Herding behavior of agents as a background of financial fluctuations

Aleksejus Kononovicius, V. Gontis, B. Kaulakys

Institute of Theoretical Physics and Astronomy, Vilnius University
aleksejus.kononovicius@gmail.com
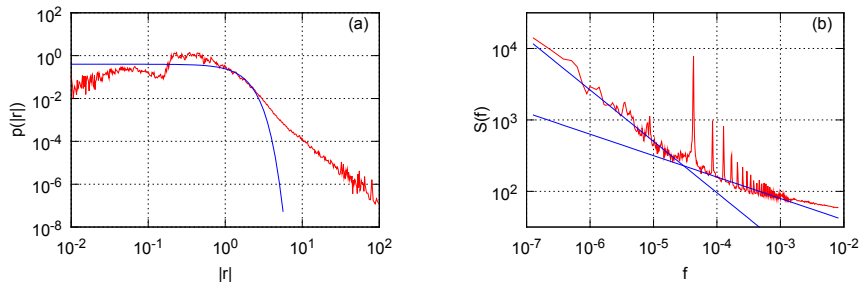
2012-07-09

Financial fluctuations

Figure: Statistical properties of the absolute one minute returns in the financial markets: (a) probability and (b) spectral density. Similar statistical properties are also observed for the trading activity.

Frequent large deviations (fat tails), long-range memory (power law spectral density and thus, due to W-K theorem, auto-correlation).
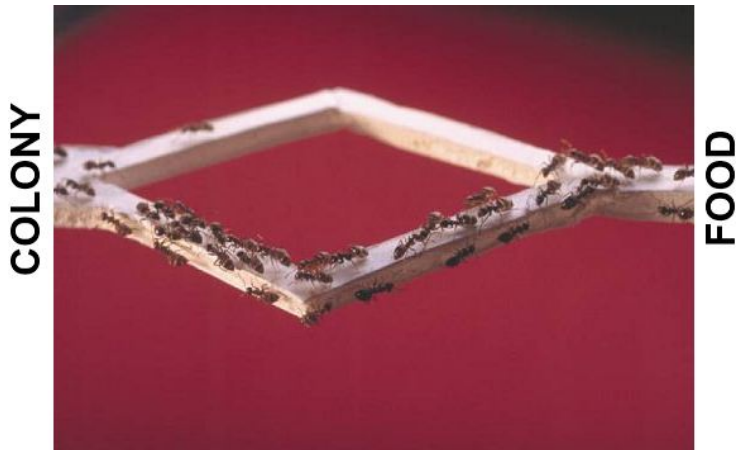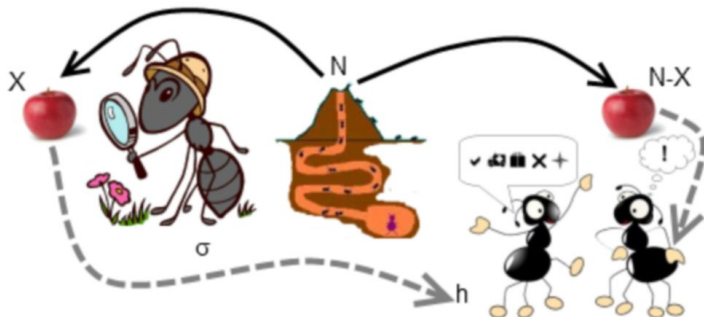
Taken from [Detrain & Deneubourg, 2006 (Physics of Life Reviews)].

Kirman's formulation of herding model

X dynamics

are determined by the one-step transition probabilities:

$$
\begin{aligned}
p(X \to X+1) &= (N-X)\sigma_1 &+& hX(N-X), \\
p(X \to X-1) &= X\sigma_2 &+& hX(N-X).
\end{aligned}
$$

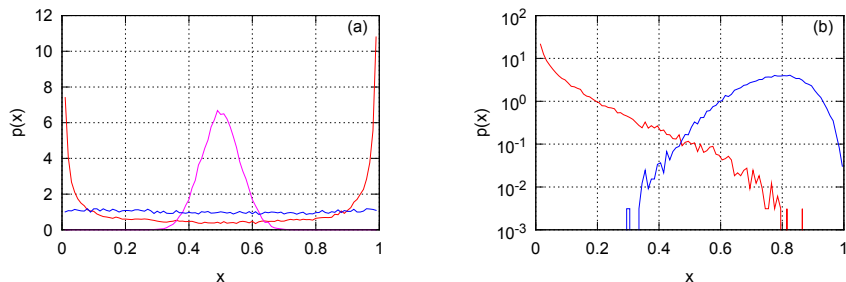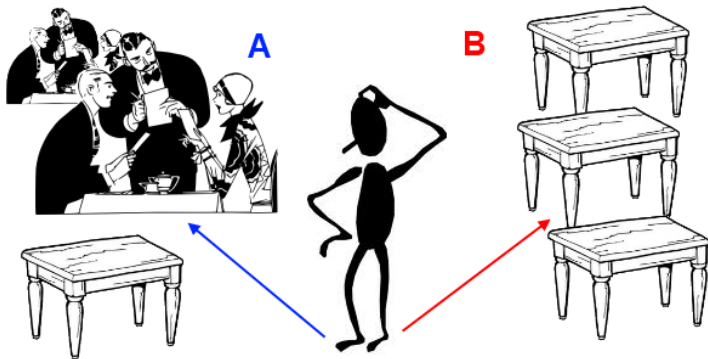# Statistical properties of Kirman's herding model

Figure: Probability density functions of $x = X/N$ in the Kirman's herding model: (a) $\sigma = \sigma_1 = \sigma_2$, (b) $\sigma_1 \neq \sigma_2$. (a) $\sigma < h$ (red curve), $\sigma = h$ (blue curve), $\sigma > h$ (magenta curve). (b) $h > \sigma_1 > \sigma_2$ (red curve), $h < \sigma_1 < \sigma_2$ (blue curve).

A

B

Which restaurant would You choose?

Experimentally it was determined that most people choose restaurant which has **more** visitors!
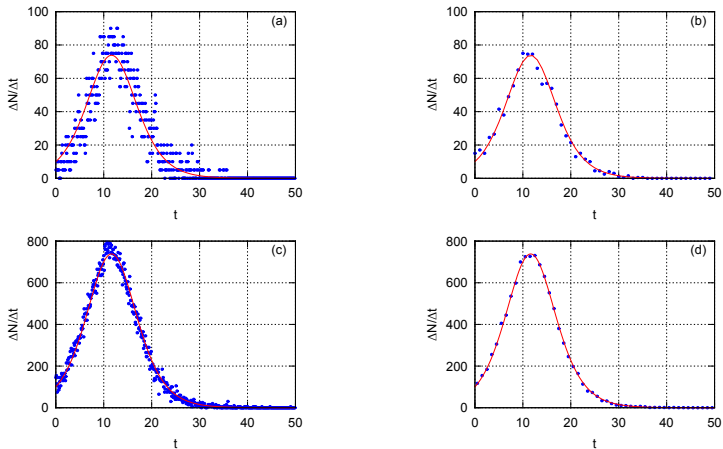
Model of human behavior? (vs Bass diffusion model)

Figure: Incoming new consumers: Bass diffusion model (red curve) againt agent based simulation using Kirman herding model (blue curves). Comparison is made for the different number of agents in the system and different discretization periods.

Defining returns

One can assume that

the two states in the population dynamics correspond to the chartist trading strategy, excess demand given by

$$D_c = -r_0 X(t)\xi(t),$$

and fundamentalist trading strategy,

$$D_f = [N - X(t)] \ln \frac{P_f}{P(t)}.$$

If market is quickly stabilized,

$$D_f + D_c = 0,$$

$$r(t) \approx r_0 \frac{X(t)}{N - X(t)} \Delta\xi(t).$$

Stochastic model for the modulating return

Stochastic model,

explicitly derived from the previous ABM (one can use birth-death process formalism), for $y = \frac{X}{N-X}$ is given by:

$$\mathrm{d}y = \left[\varepsilon_1 + y\frac{2 - \varepsilon_2}{\tau(y)}\right](1 + y)\mathrm{d}t_s + \sqrt{\frac{2y}{\tau(y)}}(1 + y)\mathrm{d}W_s,$$

$$\sigma_2 \to \frac{\sigma_2}{\tau(y)}, \quad h \to \frac{h}{\tau(y)},$$

$$\varepsilon_i = \frac{\sigma_i}{h}, \quad t_s = ht.$$

Comparison with other stochastic processes I

The SDE for $y \gg 1$ and assuming that $\tau(y) \sim y^{-\alpha}$ becomes:

$$\mathrm{d}y = (2 - \varepsilon_2)y^{2+\alpha}\mathrm{d}t_s + \sqrt{2}y^{\frac{3+\alpha}{2}}\mathrm{d}W_s.$$

Which has identical form as the general class of SDE reproducing power law statistics (PDF and PSD) [Ruseckas et al., 2011 (Phys. Rev. E)]:

$$\mathrm{d}x = \left(\eta - \frac{\lambda}{2}\right) x^{2\eta-1}\mathrm{d}t_s + x^{\eta}\mathrm{d}W_s.$$

Comparison yields $\eta = \frac{3+\alpha}{2}$, $\lambda = \varepsilon_2 + \alpha + 1$. Thus we can expect that:

$$p(y) \sim y^{-\varepsilon_2-\alpha-1}, \quad S(f) \sim f^{-\beta}, \; \beta = 1 + \frac{\varepsilon_2 + \alpha - 2}{1 + \alpha}.$$
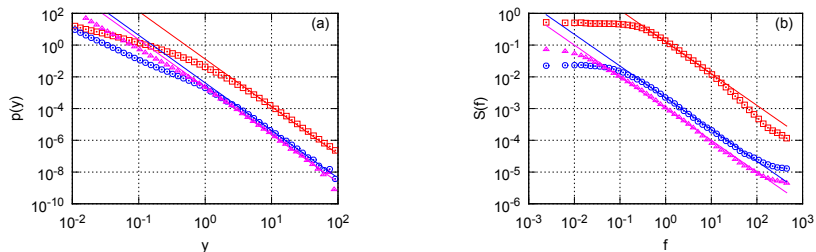
Figure: Reproducing $1/f$ noise in three cases, $\alpha = 0$ (red squares), $\alpha = 1$ (blue circles) and $\alpha = 2$ (magenta triangles). Other model parameters were set as follows: $\varepsilon_1 = 0.1$, $\varepsilon_2 = 2 - \alpha$. All model data are fitted by: (a) $\lambda = 3$, (b) $\beta = 1$.

Comparison with other stochastic processes II

The aforementioned general class of SDE

is interesting as it incorporates several widely known stochastic processes into it:

- **Bessel process** ($\eta = 0$): $\mathrm{d}x = -\frac{\lambda}{2x}\mathrm{d}t + \mathrm{d}W$.
- **Squared Bessel process** ($\eta = 0.5$): $\mathrm{d}x = \frac{1-\lambda}{2}\mathrm{d}t + \sqrt{x}\mathrm{d}W$.
- **CIR process** ($\eta = 0.5$, $m = 1$, $x_{min} = 0$, $x_{max} = 1$; the m-exponential diffusion restriction must be applied): $\mathrm{d}x = \frac{1-\lambda-x}{2}\mathrm{d}t + \sqrt{x}\mathrm{d}W$.
- **CEV process** ($\eta = \lambda/2$, $m = 2\eta - 2$, $x_{min} = 1$, $x_{max} \to \infty$; the m-exponential diffusion restriction must be applied): $\mathrm{d}x = (\eta - 1)x\mathrm{d}t + x^\eta \mathrm{d}W$.

The same processes should be also reflected by the ABM. Note that CEV and CIR processes can be obtained from the ABM by assuming that $\varepsilon_2 = 2$.
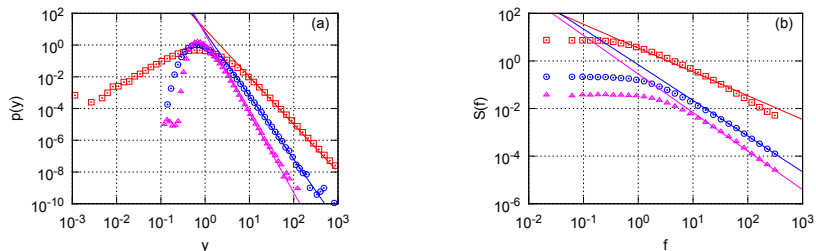
CEV-like case

Figure: Numerical results obtained from the CEV-like case, $\alpha = 0$ (red squares), $\alpha = 1$ (blue circles) and $\alpha = 2$ (magenta triangles). Other model parameters were set as follows: $\varepsilon_1 = \varepsilon_2 = 2$.

$$\mathrm{d}y = \varepsilon_1 y \mathrm{d}t_s + \sqrt{2} y^{\frac{3+\alpha}{2}} \mathrm{d}W_s, \ \ p(y) \sim y^{-3-\alpha}, \ S(f) \sim f^{-1-\frac{\alpha}{1+\alpha}}.$$
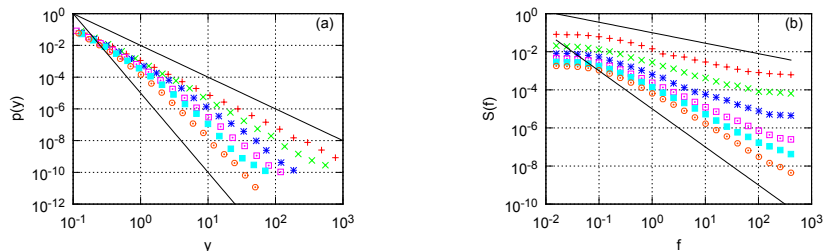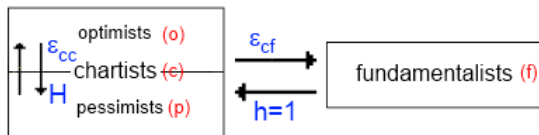
Figure: Wide spectra of obtainable λ and β values. Model parameters were set as follows: $\alpha = 1$, $\varepsilon_1 = 0.1$, $\varepsilon_2 = 0.1$ (red plus), 0.5 (green cross), 1 (blue stars), 1.5 (magenta open squares), 2 (cyan filled squares) and 3 (orange open circles). Black curves correspond to the limiting cases: (a) $\lambda_1 = 2$ and $\lambda_2 = 5$, (b) $\beta_1 = 0.5$, $\beta_2 = 2$

Formulation of the three group agent based model

Transition probabilities for $\vec{X} = \{N_f, N_p\}$,

having in mind that $N_o = N - N_f - N_p$:

$$p(\{N_f, N_p\} \to \{N_f - 1, N_p\}) = N_f(\varepsilon_{cf}/2 + N_o)\Delta t_s,$$
$$p(\{N_f, N_p\} \to \{N_f + 1, N_p\}) = N_o(\varepsilon_{cf} + N_f)\Delta t_s,$$
$$p(\{N_f, N_p\} \to \{N_f - 1, N_p + 1\}) = N_f(\varepsilon_{cf}/2 + N_p)\Delta t_s,$$
$$p(\{N_f, N_p\} \to \{N_f + 1, N_p - 1\}) = N_p(\varepsilon_{cf} + N_f)\Delta t_s,$$
$$p(\{N_f, N_p\} \to \{N_f, N_p - 1\}) = N_p(\varepsilon_{cc} + HN_o)\Delta t_s,$$
$$p(\{N_f, N_p\} \to \{N_f, N_p + 1\}) = N_o(\varepsilon_{cc} + HN_p)\Delta t_s.$$

Stochastic three group model

Stochastic model for $\vec{X} = \{n_f, \xi = \frac{n_o - n_p}{1 - n_f}\}$:

$$\mathrm{d}n_f = \frac{1 - 2n_f}{\tau(n_f, \xi)} \varepsilon_{cf} \mathrm{d}t_s + \sqrt{\frac{2(1 - n_f)n_f}{\tau(n_f, \xi)}} \mathrm{d}W_{s,1},$$

$$\mathrm{d}\xi = -\frac{2H\varepsilon_{cc}\xi}{\tau(n_f, \xi)} \mathrm{d}t_s + \sqrt{\frac{2H(1 - \xi^2)}{\tau(n_f, \xi)}} \mathrm{d}W_{s,2}.$$

Similar equations were obtained in Evolutionary Game Theory from mathematical considerations see [Traulsen et al., 2012 (Phys. Rev. E)].
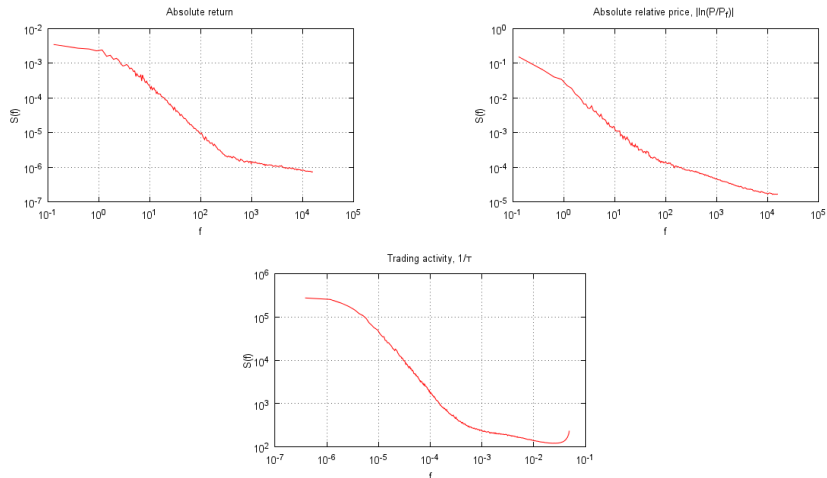
Fractured spectral density

Figure: Fractured spectral density of various observables. Yet obtained using different parameter sets.

Key points

- Nonlinear stochastic model possessing power law spectral density, $S(f) \sim 1/f^{\beta}$, can be obtained from a microscopic agent based model.
- The nonlinear herding terms in the transition probabilities are essential in reproduction of $1/f$ noise.
- Introducing variability of trading activity generalizes model and offers more modeling possibilities.
- Three group model is able to capture fractured spectral density, but still must be further developed

For further reference see [Kononovicius & Gontis, 2012 (Physica A)], [Ruseckas, Kaulakys & Gontis, 2011 (EPL)].

http://mokslasplius.lt/rizikos-fizika/en

Thank You!