Controlling the collective behavior in the agent-based herding model

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Abstract

The characteristic feature of the complex socio-economic systems is a tight coupling of the constituent parts. Social cooperation, spontaneous emergence, formation of financial bubbles, financial flash-crashes and even mass panic actually may be a result of this coupling and also certain general gimmicks of human psychology [1, 2]. In this context we can see the individuals (or firms or other socio-economic entities) as generalized agents, which are tightly coupled with other agents via the herding interactions [3, 4]. Previous empirical research, from a point of view of the behavioral biology and sociology (see recent papers by Jens Krauze [5, 6]), has shown that one can use the tight coupling to control the collective behavior of large groups of individuals. In this contribution we approach the same problem from an agent-based modeling point of view. Namely, we study the dynamics of the agent-based herding model, original proposed in [7], in which certain agents are controlled externally.

Agent-based herding model

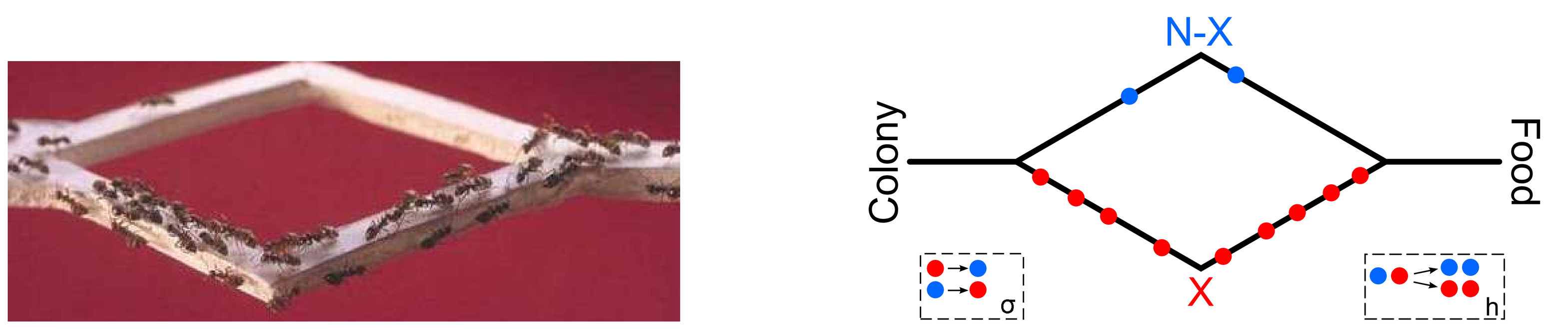

Figure 1: A photo of experiment (on the left) which inspired the agent-based herding model (schematic representation on the right). Here we have N ants, which take food to their colony. Ants may use one of the two available paths. Interestingly enough most of the time they tend to exploit only one path. This happens due to the importance of two-agent interactions, h terms, in comparison to a single-agent transitions, σ terms.

Mathematical definition of the transition probabilities requires a definition of two-agent interactions. If the agents are able to interact on the global scale, their transition probabilities will have the following form,

$$\mu(i \to j) = [\sigma_j + hX_j]\,\Delta t. \tag{1}$$

While if they are interacting only on the local scale (i.e., only with their direct neighbors), the transition probabilities take the following form,

$$\mu(i \to j) = \left[\sigma_j + \frac{h}{N} X_j\right] \Delta t. \tag{2}$$

Here i and j are indices representing the available states, Δt is a small time period (small enough for one transition to be probable).

Controlled agents and their impact

Let consider the impact of individual agents on system at a global level. It should be evident at agents acting on the local scale influence their immediate neighbors, which may (or may not) spread the control further. If system is large enough then at a certain point the influence of such agents will be stopped from spreading. On the other hand if the individual acts on the global scale, then it is seen by many agents at every time. Thus its influence spreads infinitely. So let us use controlled agents, with otherwise "inflexible" opinion (this concept was introduced in [8]), which interact on global scale, while other agents interact on local scale:

$$\mu(i \to j) = \left[\sigma_j + \frac{h}{N} X_j + hM_j\right] \Delta t, \tag{3}$$

where M_j is a number of controlled agents in the state j. Note that N no longer stands for a total number of agents in the system, now it represents a total number of other agents, which switch their opinion via endogenous interactions.

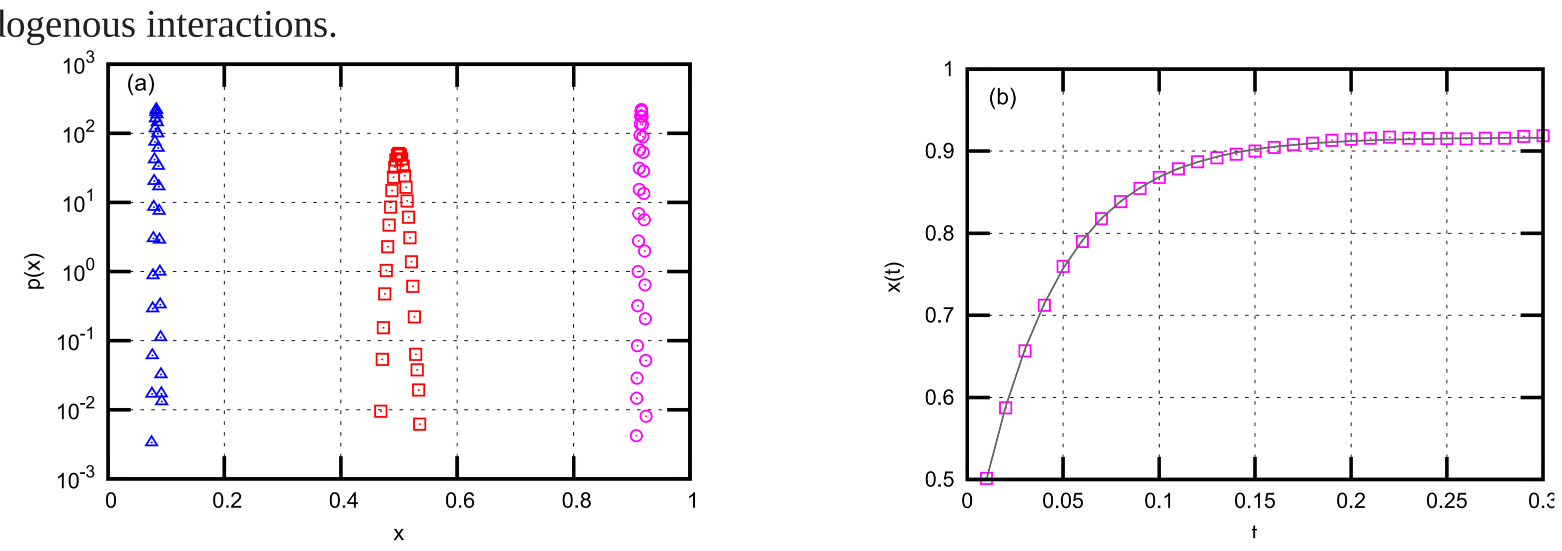

Figure 2: (a): A comparison of a numerically calculated stationary PDF with no controlled agents, $M_1 = 0$ and $M_2 = 0$ (red squares), and stationary PDF with controlled agents, $M_1 = 20$ and $M_2 = 0$ (magenta circles), $M_1 = 0$ and $M_2 = 20$ (blue triangles). (b): A convergence of agent-based model, $M_1 = 20$ and $M_2 = 0$, (magenta squares) versus the analytical prediction Eq. (5). Model parameters were set as follows: $\sigma_1 = \sigma_2 = 2$, $h = 1$, $N = 10^4$.

As you can see in the figure above an extremely small number of the controlled agents is able to make a significant impact onto the stationary distribution. This brings up an interesting question how fast is the convergence towards this stationary distribution. It is pretty straightforward (use the Master equation and one-step formalism [10]) to obtain ordinary differential equation describing the macroscopic dynamics of the considered agent-based system:

$$\dot{x} = (\sigma_1 + hM_1)(1 - x) - (\sigma_2 + hM_2)x. \tag{4}$$

The solution of this equation is an exponential function of time:

$$x(t) = \bar{x} + [x(0) - \bar{x}]e^{-[h(M_1+M_2)+\sigma_1+\sigma_2]t}. \tag{5}$$

Thus the convergence is exponential fast with the rate dependent on the individual transition rates, σ_i, and amount of controlled agents in each state, M_i. In the above $\bar{x}$ is the stationary point:

$$(\sigma_1 + hM_1)(1 - \bar{x}) - (\sigma_2 + hM_2)\bar{x} = 0, \quad \Rightarrow \quad \bar{x} = \frac{hM_1 + \sigma_1}{h(M_1 + M_2) + \sigma_1 + \sigma_2}. \tag{6}$$

Similar dynamics are observed if the ordinary agents are also interacting on the global scale [9]. The only difference is that the system dynamics are significantly more random and the stationary PDF of the model becomes power-law instead of Gaussian.

Controlling catastrophic events in the financial markets

Unlike in the society, in the financial markets any trader may make transactions with any other trader. Thus in the financial market model all of the agents should also interact on the global scale. The macroscopic dynamics of the financial markets will no longer be given by the ODE, one needs to use stochastic calculus [11, 12]. As our previous works have shown the trading in financial markets has two separate time scales - a slow process representing fundamentalist-chartist switching and a fast process representing optimism-pessimism mood fluctuations.

A slow process is a stochastic process based on modulating return, y, which is defined as a ratio between agents using chartist trading strategies and agents using fundamental trading strategies [11]. SDE for y has the following form:

$$\mathrm{d}y = \left[\sigma_1 + (2 - \sigma_2)y^{1+\alpha}\right](1 + y)\mathrm{d}t + \sqrt{2hy^{1+\alpha}}(1 + y)\mathrm{d}W. \tag{7}$$

The stationary probability density function of y is a power-law, $p(y) \sim y^{-\varepsilon_2-\alpha-1}$, thus large crashes and bubbles become probable. In order to prevent them we might use a simple rule - if absolute return y is larger than y_{lim} we introduce M agents trading based on the fundamentals.

Another possibility to prevent catastrophic events is to control the fast mood process, macroscopic dynamics of which are given by

$$\mathrm{d}\xi = \left[(1 - \xi)\sigma_{po} - (1 + \xi)\sigma_{op}\right]\mathrm{d}t + \sqrt{2h\left(1 - \xi^2\right)}\mathrm{d}W. \tag{8}$$

Let us "moderate" the time series of mood, ξ, by a similar rule as in case with y: if $\xi > \xi_{max}$, then we introduce M agents into the pessimist state, while if $\xi < \xi_{min}$, then we introduce M agents into the optimist state. A similar approach was proposed in [13], where agents using random trading strategy were used to calm the markets.

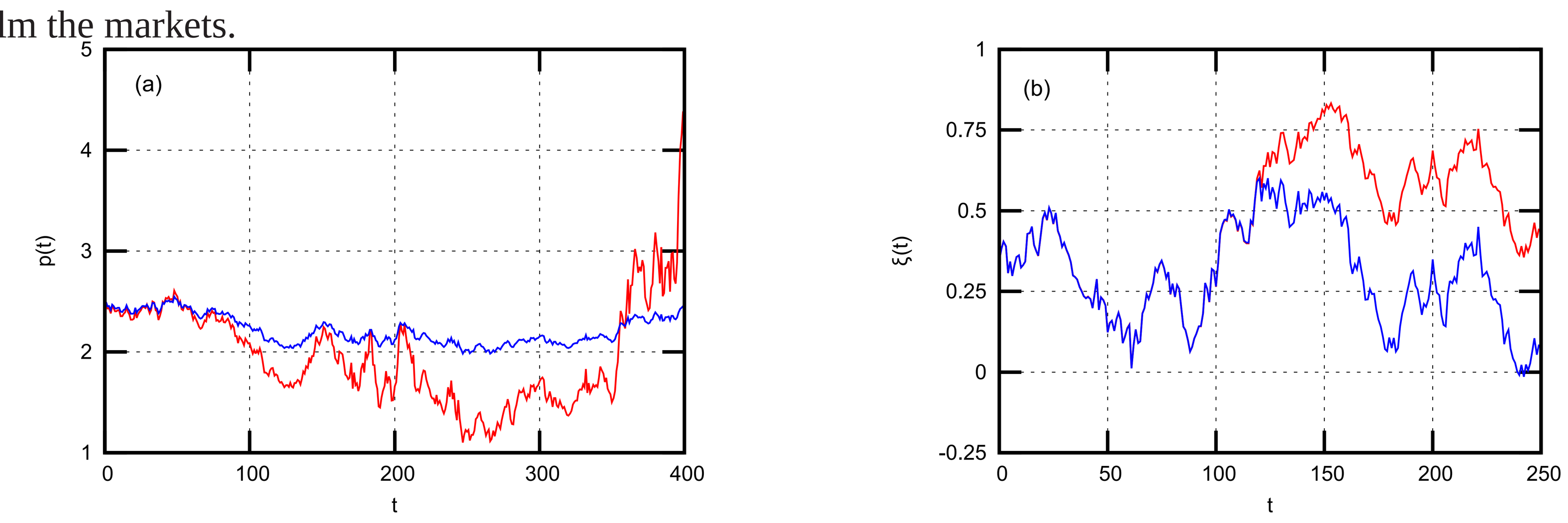

Figure 3: Original time series (red curves) and the same time series under "moderation" (blue curves). In the subfigure (a) we plot price time series obtained while using a model for y, while in the subfigure (b) we plot the chartist mood, ξ, time series. Model for y parameters were set as follows: $\sigma_1 = 0.1$, $\sigma_2 = 1$, $h = 10^{-5}$, $\alpha = 1$, $y_{lim} = 0.3$, $M = 10$. Model for ξ parameters were set as follows: $\sigma_{op} = \sigma_{po} = 1$, $h = 10^{-5}$, $\xi_{min} = -\xi_{max} = -0.5$, $M = 10$.

Note that in the figure above we see a large impact of the proposed simple policies carried out by small fixed number of agents, $M = 10$, onto the infinitely sized systems, $N \to \infty$. On the left we see that price time series become significantly smoother, while using y "moderation" policy. On the right we see that the policy does not allow for mood to reach extreme values and is effectively constrained in the $[\xi_{min}, \xi_{max}]$ range.

Conclusions

In this contribution we have approached modeling of the control of complex socio-economic systems. Namely, we have modified a well known agent-based herding model, originally introduced in [7], to include agents, whose state "inflexible" and is preset by us. The control over a small fixed number of the agents enabled us to significantly influence the behavior of the other agents, who still act based on the original rules of the model.

We find our model setup well-backed with the related experiments in the behavioral sociology and behavioral biology carried out in [5, 6]. We believe that the presented imagery of society while being simplistic still reflects certain general features of social life - ordinary people usually interact with their "neighborhood" (i.e. friends, coworkers and other acquaintances), while "leaders" are usually well known and frequently seen by broader society.

In this contribution we keep our imagery of financial markets simplistic, but it is already able to reproduce some of the results obtained from using other models for the financial markets (e.g., [13]). In future we plan to work further in this direction by considering a more sophisticated herding model of financial markets (e.g. three group modeling [12]).

Part of the research covered in this contribution was published in [9]. Some of the new material will be used in future research and scientific publications.

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