Agent-based and macroscopic modeling of the complex socio-economic systems

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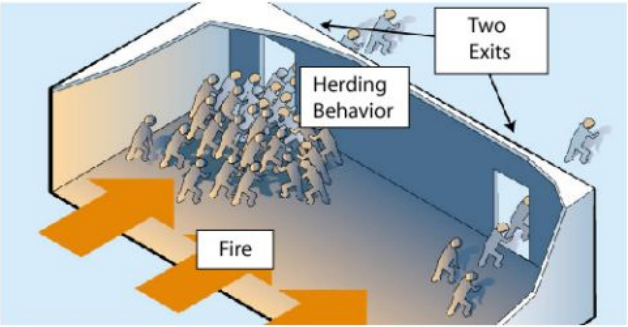
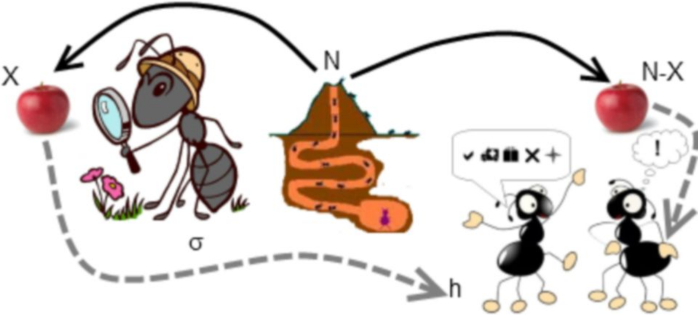
Herding behavior in social communities



Kirman's agent-based herding model



Each ant (=agent) at each time step

can switch the food source (=state) probabilistically:

$$\mu_2 = \sigma_1 + hX, \quad \mu_1 = \sigma_2 + h(N - X).$$

One-step, birth-death, process formalism

Let us assume that

the time step, Δt, is small enough for only one switch to be probable:

$$P(X \to X-1) = X\mu_1\Delta t, \quad P(X \to X+1) = (N-X)\mu_2\Delta t.$$

In this simple case we can derive

a macroscopic, stochastic, model. General form is given by:

$$\mathrm{d}x = \left[(1-x)\mu_2 - x\mu_1\right]\mathrm{d}t + \sqrt{\frac{(1-x)\mu_2 + x\mu_1}{N}}\mathrm{d}W.$$

While for the Kirman's model the exact form is given by:

$$\mathrm{d}x = \left[(1-x)\sigma_1 - x\sigma_2\right]\mathrm{d}t + \sqrt{2hx(1-x)}\mathrm{d}W.$$

ABM vs SDE

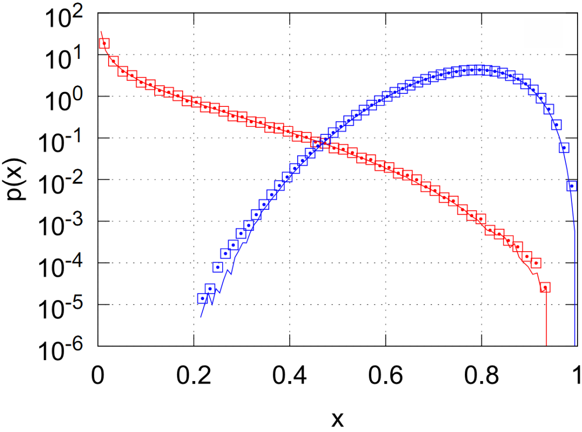


Figure: The probability density functions obtained from the macroscopic (curves) and the agent-based models (boxes) in two distinct cases. Model parameters: $\sigma_1 = 0.2$ and $\sigma_2 = 5$ (red), $\sigma_1 = 16$ and $\sigma_2 = 5$ (blue), $h = 1$ and $N = 1000$ (in all cases).

The Bass diffusion model

Let us now use the following one-agent switching probabilities:

$$\mu_2 = \sigma + \frac{h}{N} X, \quad \mu_1 = 0, \quad \Rightarrow \quad \frac{\mathrm{d}X}{\mathrm{d}t} = (N - X)\left(\sigma + \frac{h}{N} X\right).$$

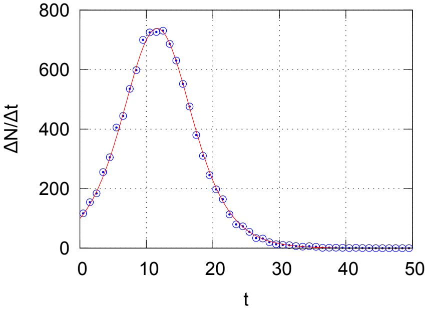


Figure: The agent-based model (circles) vs the Bass diffusion model (curve).

Global vs local interactions

Previously we have used

$$\mu_2 = \sigma_1 + hX, \quad \mu_1 = \sigma_2 + h(N - X), \qquad \text{(red)}$$

$$\mu_2 = \sigma_1 + \frac{h}{N}X, \quad \mu_1 = \sigma_2 + \frac{h}{N}(N - X). \qquad \text{(blue)}$$

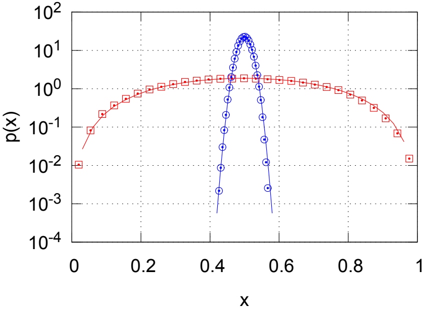


Figure: Implications of the global (red) and local (blue) interactions.

Leadership in social communities

Let us now use the following one-agent switching probabilities:

$$\mu_2 = \sigma_1 + h(M + X), \quad \mu_1 = \sigma_2 + h(N - X).$$

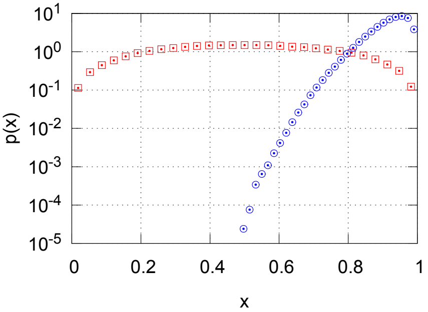


Figure: No leaders, $M = 0$, (red) and small fraction (2%) of leaders, $M = 20$, (blue).

Agent-based herding model and predator-prey model

Predator-prey model is given by

$$\frac{\mathrm{d}X_i}{\mathrm{d}t} = \left[a_i X_i - X_i \sum_j c_{ij} X_j \right], \quad \forall i.$$

The key differences are:

- the herding behavior is asymmetric in the predator-prey model,
- system size might not be fixed in the predator-prey model.

The comparison leads to:

$$\mathrm{d}x = \left[(n-x)\sigma_1 - x\sigma_2 + cx(n-x) + T_1(x,n) \right] \mathrm{d}t + \sqrt{2hx(n-x)}\mathrm{d}W,$$

$$\mathrm{d}n = \left[T_1(x,n) + T_2(x,n) \right] \mathrm{d}t.$$

In the simplest case $T_i(x,n) = T_i = a_i$.

We can assume that

agents follow chartist trading strategy,

$$D_c = r_0 N_c(t)\xi(t),$$

or a fundamentalist approach,

$$D_f = N_f(t)[\ln P_f - \ln P(t)].$$

The Walrassian scenario provides us with the absolute return, y,

$$\frac{1}{P}\frac{\mathrm{d}P}{\mathrm{d}t} = \beta[D_c + D_f], \quad p = \ln\left(\frac{P}{P_f}\right) = r_0\frac{x}{1-x}\xi = r_0 y\xi,$$

$$r_\tau(t) = p(t) - p(t-\tau) \approx r_0 y\eta, \quad \eta = \xi(t) - \xi(t-\tau).$$

Financial markets II

The macroscopic model for y is given by

$$\mathrm{d}y = \left[\varepsilon_1 + (2 - \varepsilon_2) y^{1+\alpha}\right](1+y)\mathrm{d}t_s + \sqrt{2y^{1+\alpha}}(1+y)\mathrm{d}W_s.$$

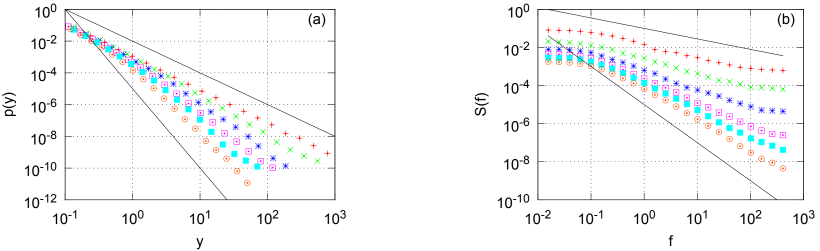


Figure: Wide spectra of obtainable probability (a) and spectral (b) density functions of absolute return, y.

Thank you!

http://mokslasplius.lt/rizikos-fizika/en/