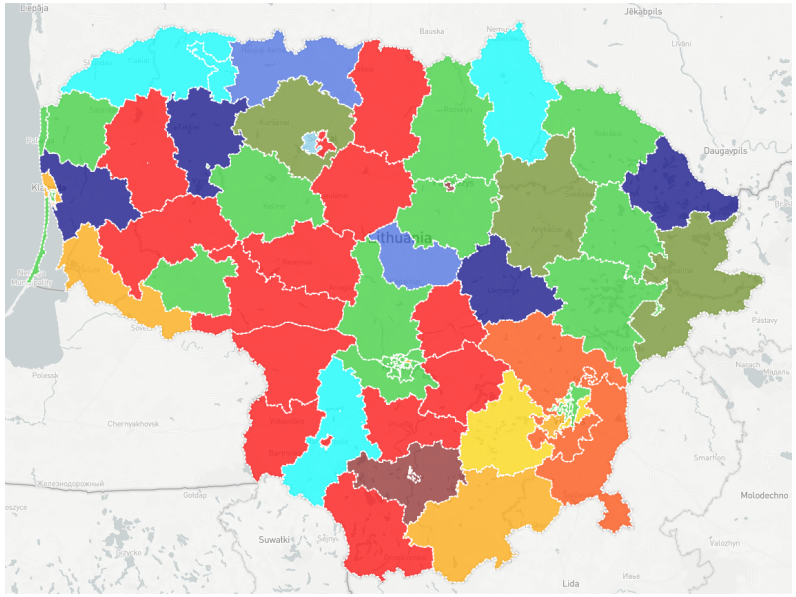
Also a big thanks to my usual colleagues Julius Ruseckas and Vygintas Gontis (ITPA, VU) for their interest in this venture.

Empirical analysis

The Lithuanian parliamentary election system

Basic facts:

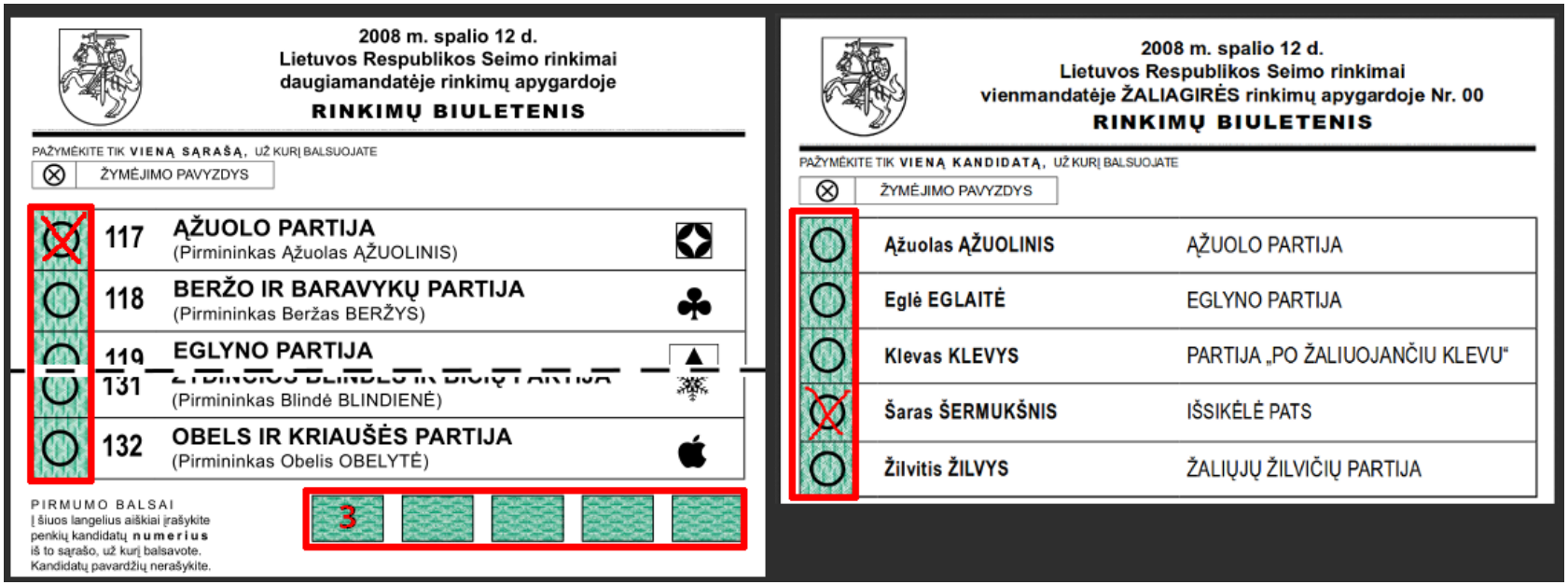
- Elections are being held each 4 years.
- All of the 141 seats are being contested.
- 71 electoral districts.
- Two-tier voting system:
  - District representative
  - Open party list



Elected district representatives by party (colors) in 2008 elections

Image source: screenshot of http://rinkimurezultatai.lt/

Example ballots



2008 m. spalio 12 d.
Lietuvos Respublikos Seimo rinkimai
daugiamandatėje rinkimų apygardoje
RINKIMŲ BIULETENIS

PAŽYMĖKITE TIK **VIENĄ SĄRAŠĄ**, UŽ KURĮ BALSUOJATE

ŽYMĖJIMO PAVYZDYS

117	**AŽUOLO PARTIJA** (Pirmininkas Ąžuolas ĄŽUOLINIS)	
118	**BERŽO IR BARAVYKŲ PARTIJA** (Pirmininkas Beržas BERŽYS)	
119	**EGLYNO PARTIJA**	
131	(Pirmininkas Blindė BLINDIENĖ)	
132	**OBELS IR KRIAUŠĖS PARTIJA** (Pirmininkas Obelis OBELYTĖ)	

PIRMUMO BALSAI
Į šiuos langelius aiškiai įrašykite penkių kandidatų **numerius** iš to sąrašo, už kurį balsavote. Kandidatų pavardžių nerašykite.

3

2008 m. spalio 12 d.
Lietuvos Respublikos Seimo rinkimai
vienmandatėje ŽALIAGIRĖS rinkimų apygardoje Nr. 00
RINKIMŲ BIULETENIS

PAŽYMĖKITE TIK **VIENĄ KANDIDATĄ**, UŽ KURĮ BALSUOJATE

ŽYMĖJIMO PAVYZDYS

Ąžuolas ĄŽUOLINIS	AŽUOLO PARTIJA
Eglė EGLAITĖ	EGLYNO PARTIJA
Klevas KLEVYS	PARTIJA „PO ŽALIUOJANČIU KLEVU“
Šaras ŠERMUKŠNIS	IŠSIKĖLĖ PATS
Žilvitis ŽILVYS	ŽALIŲJŲ ŽILVIČIŲ PARTIJA

We analyze

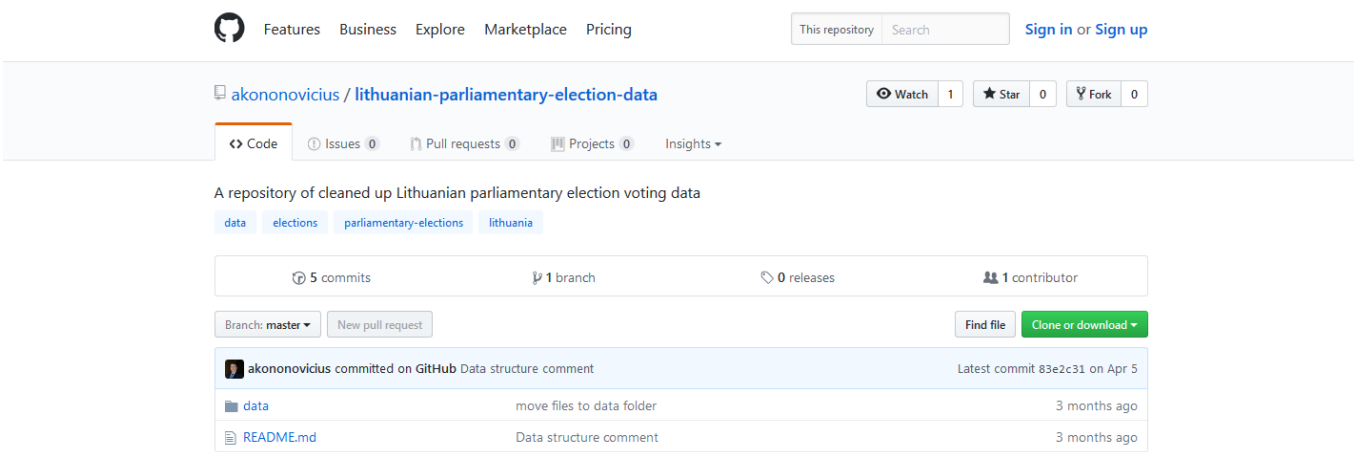
- voting for major parties (their lists) in 1992, 2008, 2012 elections.
- results at the smallest scale available (polling stations).

Image source: Central Electoral Commission

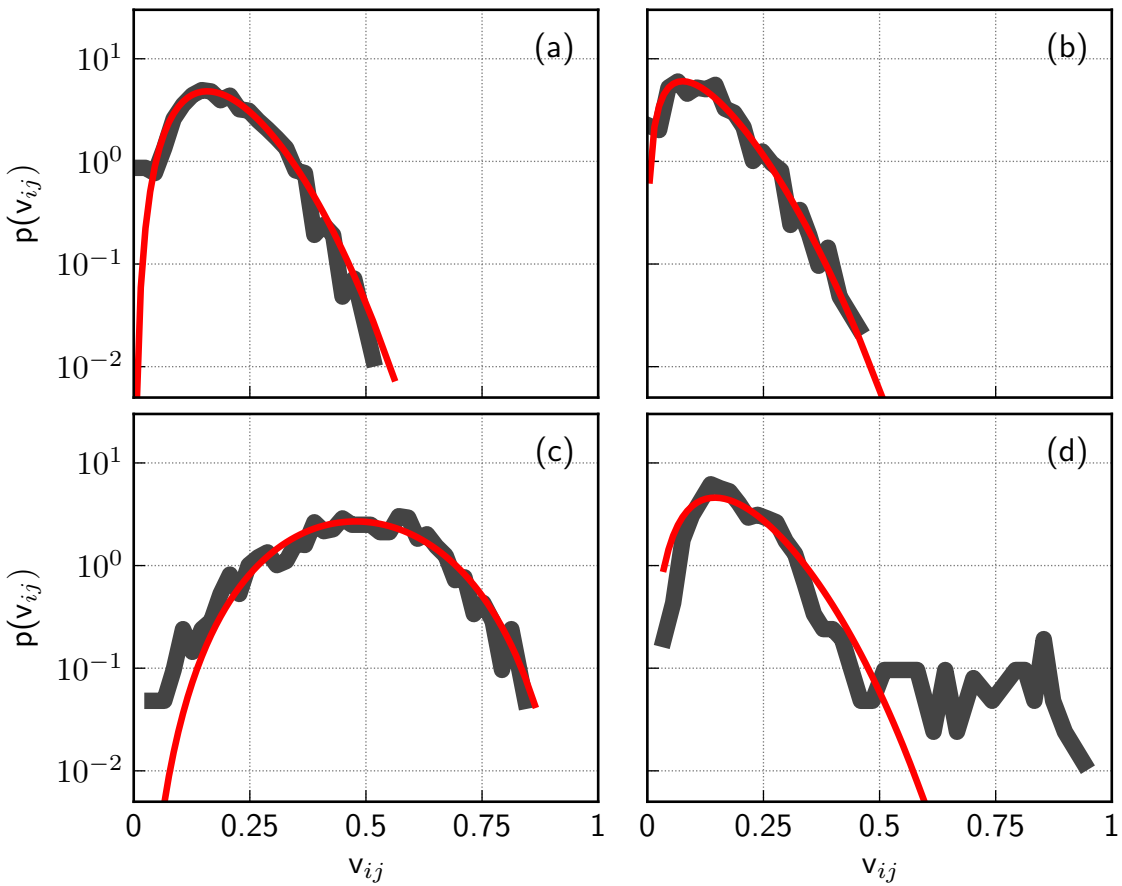
Data availability

Freely available from:

- **Central Electoral Commission**: https://rinkejopuslapis.lt
- **Baltic Institute of Advanced Technology**: http://rinkimurezultatai.lt
- **My github repository**: https://github.com/akononovicius/lithuanian-parliamentary-election-data



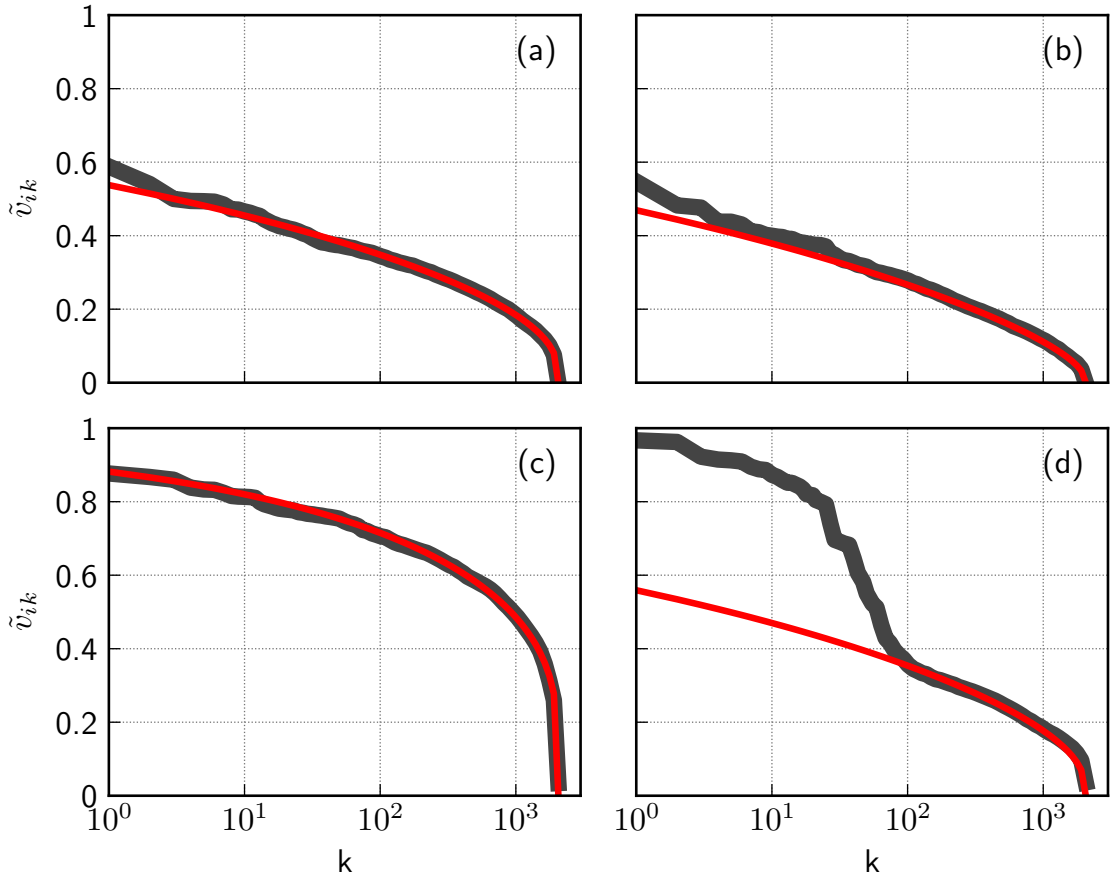
1992 election results I
Vote-share PDF



Vote-share PDF (gray curve) of four parties with average vote-share above 5% (a)-(c) and all other smaller parties combined (d). Fits (red curve) are provided assuming Beta distribution.

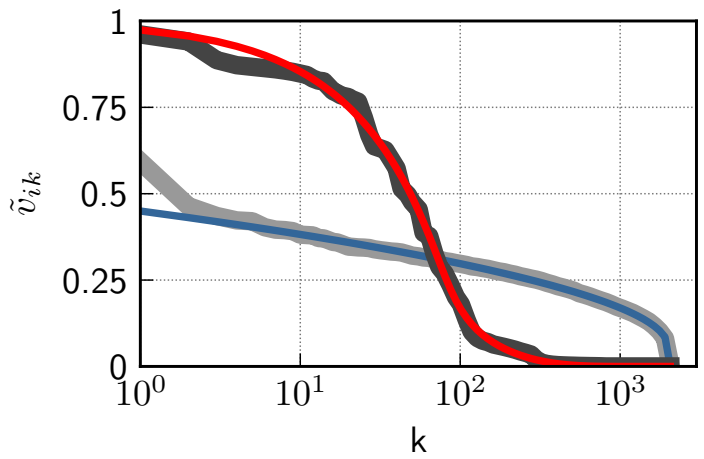
1992 election results II

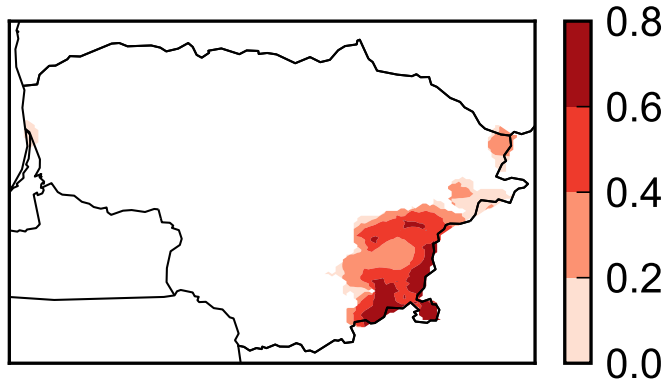
Vote-share rank-size distribution



Rank-size distribution (gray curve) of four parties with average vote-share above 5% (a)-(c) and all other smaller parties combined (d). Fits (red curve) are provided assuming Beta distribution.

Explaining the "outliers"

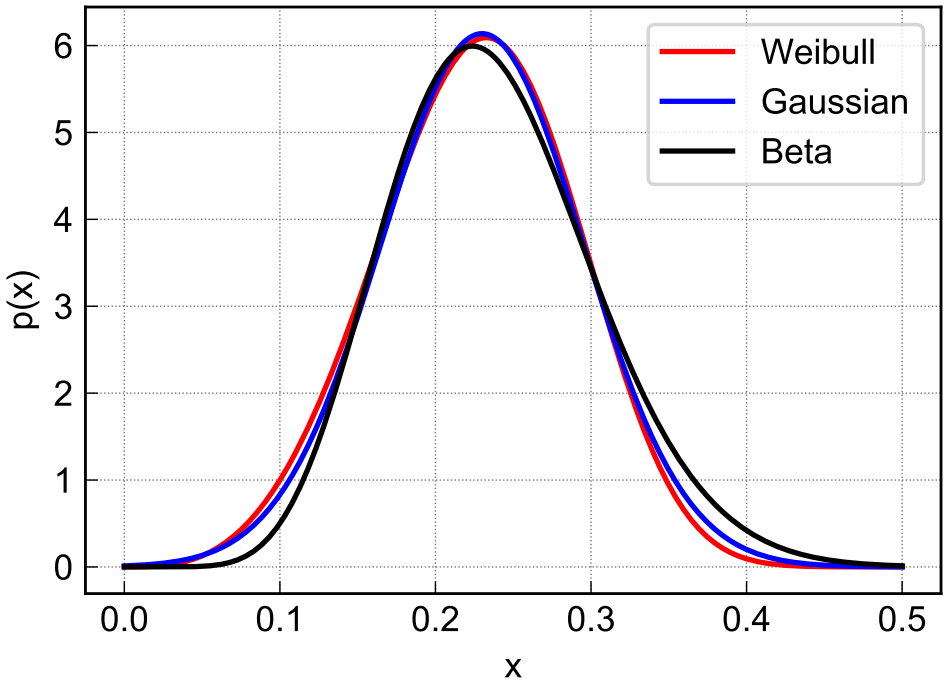




All outliers are present in vote-share data of a single party, which represents Lithuanian ethnic minorities (darker curve), while other small parties have no outliers (brighter curve). Fit (red curve) is provided assuming a mixture of two Beta distributions, fit (blue curve) is provided assuming Beta distribution.

Similar observation made in T. Fenner *et al.*, arXiv:1609.04282 [physics.soc-ph].

Beta distribution and oft-used alternatives

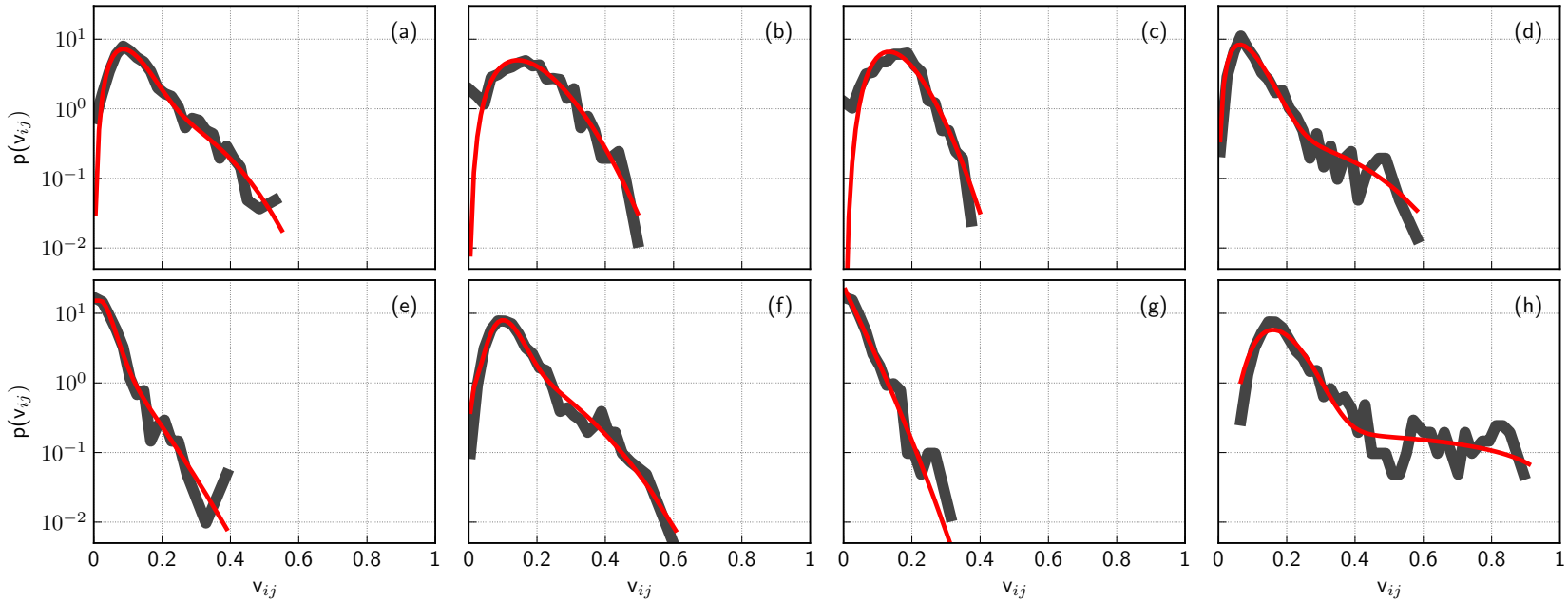


Comparison of Weibull ($\lambda = 0.25$, $k = 4$), Gaussian ($\mu = 0.23$ and $\sigma = 0.065$) and Beta ($\varepsilon_1 = 9.5$, $\varepsilon_2 = 30.5$) distributions.

R. F. da Paz *et al.*, Springer Proc Math Stat, 2015.

J. Fernndez-Gracia *et al.*, Phys Rev Lett **112**, 2013.

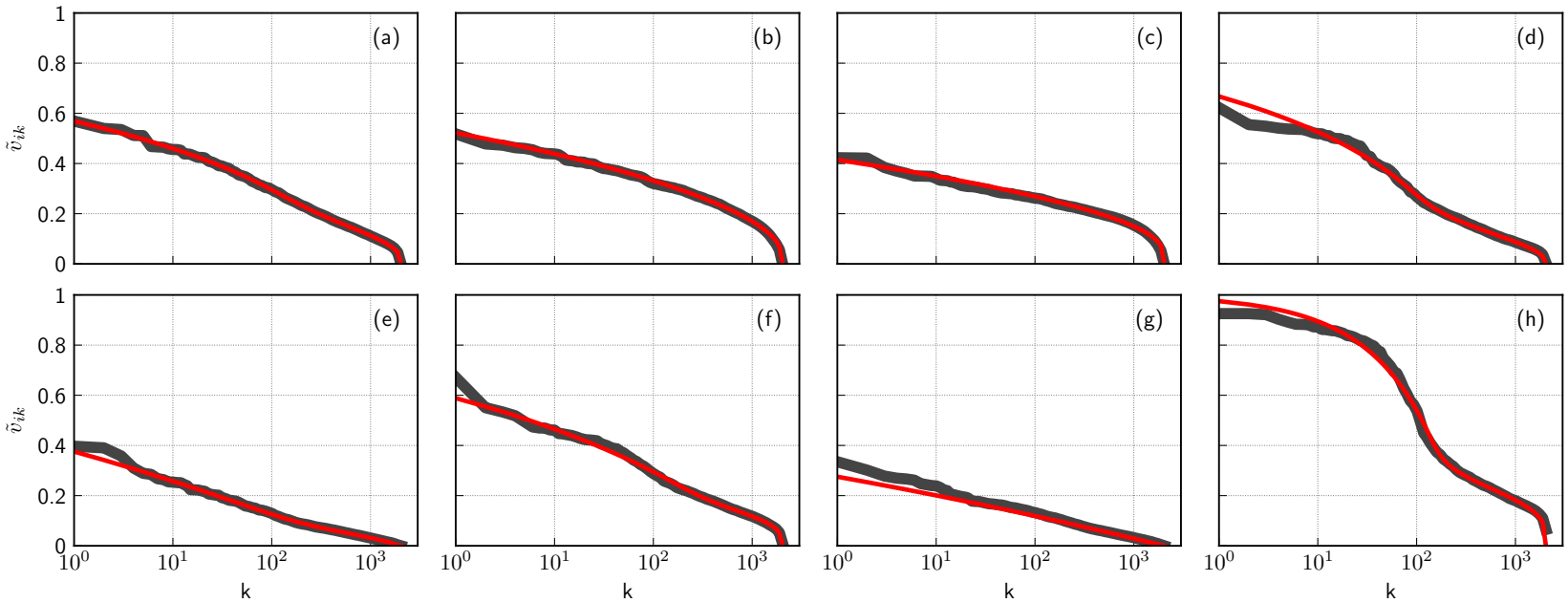
2008 election results I
Vote-share PDF



Vote-share distribution (gray curve) of seven parties with average vote-share above 5% (a)-(g) and all other smaller parties combined (h). Fits (red curve) are provided assuming a mixture of two Beta distributions.

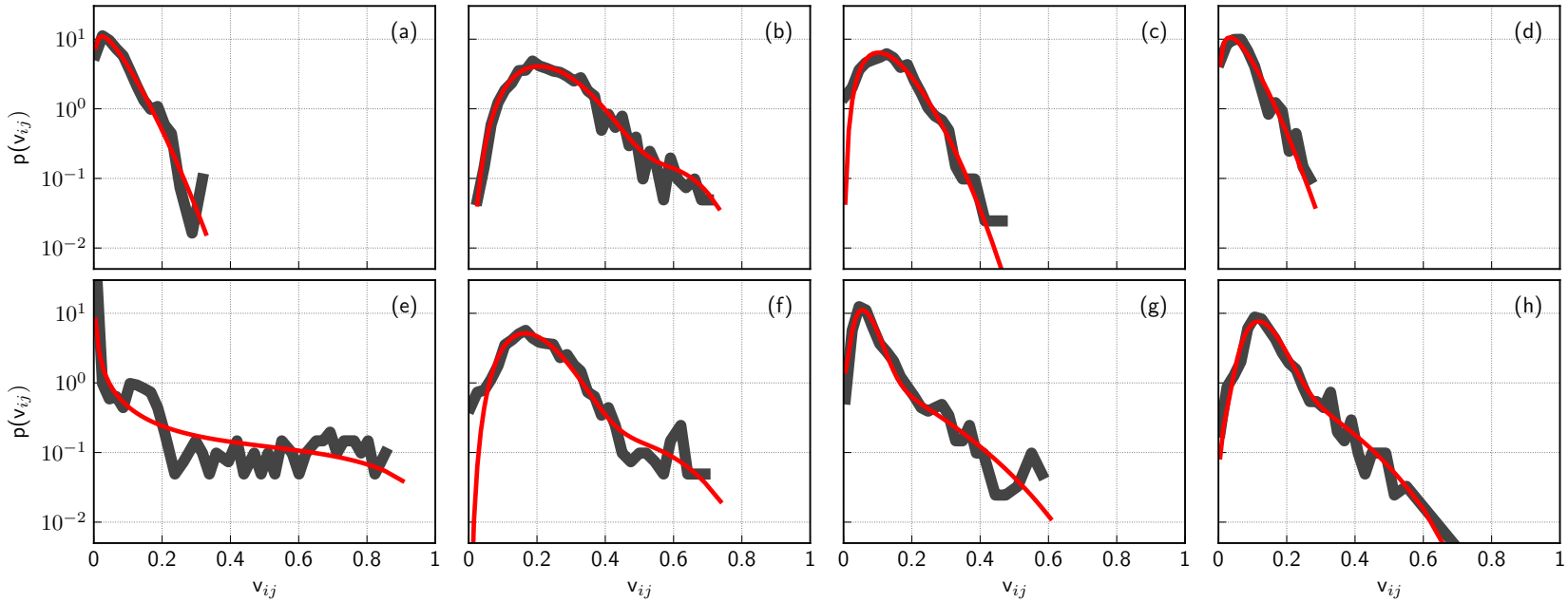
2008 election results II

Vote-share rank-size distribution



Rank-size distribution (gray curve) of seven parties with average vote-share above 5% (a)-(g) and all other smaller parties combined (h). Fits (red curve) are provided assuming a mixture of two Beta distributions.

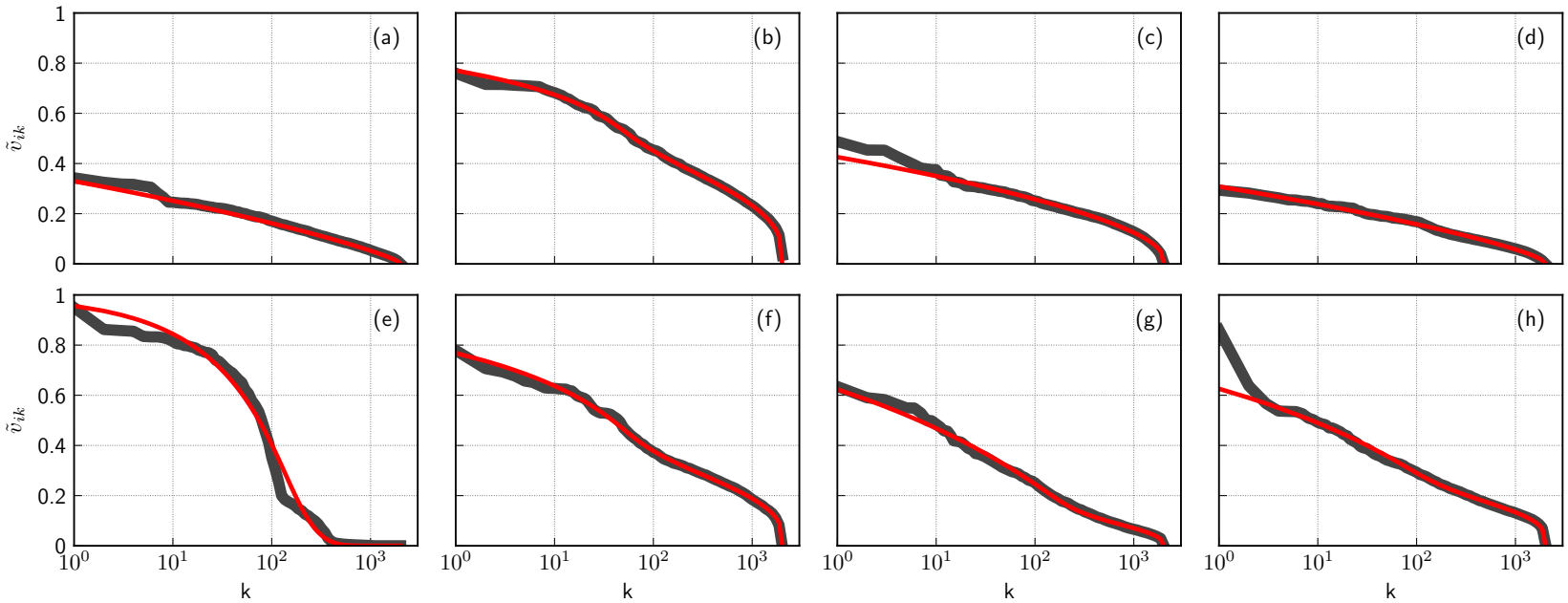
2012 election results I
Vote-share PDF



Vote-share distribution (gray curve) of seven parties with average vote-share above 5% (a)-(g) and all other smaller parties combined (h). Fits (red curve) are provided assuming a mixture of two Beta distributions.

2012 election results II

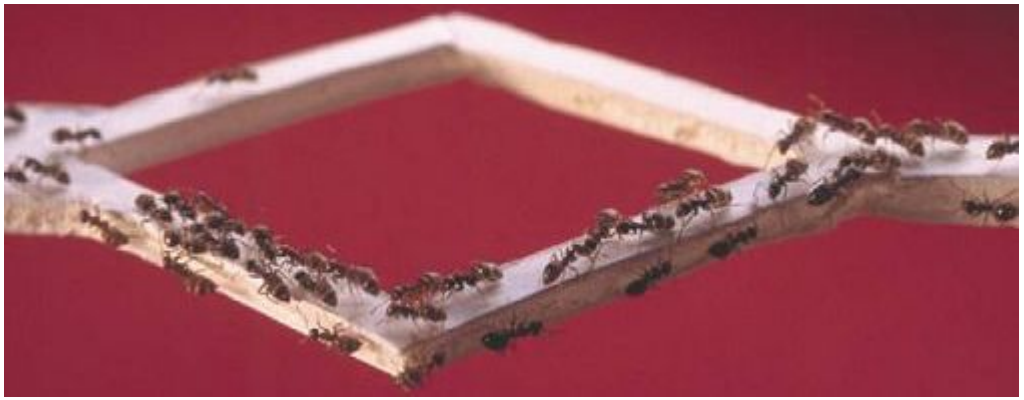
Vote-share rank-size distribution

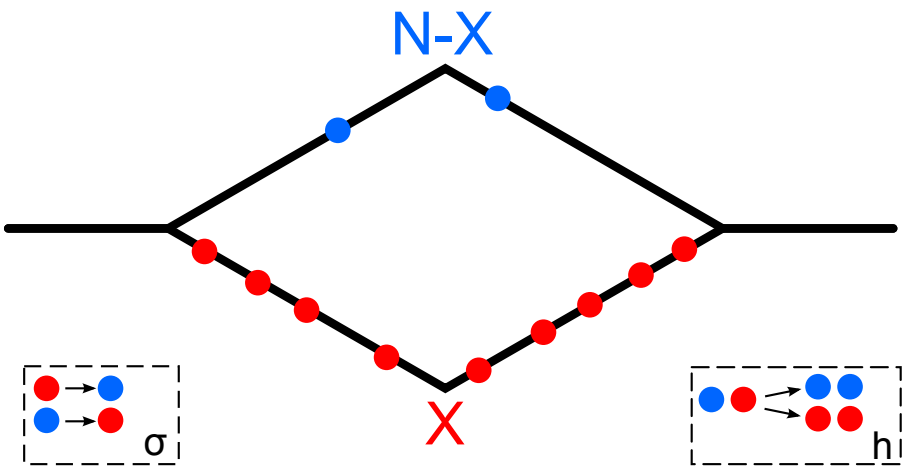


Rank-size distribution (gray curve) of seven parties with average vote-share above 5% (a)-(g) and all other smaller parties combined (h). Fits (red curve) are provided assuming a mixture of two Beta distributions.

Agent-based model of imitative voting

Imitative (herding) behavior in social insects





Upper image taken from Detrain & Deneubourg, PLR 3 (2006)

Formulation of the two-state model

1. Pick one random agent.
2. If agent is "red", then agent switches to "blue" with probability

$$P_{r\to b} = [\varepsilon_b + (N - X)]h\Delta t,$$

otherwise the agent is "blue", the switching probability to "red" is

$$P_{b\to r} = [\varepsilon_r + X]h\Delta t.$$

Stationary distribution of $x = X/N$ is Beta,

$$p(x) \propto x^{\varepsilon_r - 1}(1 - x)^{\varepsilon_b - 1}.$$

Kirman, QJE 108, 137-156 (1993)

Formulation of the M-state model

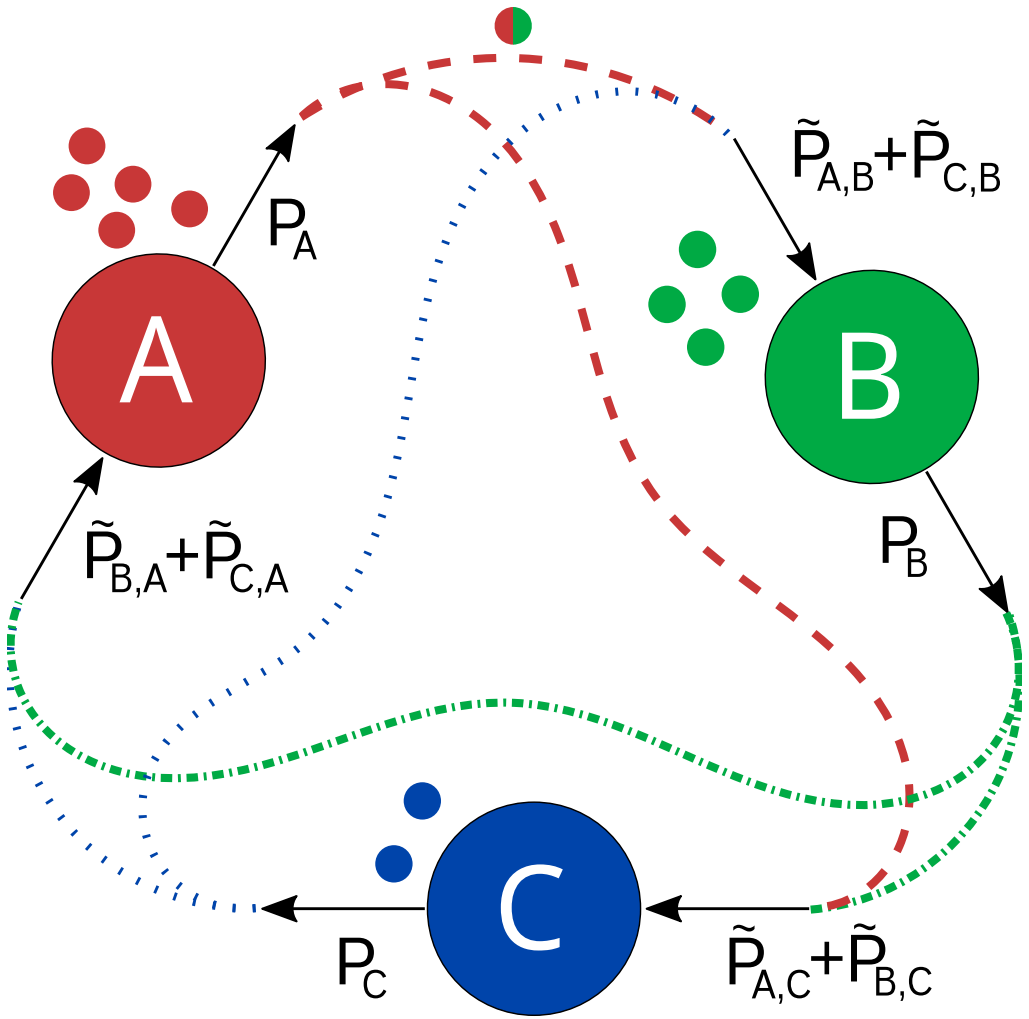
1. Pick a random agent.
2. If agent votes for i party, the probability to switch to any other party is given by:

$$P_i = \sum_{j \neq i} \left[\varepsilon_j + X_j\right] h\Delta t = \left[\varepsilon_{-i} + (N - X_i)\right] h\Delta t.$$

3. If agent decides to switch, the party is picked proportionaly based on $\tilde{P}_{i,j} \propto \varepsilon_j + X_j$.

Should be equivalent to a noisy multi-state Voter model.

Illustration of the three-state case



$$P_A = [\varepsilon_{-A} + (N - X_A)]\, h\Delta t = [\varepsilon_B + X_B + \varepsilon_C + X_C]\, h\Delta t = \tilde{P}_{A,B} + \tilde{P}_{A,C}.$$

Stationary distribution of the M-state model

Due to similarity to the two-state model, we expect that each $x_i = X_i/N$ is distributed according to Beta distribution with parameters ε_i and ε_{-i}.

Thus stationary distribution of vote-share vector, $\vec{x} = \{x_1, \ldots, x_M\}$, should be Dirichlet distribution:

$$p(\vec{x}) \propto \prod_{i=1}^{M} x_i^{\varepsilon_i - 1}.$$

Reproducing results of 1992 elections

Modeling implications and the actual data

Party	$\hat{\varepsilon}_i$	$\hat{\varepsilon}_{-i}$	R^2_{PDF}	R^2_{RS}
SK	3.9	16.6	0.95	0.994
LKDP	2.2	16	0.92	0.995
LDDP	5.7	6.1	0.91	0.998
Other	3.3	14.4	0.91	0.86
	15.1			

A restriction follows from the model:

$$\varepsilon_{-i} = \sum_{j=1}^{M} \varepsilon_j - \varepsilon_i,$$

which does not hold for the data. Over-fitting?

Bayesian inference of ε_i

We split data of 1992 elections into two sets:

- minority party vote share $> 20\%$ (94 polling stations),
- minority party vote share $< 20\%$ (1966 polling stations).

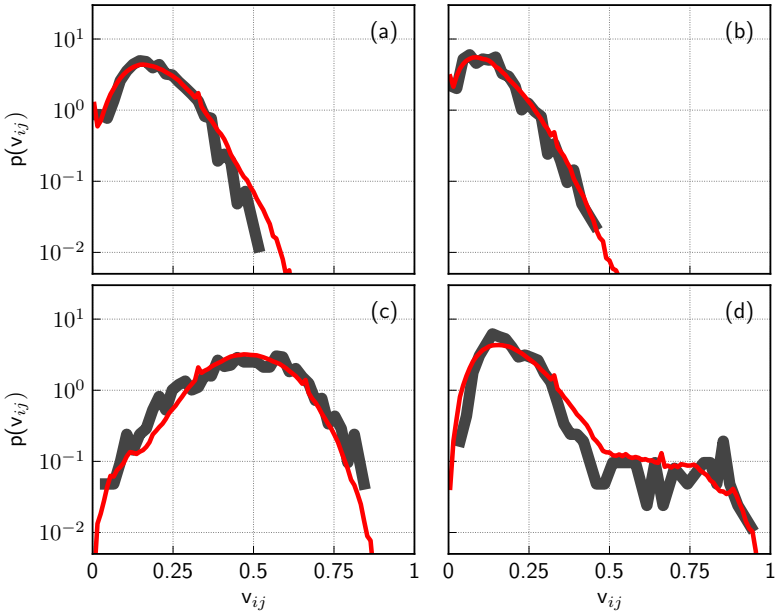
$> 20\%$ polling stations

Party	ε_i
SK	0.65 ± 0.1
LKDP	0.35 ± 0.05
LDDP	2.5 ± 0.2
Other	4.7 ± 0.4
	8.2

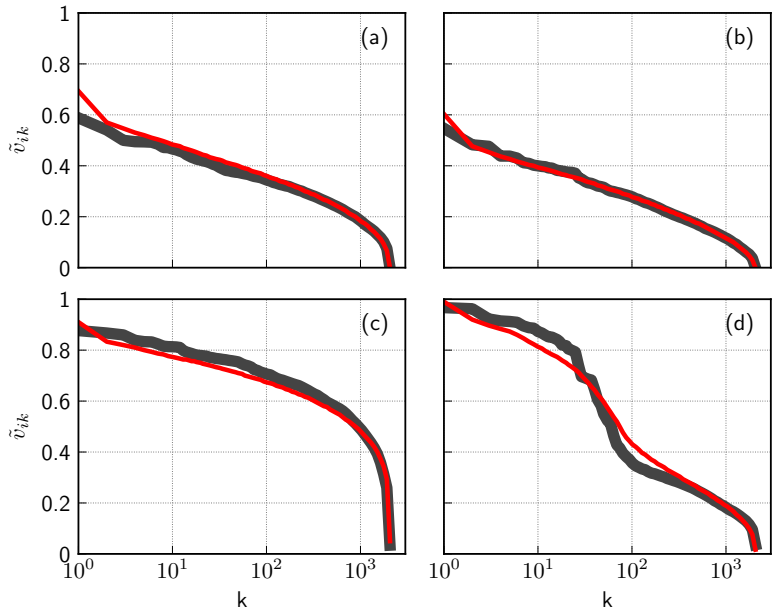
$< 20\%$ polling stations

Party	ε_i
SK	3.8 ± 0.1
LKDP	2.55 ± 0.1
LDDP	9.3 ± 0.2
Other	3.7 ± 0.1
	19.35

Reproducing 1992 elections



Vote-share PDF of the three main parties (a)-(c) and the other party (d).



Rank-size distribution of the three main parties (a)-(c) and the other party (d).

To summarize...

Quick summary

- We have shown that vote-share distributions are well approximated by a mixture of two Beta distributions.
- We have presented a simple model, which reproduces Beta and, more generally, Dirichlet distribution.
- We have used Bayesian inference to infer model parameters from the 1992 election results.
- We have used the inferred parameters to reproduce the 1992 election results.

A. Kononovicius, arXiv:1704.02101 [physics.soc-ph]

Thank You!

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