Leadership phenomenon in the agent-based herding model

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Introduction

It is often thought that endogenous interactions are behind many big spontaneous events observed in the complex socio-economic systems. Social cooperation, spontaneous emergence, formation of financial bubbles, financial flash-crashes and even mass panic may be caused by the endogenous dynamics and certain general similarities of human psychology [1, 2]. The tendency to rely on others, importance of social interactions, tight coupling suggests that it might be possible to influence the collective behavior of the complex socio-economic systems.

One of the suitable frameworks to test this idea is known as agent-based modeling [3, 4]. In this framework a generalized entity, known as agent, is used in place of real-life interacting parts of the modeled system. While the real-life interactions might be very complex, agents are frequently assumed to interact by following very simple rules. Despite the underlying simplification the complex collective behavior emerges as a result of the interactions between these agents.

In this contribution we consider agent-based herding model proposed by Alan Kirman [5], which explains herding behavior in ant colonies and is also successfully applied to other socio-economic scenarios [6]. We extended the considered model by introducing inflexible controlled agents. Similar setup was studied in [7].

Agent-based herding model

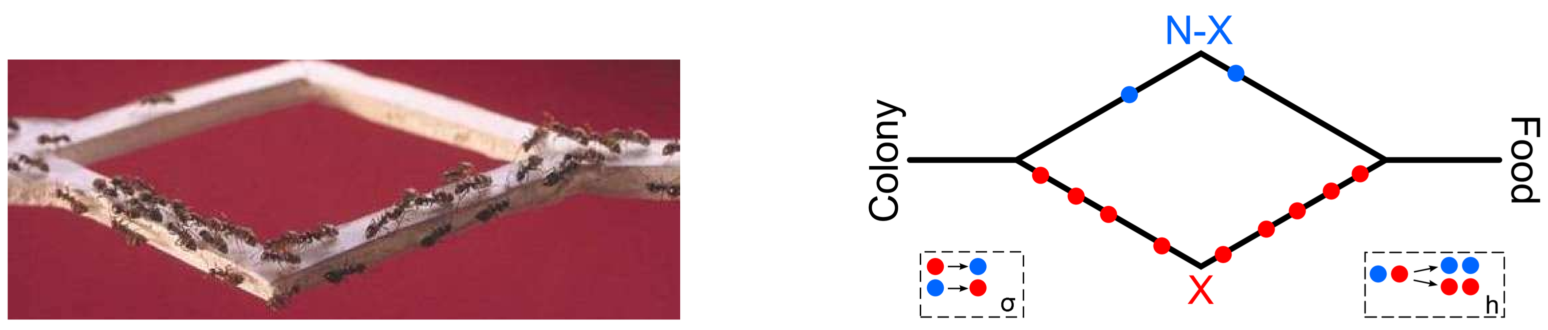

Figure 1: A photo of experiment (on the left) which inspired the agent-based herding model (schematic representation on the right). Here we have N ants, which take food to their colony. Ants may use one of the two available paths. Interestingly enough most of the time they tend to exploit only one path. This happens due to the importance of two-agent interactions, h terms, in comparison to a single-agent transitions, σ terms.

Mathematical definition of the transition probabilities requires a definition of two-agent interactions. If the agents are able to interact on the global scale, their transition probabilities will have the following form,

$$\mu(i \rightarrow j) = [\sigma_j + hX_j]\,\Delta t. \tag{1}$$

While if they are interacting only on the local scale (i.e., only with their direct neighbors), the transition probabilities take the following form,

$$\mu(i \rightarrow j) = \left[\sigma_j + \frac{h}{N} X_j\right] \Delta t. \tag{2}$$

Here i and j are indices representing the available states, Δt is a small time period (small enough for one transition to be probable).

Leaders and their impact

Let discuss the concept of leadership. First of all leaders unlike ordinary people are well known and frequently seen on the media, thus leaders, in our setup, will interact with ordinary agents globally. Ordinary agents being not so well known and not so frequently seen, will be assumed to interact on the local scale. Another important assumption about the leadership is that leaders are truly informed about the desirable outcome, thus they have "inflexible" opinion (which we control externally). A similar setup is modeled in [7], but this previous approach considers only ordinary people with inflexible opinion interacting only on the local scale. Taking this discussion into account we can define per-agent transition probabilities (for ordinary agents) as:

$$\mu(i \rightarrow j) = \left[\sigma_j + \frac{h}{N} X_j + hM_j\right] \Delta t. \tag{3}$$

Where M_j is a number of leaders in the state j. Note that N no longer stands for a total number of agents in the system, now it represents a total number of ordinary agents, which are able to switch their opinion via endogenous interactions.

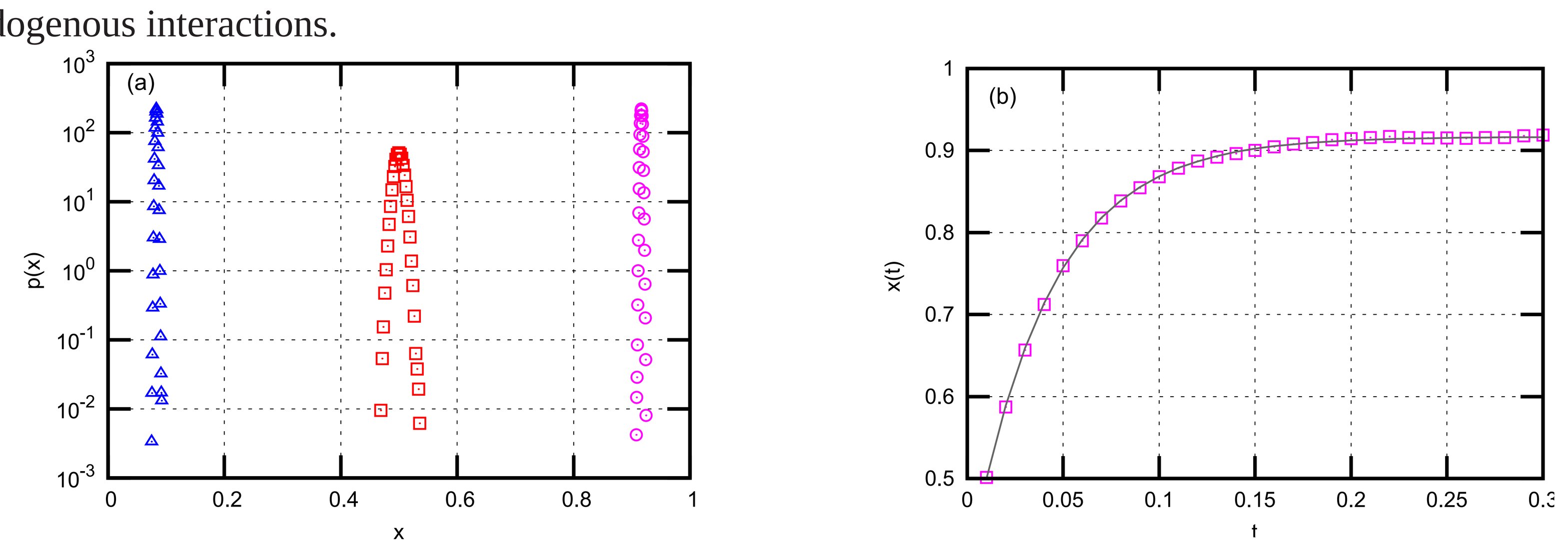

Figure 2: (a): A comparison of a numerically calculated stationary PDF with no controlled agents, $M_1 = 0$ and $M_2 = 0$ (red squares), and stationary PDF with controlled agents, $M_1 = 20$ and $M_2 = 0$ (magenta circles), $M_1 = 0$ and $M_2 = 20$ (blue triangles). (b): A convergence of agent-based model, $M_1 = 20$ and $M_2 = 0$, (magenta squares) versus the analytical prediction Eq. (5). Model parameters were set as follows: $\sigma_1 = \sigma_2 = 2$, $h = 1$, $N = 10^4$.

As you can see in the figure above an extremely small number of leaders is able to make a significant impact onto the stationary distribution. This brings up an interesting question how fast is the convergence towards this stationary distribution. It is pretty straightforward (use the Master equation and one-step formalism [8]) to obtain ordinary differential equation describing the macroscopic dynamics of the considered agent-based system:

$$\dot{x} = (\sigma_1 + hM_1)(1 - x) - (\sigma_2 + hM_2)x. \tag{4}$$

The solution of this equation is an exponential function of time:

$$x(t) = \bar{x} + [x(0) - \bar{x}]e^{-[h(M_1+M_2)+\sigma_1+\sigma_2]t}. \tag{5}$$

Thus the convergence is exponential fast with the rate dependent on the individual transition rates, σ_i, and amount of leaders in each state, M_i. In the above $\bar{x}$ is the stationary point:

$$(\sigma_1 + hM_1)(1 - \bar{x}) - (\sigma_2 + hM_2)\bar{x} = 0, \quad \Rightarrow \quad \bar{x} = \frac{hM_1 + \sigma_1}{h(M_1 + M_2) + \sigma_1 + \sigma_2}. \tag{6}$$

Preventing catastrophic events in the financial markets

Unlike in the society, in the financial markets any trader may make transactions with any other trader. Thus in the financial market model all of the agents should also interact on the global scale. The macroscopic dynamics of the financial markets will no longer be given by the ODE, one needs to use stochastic calculus [6, 9, 10]. As our previous works have shown the trading in financial markets has two separate time scales - a slow process representing fundamentalist-chartist switching and a fast process representing optimism-pessimism mood fluctuations.

A slow process is a stochastic process based on modulating return, y, which is defined as a ratio between agents using chartist trading strategies and agents using fundamental trading strategies [9]. SDE for y has the following form:

$$\mathrm{d}y = \left[\sigma_1 + (2 - \sigma_2)y^{1+\alpha}\right](1 + y)\mathrm{d}t + \sqrt{2hy^{1+\alpha}}(1 + y)\mathrm{d}W. \tag{7}$$

The stationary probability density function of y is a power-law, $p(y) \sim y^{-\varepsilon_2 - \alpha - 1}$, thus large crashes and bubbles become probable. In order to prevent them we might use a simple rule - if absolute return y is larger than y_{lim} we introduce M agents trading based on the fundamentals.

Another possibility to prevent catastrophic events is to control the fast mood process, macroscopic dynamics of which are given by

$$\mathrm{d}\xi = \left[(1 - \xi)\sigma_{po} - (1 + \xi)\sigma_{op}\right]\mathrm{d}t + \sqrt{2h\left(1 - \xi^2\right)}\mathrm{d}W. \tag{8}$$

Let us "moderate" the time series of mood, ξ, by a similar rule as in case with y: if $\xi > \xi_{max}$, then we introduce M agents into the pessimist state, while if $\xi < \xi_{min}$, then we introduce M agents into the optimist state. A similar approach was proposed in [11], where agents using random trading strategy were used to calm the markets.

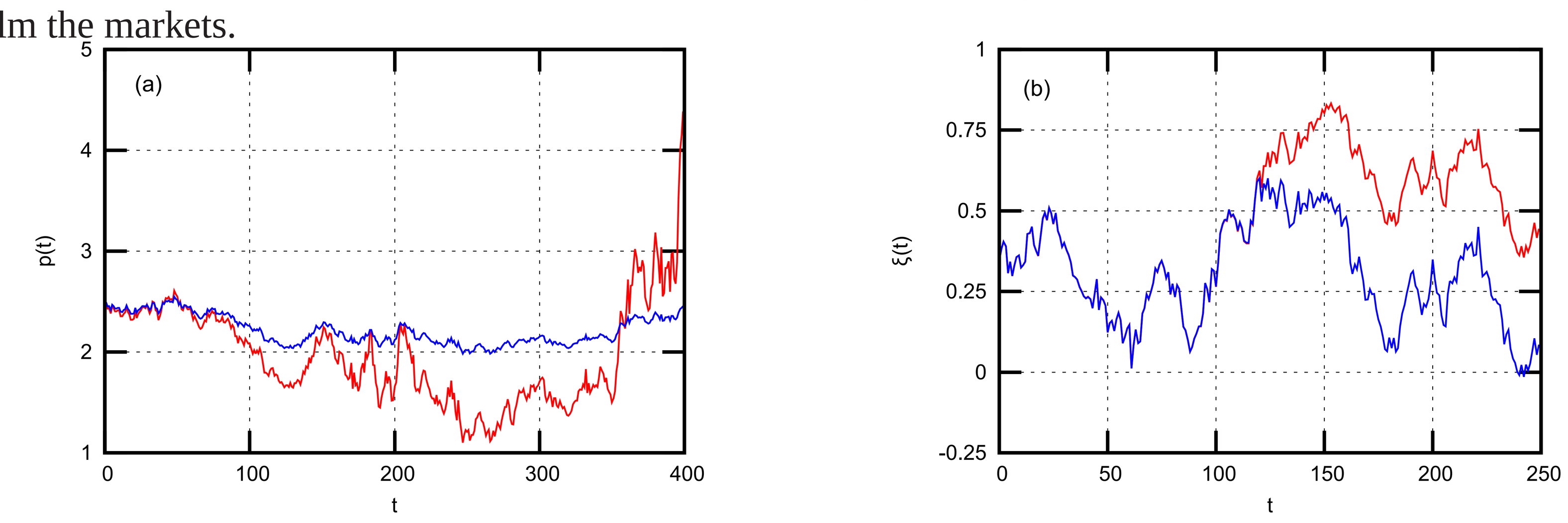

Figure 3: Original time series (red curves) and the same time series under "moderation" (blue curves). In the subfigure (a) we plot price time series obtained while using a model for y, while in the subfigure (b) we plot the chartist mood, ξ, time series. Model for y parameters were set as follows: $\sigma_1 = 0.1$, $\sigma_2 = 1$, $h = 10^{-5}$, $\alpha = 1$, $y_{lim} = 0.3$, $M = 10$. Model for ξ parameters were set as follows: $\sigma_{op} = \sigma_{po} = 1$, $h = 10^{-5}$, $\xi_{min} = -\xi_{max} = -0.5$, $M = 10$.

Note that in the figure above we see a large impact of the proposed simple policies carried out by small fixed number of agents, $M = 10$, onto the infinitely sized systems, $N \rightarrow \infty$. On the left we see that price time series become significantly smoother, while using y "moderation" policy. On the right we see that the policy does not allow for mood to reach extreme values and is effectively constrained in the $[\xi_{min}, \xi_{max}]$ range.

Conclusions

In this contribution we have approached modeling of the leadership phenomenon in complex socio-economic systems. Namely, we have modified a well known agent-based herding model, originally introduced in [5], to include agents, whose state "inflexible" and is preset by us. The control over a small fixed number of the agents enabled us to significantly influence the behavior of the other agents, who still act based on the original rules of the model.

We find our model setup well-backed with the related experiments in the behavioral sociology and behavioral biology carried out in [12, 13]. We believe that the presented imagery of society while being simplistic still reflects certain general features of social life - ordinary people usually interact with their "neighborhood" (i.e. friends, coworkers and other acquaintances), while "leaders" are usually well known and frequently seen by broader society.

In this contribution we keep our imagery of financial markets simplistic, but it is already able to reproduce some of the results obtained from using other models for the financial markets (e.g., [11]). In future we plan to work further in this direction by considering a more sophisticated herding model of financial markets (e.g. three group modeling [10]).

References

[1] G. Akerlof, J. Shiller, *Animal Spirits: How Human Psychology Drives the Economy, and Why It Matters for Global Capitalism* (Princeton University Press, 2009).

[2] J. P. Bouchaud, Crises and collective socio-economic phenomena: Simple models and challenges, Journal of Statistical Physics **151**, 567-606 (2013).

[3] R. Conte et al., Manifesto of computational social science, European Physics Journal Special Topics **214**, 325-346 (2012).

[4] M. Cristelli et al., Critical Overview of Agent-Based Models for Economics, Proceedings of the School of Physics "E. Fermi" **CLXXVI**, 235 - 282 (2012).

[5] A. P. Kirman, Ants, rationality and recruitment, Quarterly Journal of Economics **108**, 137-156 (1993).

[6] A. Kononovicius, V. Daniunas, Agent-based and macroscopic modeling of the complex socio-economic systems, Social Technologies **3**, 85-103 (2013).

[7] S. Galam, F. Jacobs, The role of inflexible minorities in the breaking of democratic opinion dynamics, Physica A **381**, 366 - 376 (2007).

[8] N. G. van Kampen, *Stochastic process in Physics and Chemistry* (North Holland, Amsterdam, 2007).

[9] A. Kononovicius, V. Gontis, Agent based reasoning for the non-linear stochastic models of long-range memory, Physica A **391**, 1309-1314 (2012).

[10] A. Kononovicius, V. Gontis, Three state herding model of the financial markets, EPL **101**, 28001 (2013).

[11] A. E. Biondo et al., Stopping Financial Avalanches By Random Trading, Physical Review E **88**, 062814 (2013).

[12] J. R. G. Dyer et al., Leadership, consensus decision making and collective behaviour in humans, Philosophical Transactions of the Royal Society B **364**, 781-789 (2009).

[13] J. Krause et al., Interactive robots in experimental biology, Trends in Ecology and Evolution **26**, 369-375 (2011).