Microscopic herding model leading to long-range processes and $1/f$ noise with application to absolute return in financial markets

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2012-03-29

$1/f$ (1-over-f), or flicker, or pink, noise

It's a type of noise

with spectral density, $S(f)$, behaving as

$$S(f) \sim 1/f^{\beta},$$

where $\beta \approx 1$. This type of noise is interesting as it lies in-between white noise and Brownian noise. Furthermore it is observed in a wide variety of **physical, biological, economic, social systems**.

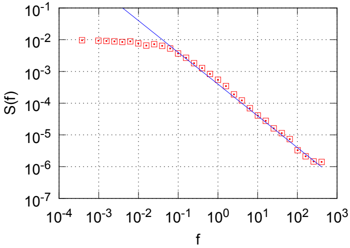
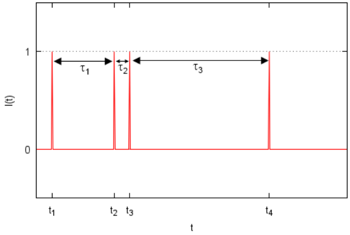


Figure: Sample spectral density, $S(f) \sim 1/f$.

Point process model of $1/f^\beta$ noise I: discrete case

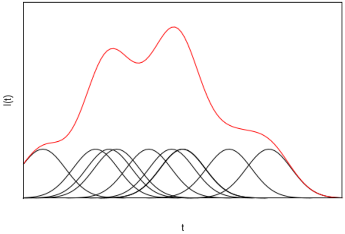


Iterative equation for the inter-event time:

$$\tau_{k+1} = \tau_k + \gamma \tau_k^{2\mu-1} + \sigma \tau_k^{\mu} \zeta_k, \quad S_\tau(f) \sim 1/f^\beta,\ \beta = 1 + \frac{2(\mu\sigma^2 - \gamma)}{\sigma^2(2\mu - 3)}.$$

Recently used to model musical rhythm [Levitin et al., 2012 (PNAS)].

Point process model of $1/f^\beta$ noise II: continuous case



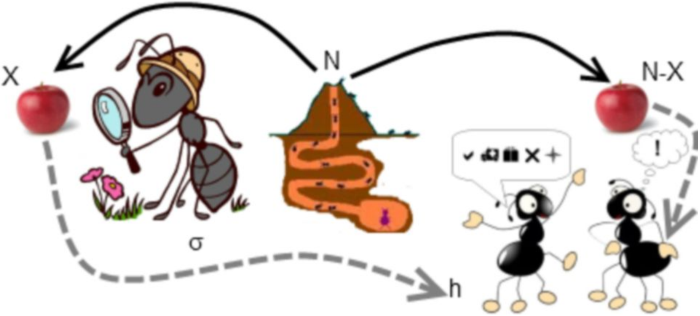
While SDE for signal intensity, x:

$$\mathrm{d}x = \left(\eta - \frac{\lambda}{2}\right) x^{2\eta-1}\mathrm{d}t_s + \sigma x^{\eta}\mathrm{d}W_s.$$

$$p(x) \sim x^{-\lambda}, \quad S_x(f) \sim 1/f^{\beta},\ \beta = 1 + \frac{\lambda - 3}{2(\eta - 1)}.$$

This model can be further extended to reproduce sophisticated behavior of the financial markets (see [Gontis et al., 2010 (Sciyo)]).

Kirman's herding model



X dynamics

are determined by the one-step transition probabilities:

$$p(X \to X+1) = (N-X)\sigma_1 + hX(N-X),$$
$$p(X \to X-1) = X\sigma_2 + hX(N-X).$$

Experiment by Deneubourg

Taken from [Detrain & Deneubourg, 2006 (Physics of Life Reviews)].

Towards herding model for the financial markets I

One can assume that

the two states in the population dynamics correspond to the chartist trading strategy, excess demand given by

$$D_c = -r_0 X(t) \xi(t),$$

and fundamentalist trading strategy,

$$D_f = [N - X(t)] \ln \frac{P_f}{P(t)}.$$

If market is stabilized,

$$D_f + D_c = 0,$$

$$r(t) \approx r_0 \frac{X(t)}{N - X(t)} \Delta \xi(t).$$

Towards herding model for the financial markets II

Stochastic model (explicitly derived from the previous ABM)

for $y = \frac{X}{N-X}$ is given by:

$$\mathrm{d}y = \left[\varepsilon_1 + y\frac{2-\varepsilon_2}{\tau(y)}\right](1+y)\mathrm{d}t_s + \sqrt{\frac{2y}{\tau(y)}}(1+y)\mathrm{d}W_s,$$

$$\sigma_2 \rightarrow \frac{\sigma_2}{\tau(y)}, \quad h \rightarrow \frac{h}{\tau(y)},$$

$$\varepsilon_i = \frac{\sigma_i}{h}, \quad t_s = ht.$$

Towards herding model for the financial markets III

The SDE for $y \gg 1$ and assuming that $\tau(y) \sim y^{-\alpha}$ becomes:

$$\mathrm{d}y = (2 - \varepsilon_2) y^{2+\alpha} \mathrm{d}t_s + \sqrt{2} y^{\frac{3+\alpha}{2}} \mathrm{d}W_s.$$

The comparison with

$$\mathrm{d}x = \left(\eta - \frac{\lambda}{2} \right) x^{2\eta - 1} \mathrm{d}t_s + x^{\eta} \mathrm{d}W_s,$$

yields: $\eta = \frac{3+\alpha}{2}$, $\lambda = \varepsilon_2 + \alpha + 1$. Thus we can expect that:

$$p(y) \sim y^{-\varepsilon_2 - \alpha - 1}, \quad S_y(f) \sim 1/f^{\beta},\, \beta = 1 + \frac{\varepsilon_2 + \alpha - 2}{1 + \alpha}.$$

Reproducing $1/f$ noise

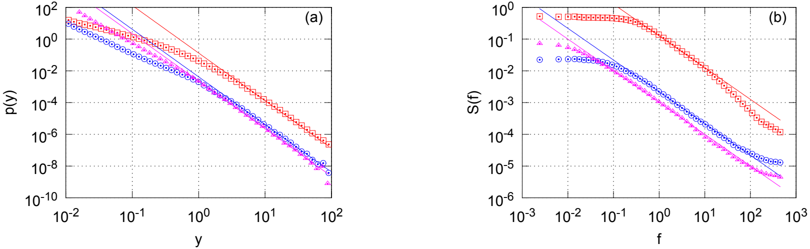


Figure: Reproducing $1/f$ noise in three cases, $\alpha = 0$ (red squares), $\alpha = 1$ (blue circles) and $\alpha = 2$ (magenta triangles). Other model parameters were set as follows: $\varepsilon_1 = 0.1$, $\varepsilon_2 = 2 - \alpha$. All model data are fitted by: (a) $\lambda = 3$, (b) $\beta = 1$.

The multifractal spectrum of the model

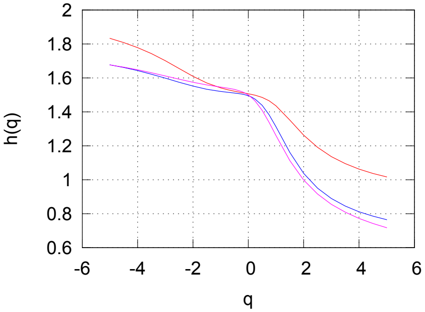


Figure: Checking for multifractality, using MF-DFA method, in the aforementioned distinct cases.

Note:

- $h(2) \approx 1$ for $\alpha = 1$ and $\alpha = 2$; $\alpha = 0$ case in the crossover region.
- model posses multifractality.

CEV-like case

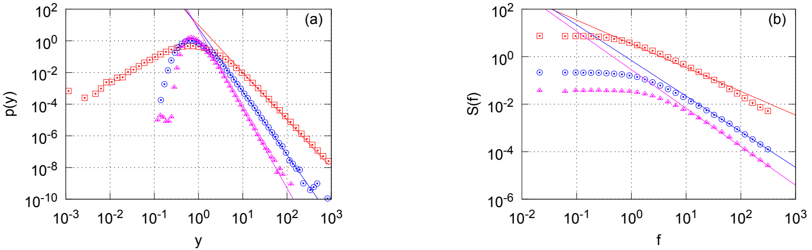


Figure: Numerical results obtained from the CEV-like case, $\alpha = 0$ (red squares), $\alpha = 1$ (blue circles) and $\alpha = 2$ (magenta triangles). Other model parameters were set as follows: $\varepsilon_1 = \varepsilon_2 = 2$.

$$\mathrm{d}y = \varepsilon_1 y \mathrm{d}t_s + \sqrt{2} y^{\frac{3+\alpha}{2}} \mathrm{d}W_s, \quad p(y) \sim y^{-3-\alpha}, \, S_y(f) \sim 1/f^{1+\frac{\alpha}{1+\alpha}}.$$

Variety of reproducible λ and β values

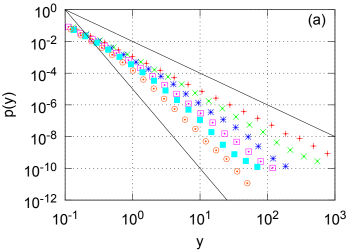


Figure: Wide spectra of obtainable λ and β values. Model parameters were set as follows: $\alpha = 1$, $\varepsilon_1 = 0.1$, $\varepsilon_2 = 0.1$ (red plus), 0.5 (green cross), 1 (blue stars), 1.5 (magenta open squares), 2 (cyan filled squares) and 3 (orange open circles). Black curves correspond to the limiting cases: (a) $\lambda_1 = 2$ and $\lambda_2 = 5$, (b) $\beta_1 = 0.5$, $\beta_2 = 2$

Key points

- Nonlinear stochastic model possessing power law spectral density, $S(f) \sim 1/f^{\beta}$, can be obtained from a microscopic agent based model.
- The nonlinear herding terms in the transition probabilities are essential in reproduction of $1/f$ noise.
- Introducing variability of herding interaction generalizes the model and enables more possibilities to reproduce $1/f$ noise.

For further reference see [Kononovicius & Gontis, 2012 (Physica A)], [Ruseckas, Kaulakys & Gontis, 2011 (EPL)].

Physics of Risk

Models of competition and conflicts in economics

http://mokslasplius.lt/rizikos-fizika/en

Thank You!