Bursting behavior of the non-linear stochastic models applicable to the financial markets

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Stochastic models for the financial markets

gCEV process,

$$\mathrm{d}x = \mu x \mathrm{d}t + \sigma x^{\eta} \mathrm{d}W,$$

is considered by the mainstream economists and mathematicians with $\eta < 1$. It possesses many desired features such as "fairness" and non-explosiveness.

Yet we think that the markets tend to explode,

namely that the case of $\eta > 1$ must be considered in order to efficiently model financial markets. We propose to use a general class of SDE:

$$\mathrm{d}x = \left(\eta - \frac{\lambda}{2}\right) x^{2\eta-1} \mathrm{d}t + x^{\eta} \mathrm{d}W, \quad \eta > 1,$$

to model absolute returns and trading activity in the financial markets.

General class of SDE

Usefulness of the proposed class

is not limited to the financial markets. It can be applied to any systems exhibiting power-law statistical properties:

$$S(f) \sim 1/f^{\beta}, \; \beta = 1 + \frac{\lambda - 3}{2(\eta - 1)}, \quad p(x) \sim x^{-\lambda}.$$

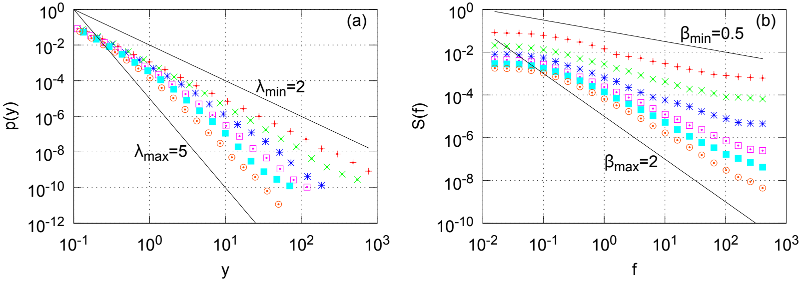

Figure PDF and PSD of the SDE with $\eta = 2$.

Explosions are just "bursts"

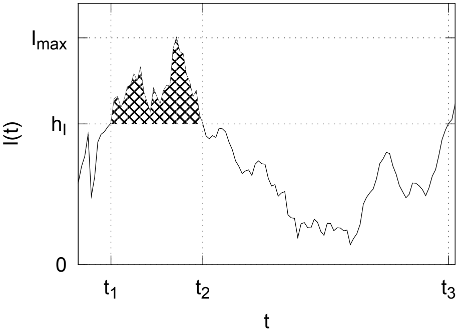

Figure Time series exhibiting bursty behavior, $I(t)$. Here h_I is threshold value, above which bursts are detected, t_i is the three visible threshold passage events. Which are used to define burst duration, $T = t_2 - t_1$, inter-burst time, $\theta = t_3 - t_2$, and waiting time, $\tau = t_3 - t_1$. The burst's peak value is measured as $I'_{max} = I_{max} - h_I$. The highlighted area is defined as the size of the burst, S.

Analytical treatment of the bursting behavior

Some are trivial, or clearly dependent on others, e.g.

it should be evident that the PDF for I'_{max} is given by:

$$p^{(max)}(I'_{max}) \sim p^{(I)}(I'_{max} + h_I),$$

or the PDF for τ:

$$p^{(\tau)}(\tau) = p^{(\tau)}(\theta + T) \sim p^{(\theta)}(\theta) * p^{(T)}(T),$$

or size:

$$S \propto T \cdot I'_{max}.$$

The others need to be treated in detail.

Burst duration PDF, $p(T)$, and inter-burst times PDF, $p(\theta)$, might be obtained via the **hitting time framework**.

Obtaining burst duration PDF, $p(T)$

There are some issues:

- The general class of SDE is highly non-linear.
- The PDF of hitting times, $\rho(T)$, of a stochastic process, $y(t)$, starting on the boundary, $y_0 \to h_y$, is $\delta(T)$.
- The $\rho(T)$ involves $\sum_{i=1}^{\infty}$ over Bessel-J zeros.

This yields a PDF with the following asymptotic behavior

$p(T) \sim T^{-3/2}$ for $T \ll T_c$ and $p(T) \sim \frac{\exp(-\alpha T)}{T}$ for $T \gg T_c$.

$p(T)$ for the general class of SDE

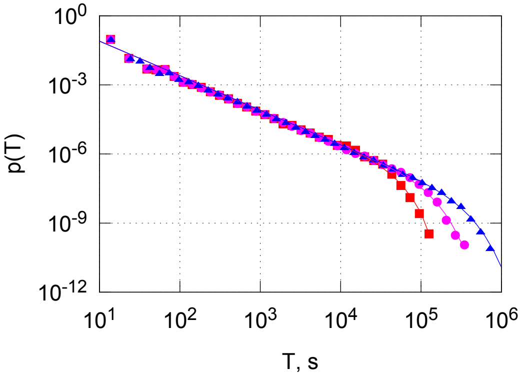

Figure $p(T)$ of the time series obtained by solving the general class of SDE (points) by the analytically obtained $p(T)$ (lines). Squares: $\eta = 2.5$; circles: $\eta = 2$; triangles: $\eta = 1.5$.

$p(T)$ for the absolute return

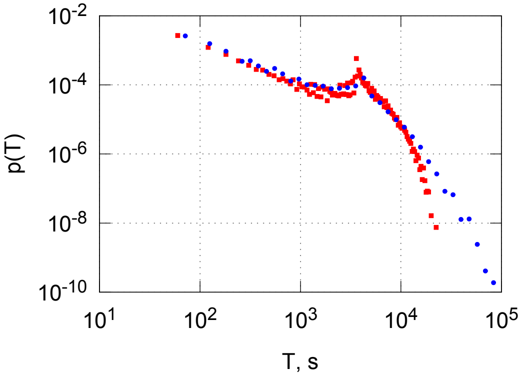

Figure $p(T)$ of the empirical absolute one minute return (blue circles) and a sophisticated model of the absolute return (red squares). The parameters of the general class of SDE and a sophisticated model are the same.

$p(T)$ for the trading activity

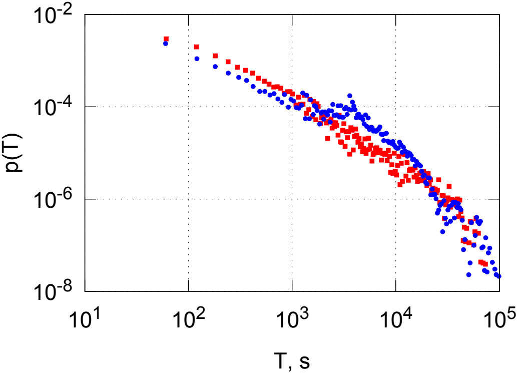

Figure $p(T)$ of the empirical trading activity, deals per one minute, (blue circles) and a sophisticated model of the trading activity (red squares).

Scaling of the burst geometry of the absolute return

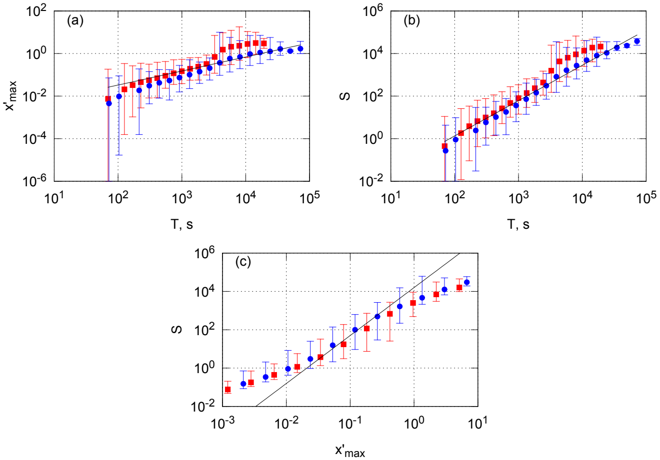

Figure The empirical (red squares) and the absolute return model's (blue circles) scatter plots of the burst related observables. Black lines provide power-law fits: $\alpha_{t-max} = 0.66$, $\alpha_{t-s} = 1.66$, $\alpha_{max-s} = 2.5$.

Scaling of the burst geometry of the trading activity

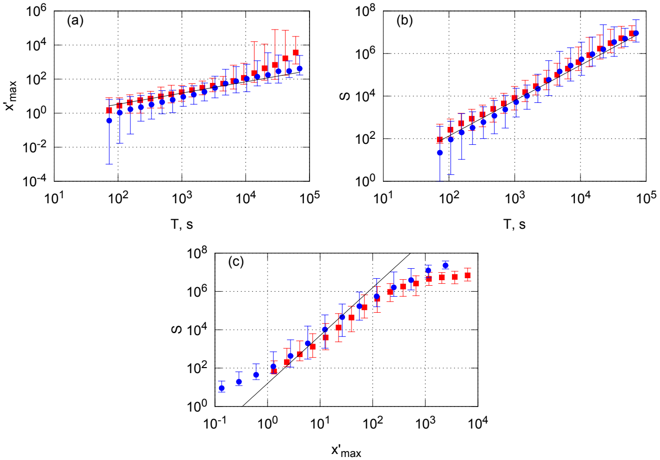

Figure The empirical (red squares) and the trading activity model's (blue circles) scatter plots of the burst related observables. Black lines provide power-law fits: $\alpha_{t-max} = 0.66$, $\alpha_{t-s} = 1.66$, $\alpha_{max-s} = 2.5$.

To conclude...

We have shown that:

- even "explosive", $\eta > 1$, stochastic models can be treated analytically to obtain their bursting behavior,
- burst duration PDF is power-law, with power of -1.5, for $T < T_c$, with exponential cutoff for $T > T_c$,
- the geometry of the bursts scales as a power-law,
- empirical bursting behavior is well reproduce by the considered models.

Future plans include:

- empirical and analytical study of the inter-burst statistics.
- analytical treatment of the "dependent" observables.

Thank you for your attention!

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