Non-extensive and extensive statistics in the agent-based herding model

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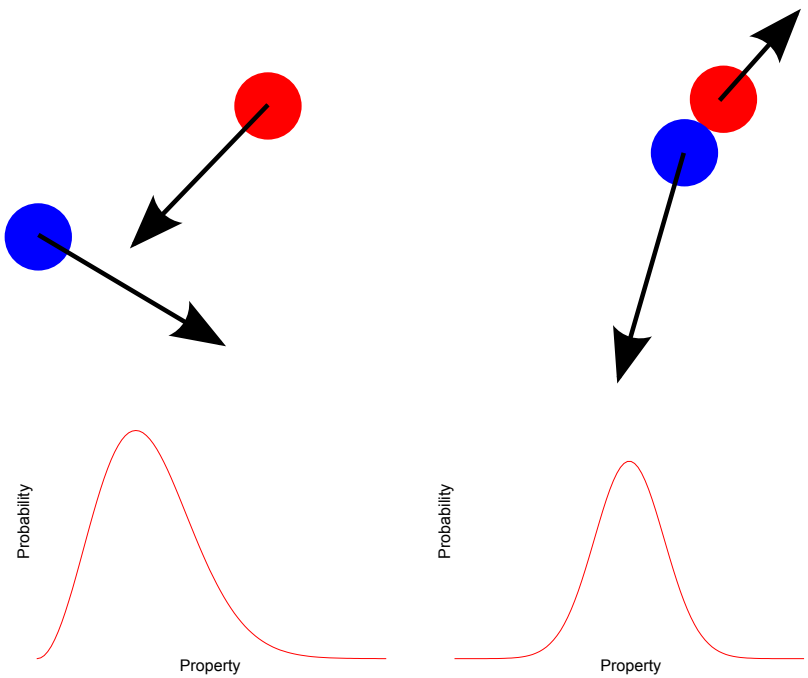
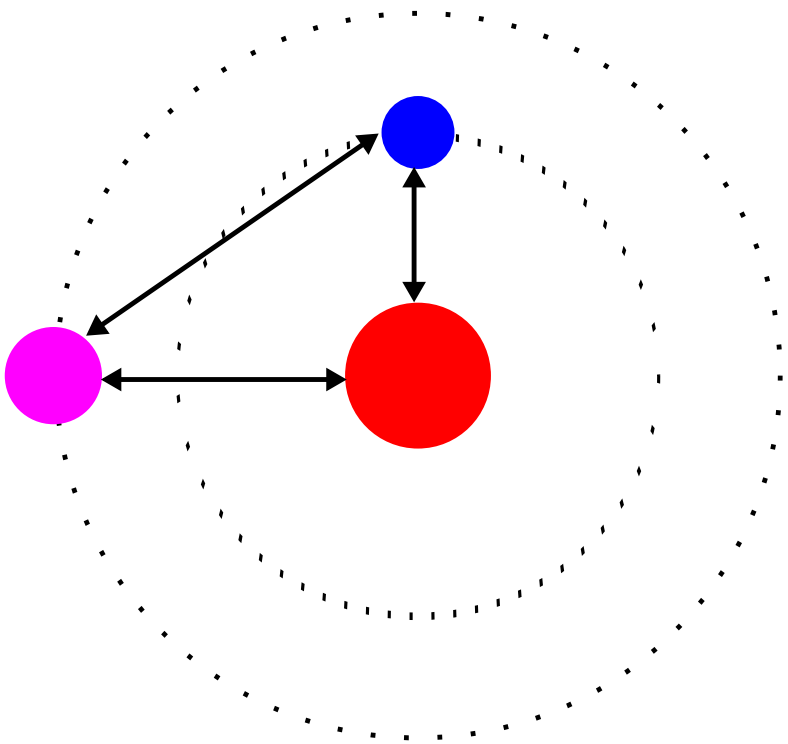
Ideal gases - emergence of Boltzmann-Gibbs statistics



- Molecules are assumed to interact only via collisions
- Thus the interactions are short range (local)
- Ideal gases are additive and extensive
- Statistical properties of such systems are usually characterized by the exponential distributions

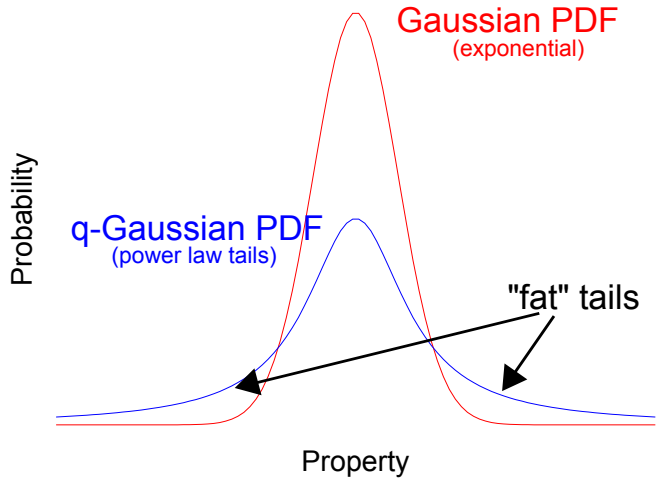
Long range (global) interactions - $F \sim r^{-\alpha}$, $\alpha < d$



Among the simplest examples:

- Gravitational forces - $F = G\frac{m_1 m_2}{r^2} \sim r^{-2}$,
- Coulomb forces - $F = \frac{q_1 q_2}{4\pi\epsilon\epsilon_0 r^2} \sim r^{-2}$.

Such systems may suffer from non-additivity and non-extensiveness.



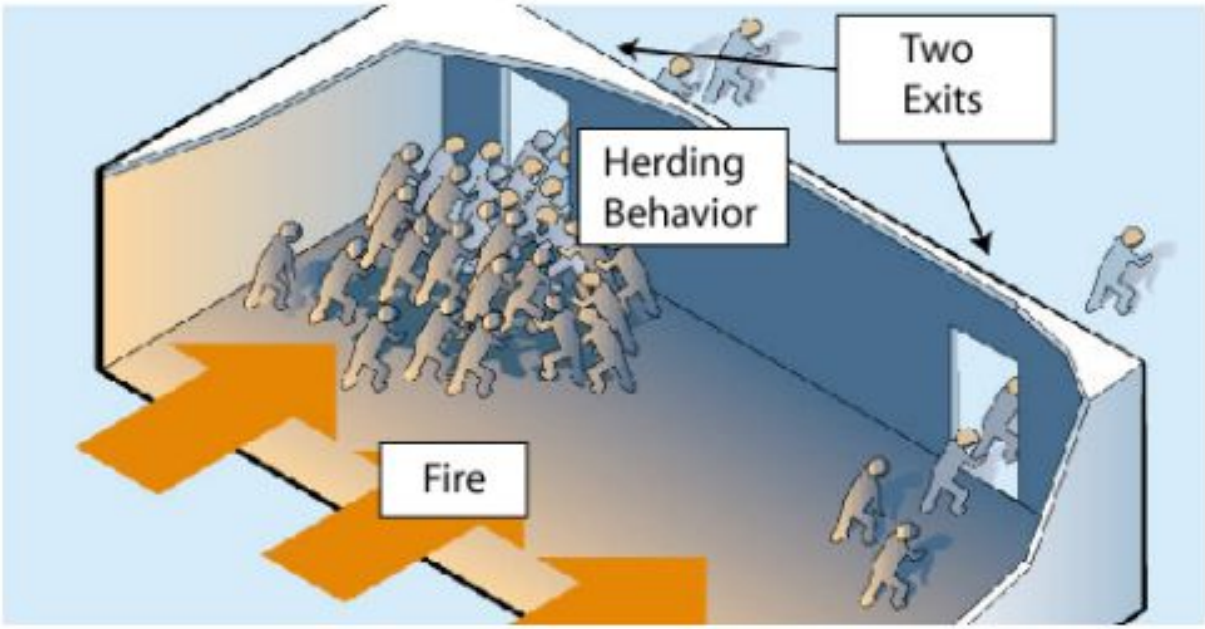
“Molecules are like so many individuals,

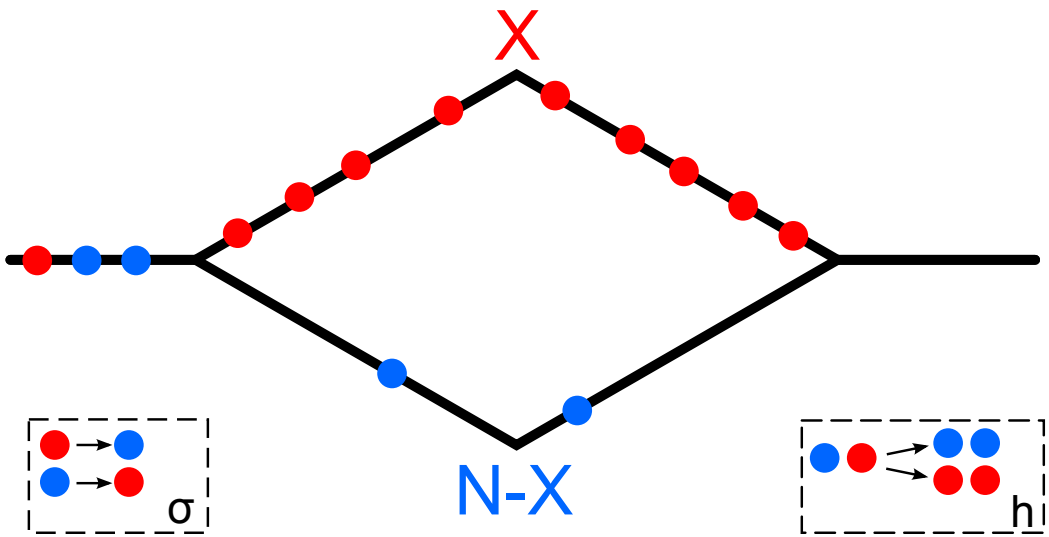
having the most various states of motion” (L. Boltzmann)

Short range interaction

Long range interaction

Agent-based herding model for social systems



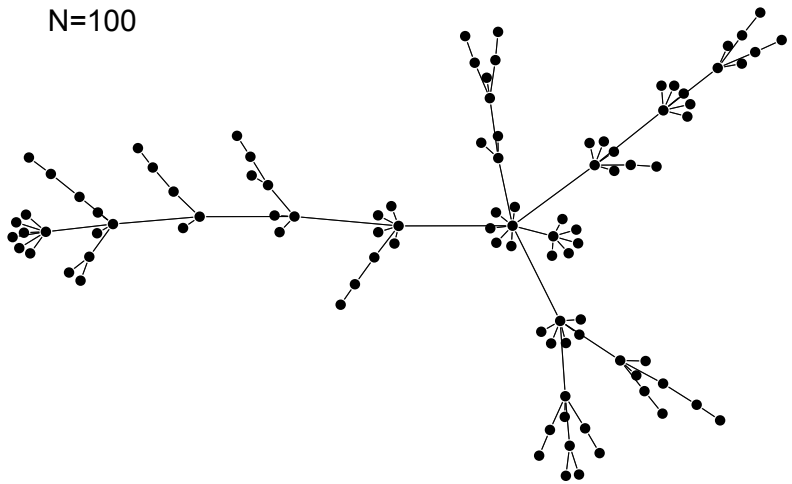


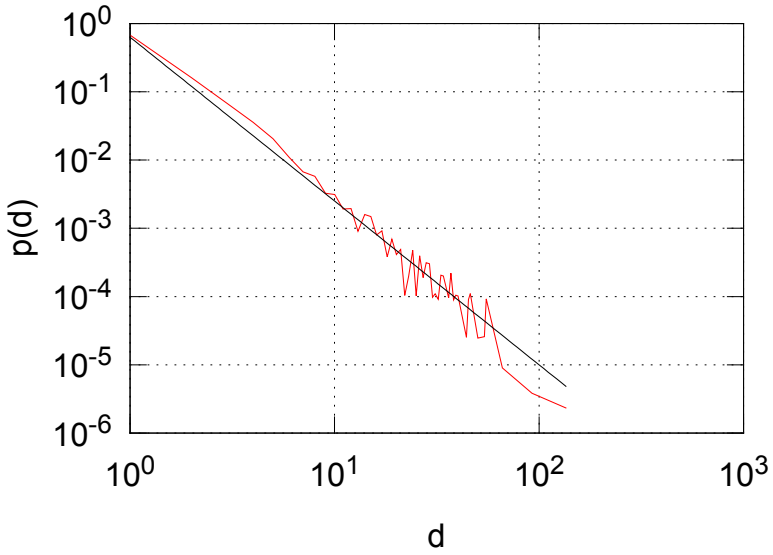
Per agent transition probability:

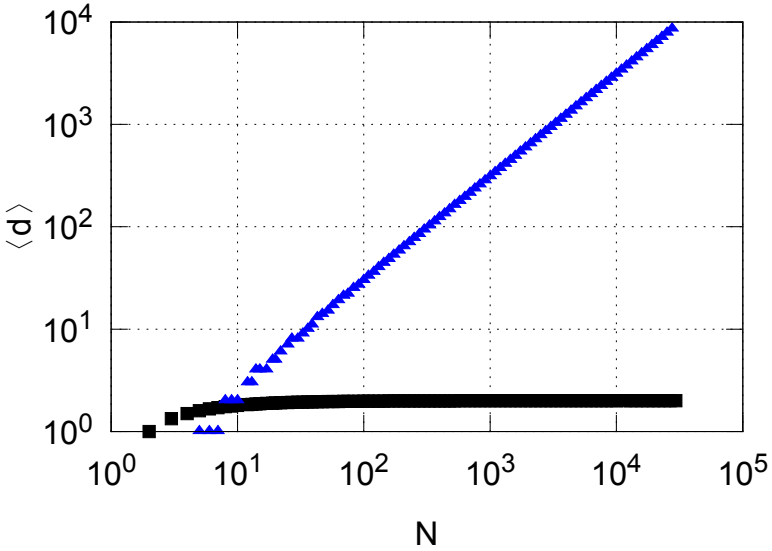
$$\mu(i \to j) = \sigma_j + \frac{h}{N^{1-\gamma}} X_j.$$

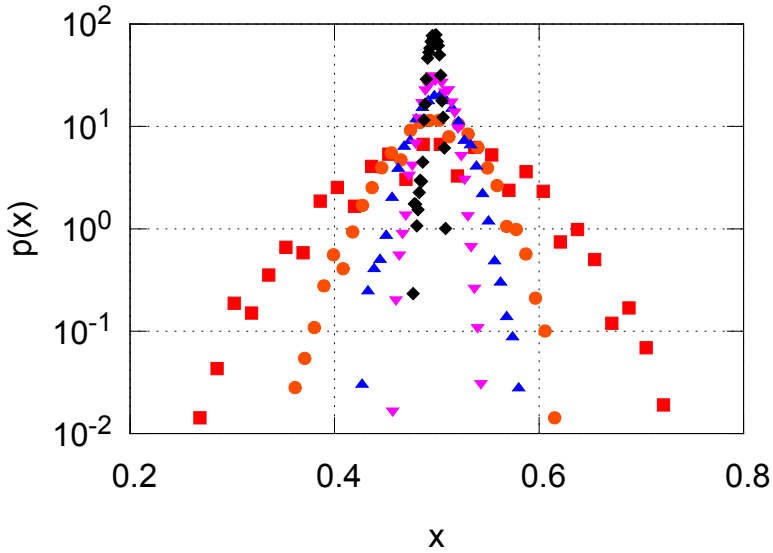
Previously considered cases - $\gamma = 0$ and $\gamma = 1$.

Short range ($\gamma = 0$) interactions

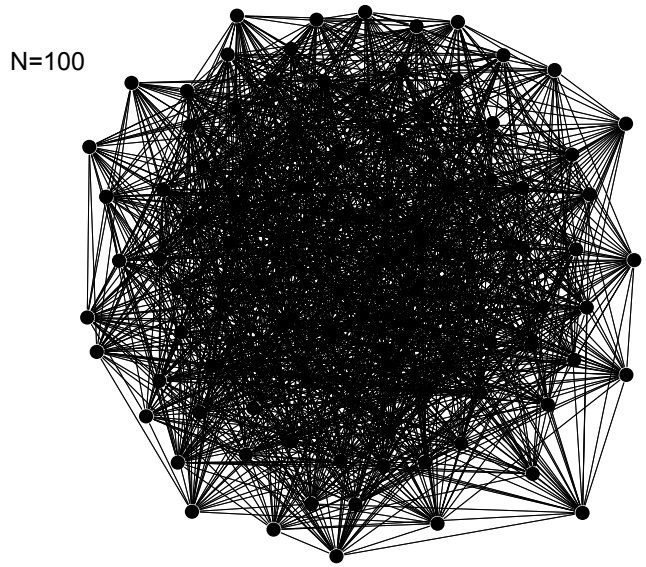


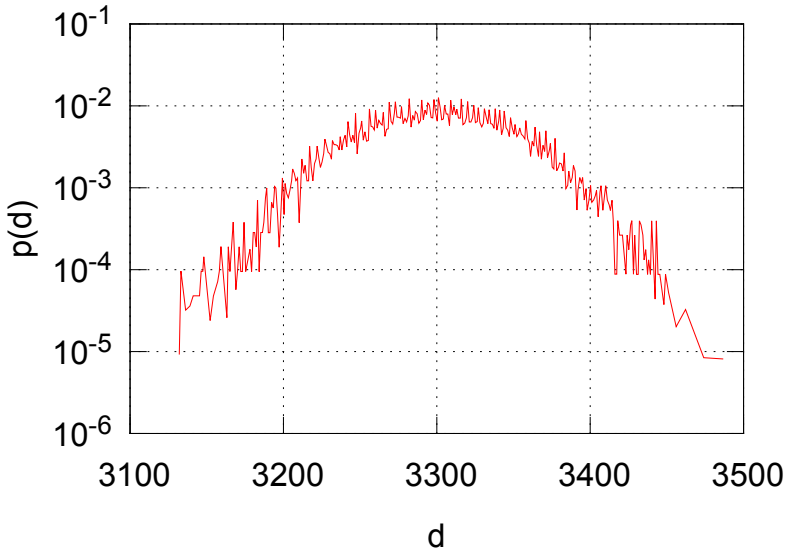


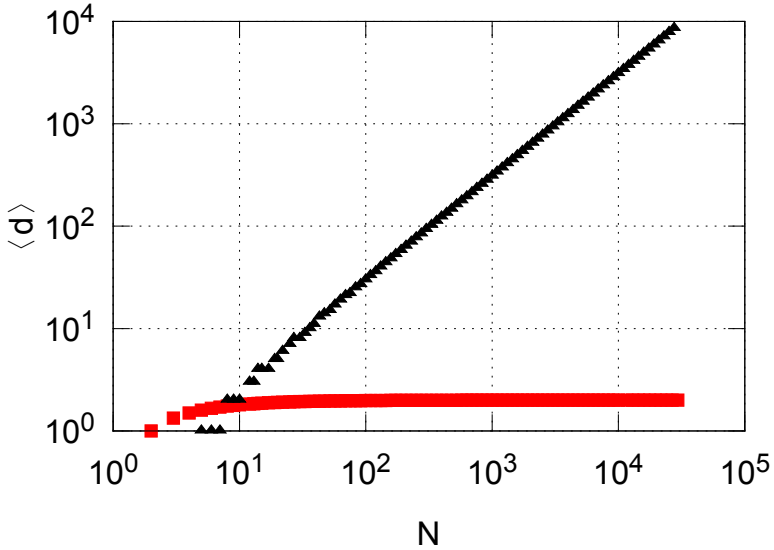


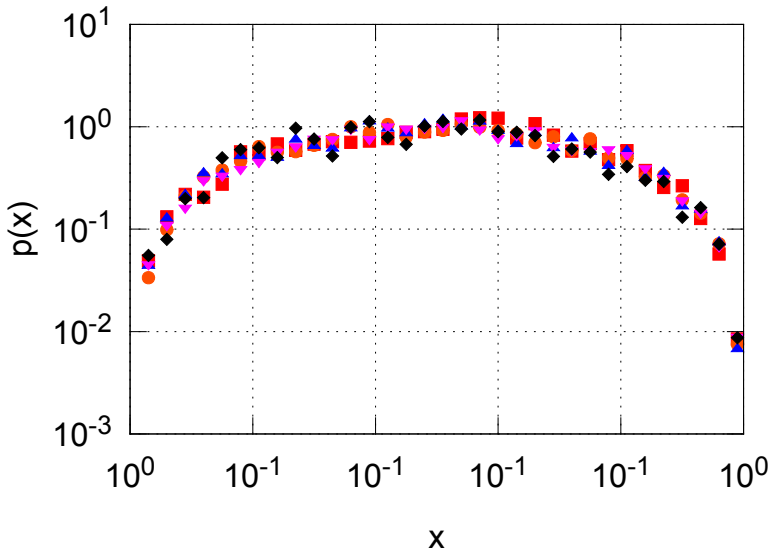


Long range ($\gamma = 1$) interactions

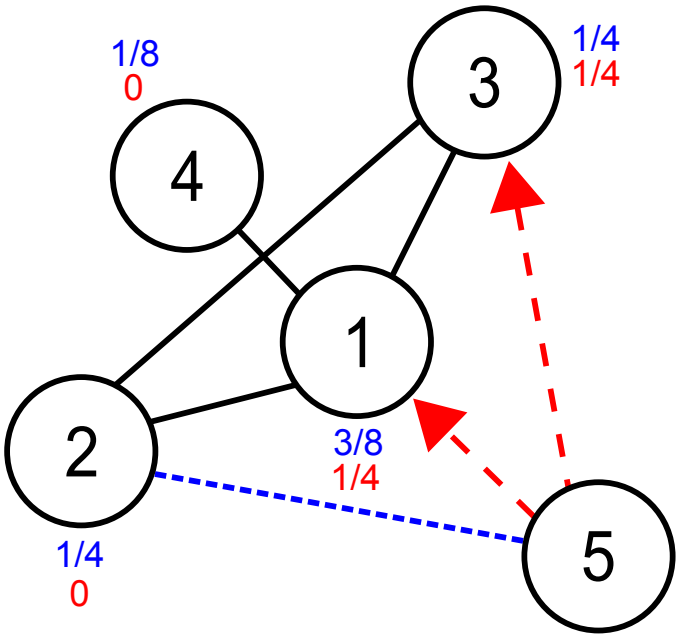


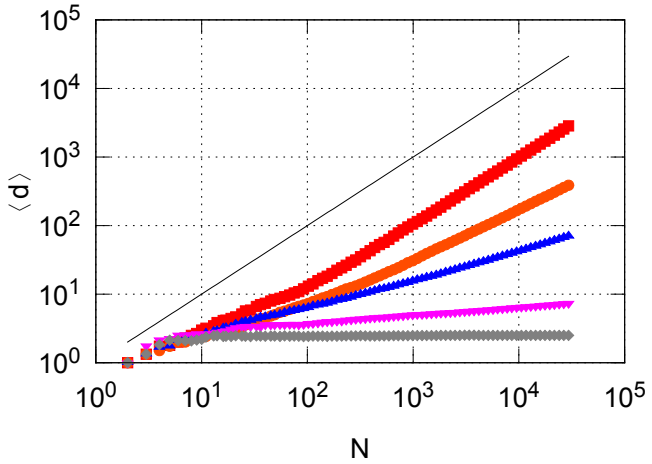






Hybrid network model, $\gamma \in [0, 1]$





Use "rich get richer" scheme to form single initial edge:

$$p_i = \frac{d_i^\beta}{\sum_i d_i^\beta}.$$

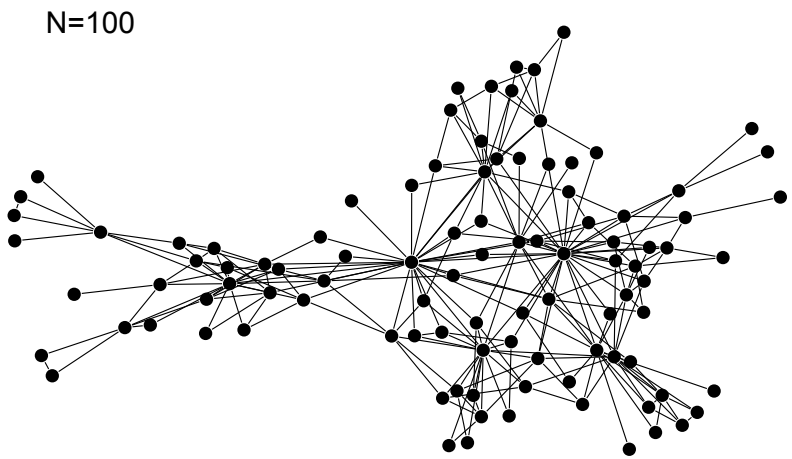
In the figure on the left we assume $\beta = 1$.

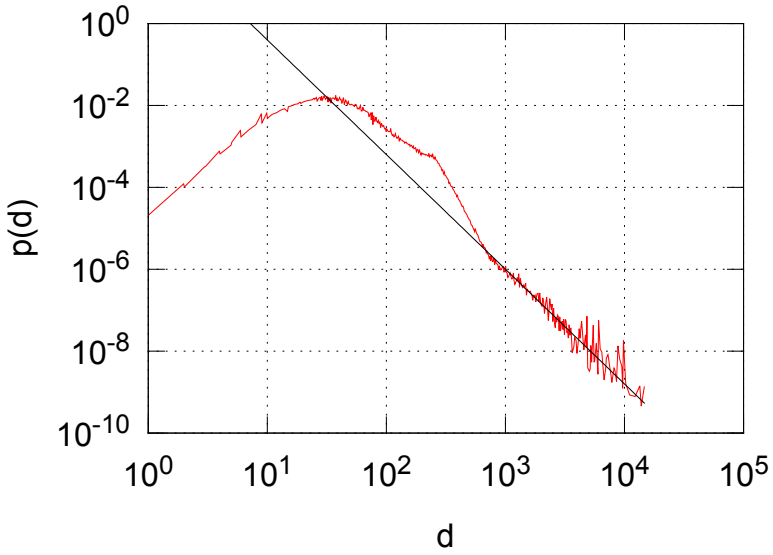
Make connections to neighbors of the initially selected node, i:

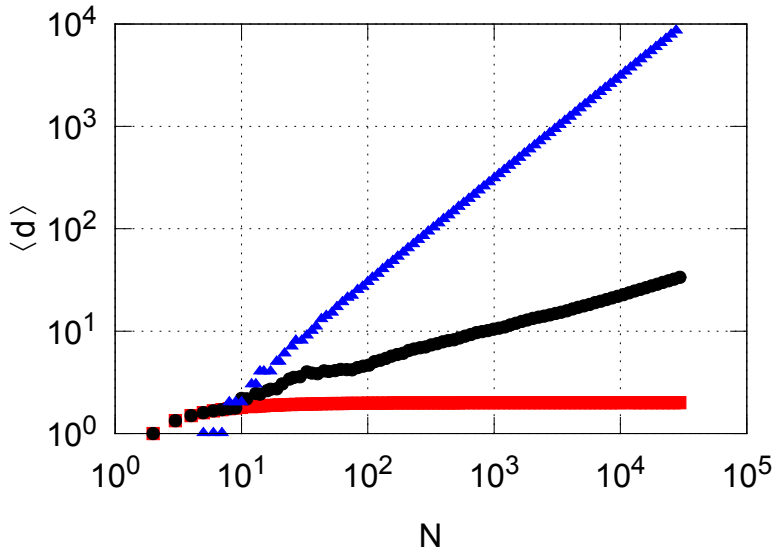
$$p_i = p_0 d_i^{-\delta}.$$

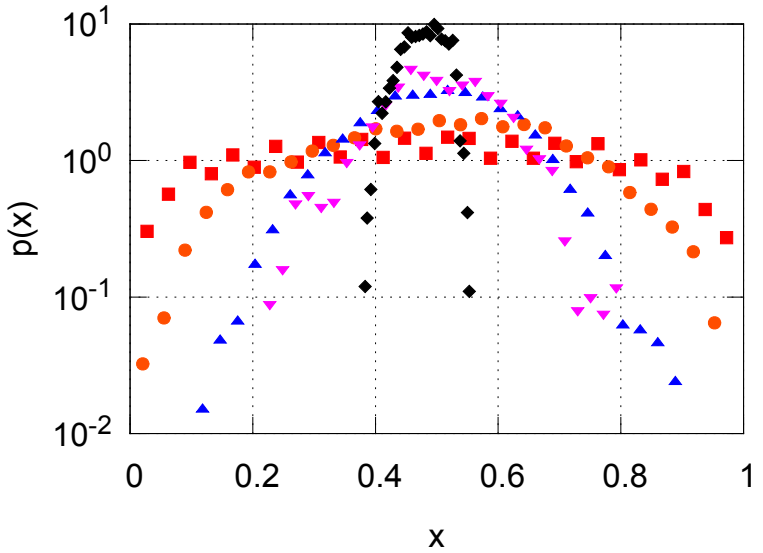
In the figure on the left we assume $p_0 = 0.31$, $\delta = 0.31$.

Medium range ($\gamma = 0.35$) interactions



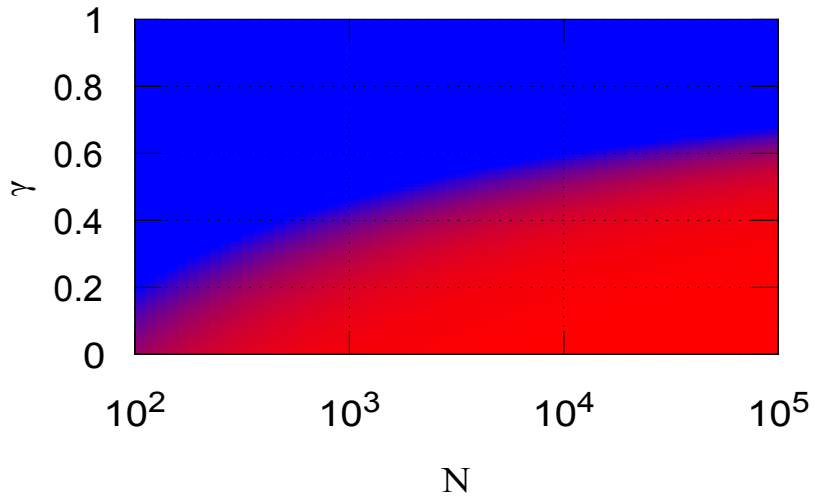




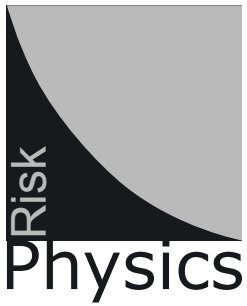


Summary

- We proposed a hybrid network model with tunable scaling of average degree, $\langle d \rangle \sim N^{\gamma}$ with $\gamma \in [0, 1]$.
- The hybrid network model allows continuous transition between two well-known network types - “random graph” and “scale-free network”.
- The hybrid network model allows us to observe transition between the non-extensive and extensive statistics in well-known agent-based herding model.



Thank you!



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