# Agent based reasoning of the nonlinear stochastic models

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## Abstract

Recently we introduced a double stochastic process driven by the nonlinear scaled stochastic differential equation reproducing the main statistical properties of the return, observed in the financial markets [1, 2]. The proposed model is based on the class of nonlinear stochastic differential equations, providing the long-range processes, the power-law behavior of spectra and the power-law distributions of the probability density [3, 4]. Stochastic framework mainly gives only a macroscopic insight into the modeled system, while microscopic behavior currently is also of big interest. In this contribution we will provide a version of agent based herding model with transition to the nonlinear stochastic equations of trading activity and return in financial markets.

## Kirman's ant colony model

In 1993 Alan Kirman proposed a model of herding behavior in ant colonies [5], which explained interesting entomological observations - ant colony at a given time exploits single food source even if second, identical, food source is available. Evidently herding behavior is very important in the colony, but without at least minor individuality of ants one would not able to explain switches between the food sources, which occur from time to time. Thus Markovian system state, defined as number of ants using one of the food sources, one step switch probability,

$$p(X \to X+1) = (N-X)(\sigma_1 + hX)\Delta t, \quad (1)$$
$$p(X \to X-1) = X[\sigma_2 + h(N-X)]\Delta t, \quad (2)$$

is composed of two terms $\sigma_i$ (invidual behavior) terms and $h$ (herding behavior) terms. Original Kirman model [5] was defined in event time scale, thus they did not include $\Delta t$ term. This term was introduced by Alfarano [6] in order to be able to obtain stochastic differential equations for Kirman model

$$\mathrm{d}x = [\sigma_1(1-x) - \sigma_2 x]\mathrm{d}t + \sqrt{2hx(1-x)}\mathrm{d}W, \quad (3)$$

here $x = \frac{X}{N}$, $W$ is Wiener process.

Though this model was inspired by ant colony behavior, it is applied towards financial markets (see [5, 6, 7]). One can define absolute return as ratio of chartist, assumed to be $x$ and fundamentalist, consequently assumed to $1-x$, population fractions [6],

$$y(t) = \frac{x(t)}{1-x(t)}. \quad (4)$$

Using Ito formula for variable substitution [8] one can derive stochastic differential equation for $y$,

$$\mathrm{d}y = (\sigma_1 - y[\sigma_2 - 2h])(1+y)\mathrm{d}t + \sqrt{2hy}(1+y)\mathrm{d}W. \quad (5)$$

Though solutions of Eq. 5 can have $1/f$ power spectral density, but it appears to be impossible to recover more sophisticated (i.e. fractured) spectral density.

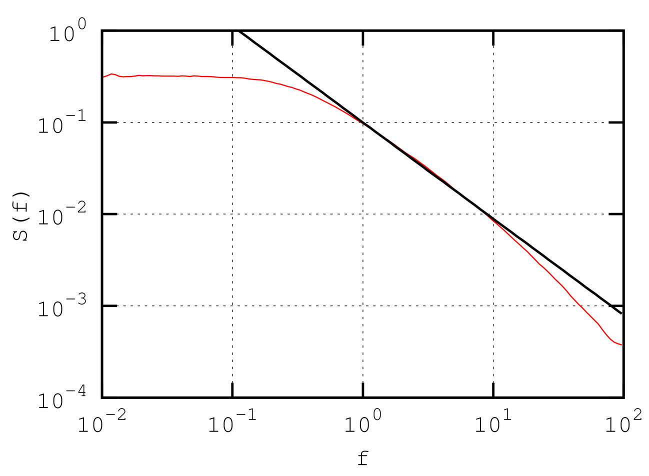


Figure 1: Spectral density obtained from the stochastic model, Eq. 5. Red line represents numerically obtained result, while the black line, $\beta = 1.05$, fits it. Model parameters were set as follows: $\sigma_1 = 0$, $\sigma_2 = 2$, $h = 1$, $r_0 = 1$, $T = 5 \cdot 10^{-3}$.

Thus in order to be able to reproduce fracture in spectral density Kirman's model must be improved. In this next sections we will propose some modifications.

## Modification of Kirman's model: variable event time scale

It is known that trading activity in the financial markets is not constant, and that it is positively correlated with the returns. The original transition probabilities, Eq. 1 and 2, assume that characteristic transition times are constant. One can introduce variability by re-expressing transition probabilities as

$$p(X \to X+1) = (N-X)\left(\sigma_1 + \frac{hX}{\tau(X)}\right)\Delta t, \quad (6)$$
$$p(X \to X-1) = X\frac{\sigma_2 + h(N-X)}{\tau(X)}\Delta t, \quad (7)$$

here $\tau(X)$ is time scale variability scenario. Note that $\sigma_1$ is not divided by $\tau(X)$ as one can assume that fundamentalists base their decisions on strategy and not market activity itself. In such case stochastic differential equation for absolute return, $y$, becomes

$$\mathrm{d}y = \left(\sigma_1 - y\frac{\sigma_2 - 2h}{\tau(y)}\right)(1+y)\mathrm{d}t + \sqrt{\frac{2hy}{\tau(y)}}(1+y)\mathrm{d}W. \quad (8)$$

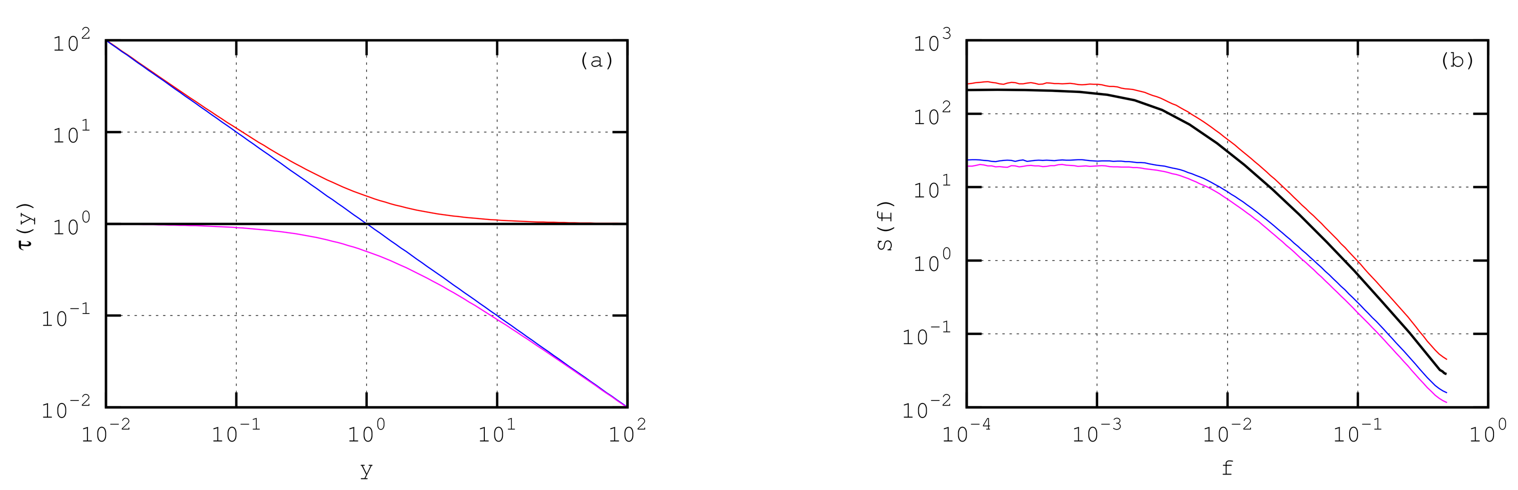


Figure 2: Applied $\tau(y)$ scenarios (a) and obtained power spectral densities (b) while using the model, Eq. 8. Model parameters: $\sigma_1 = \sigma_2 = 0.009$, $h = 0.003$, $T = 1$.

As you can see in Figure 2, using simple and thus easily justifiable $\tau(y)$ scenarios we were unable to obtain new quality of spectral density. Though it is evident that one might use more complex scenarios $\tau(y)$ in order to obtain needed quality. Comparison of Eq. 8 with stochastic differential equation for return from [1] suggest that sigmoid $\tau(y)$ scenario could provide fractured spectral density. But the problem is that this scenario can not be justified by simple logic.

Despite lack of logical reasoning behind the idea of sigmoid $\tau(y)$, this idea proves to be useful for further research as it suggests that there could be two processes, which happen on a different characteristic time scales. One of the processes being active then returns are large, while another then returns are small.

## Modification of Kirman's model: two process model

Interestingly enough in [6] absolute return, $y$, is multiplied by noise term, $\eta$ in order to obtain actual returns, $r$. This noise term is assumed to describe change in average chartist opinion during small time window $T$. But chartist opinions can be also modeled using Kirman model! Thus let's now redefine absolute return as

$$r_T(t) = r_0 \left| y(t)\xi(t) - y(t-T)\xi(t-T) \right|, \quad (9)$$

where $\xi(t)$ is an average chartist trader opinion. As $\xi(t) = 1 - 2x_{pes}$, $\xi(t)$ can be modeled by modeling $x_{pes}(t)$, but it also can be obtained directly by using stochastic differential equation

$$\mathrm{d}\xi = (a_1(1-\xi) - a_2(1+\xi))\mathrm{d}t + \sqrt{2b(1-\xi^2)}\mathrm{d}W, \quad (10)$$

here $a_i$ correspond to $\sigma_i$ and $b$ to $h$ from the previous equations. Alfarano [7] has obtained similar, yet less general (in case of $a_1 = a_2$), equation.

By combining Eq. 5 and 10 through Eq. 9 we were able to reproduce fracture in spectral density of $r$ (see Figure 3). Range of frequencies whose power behaves according to second power law can be expanded by introducing $\tau(y) = y^{-2}$ scenario into $\xi$ process (see Figure 4).

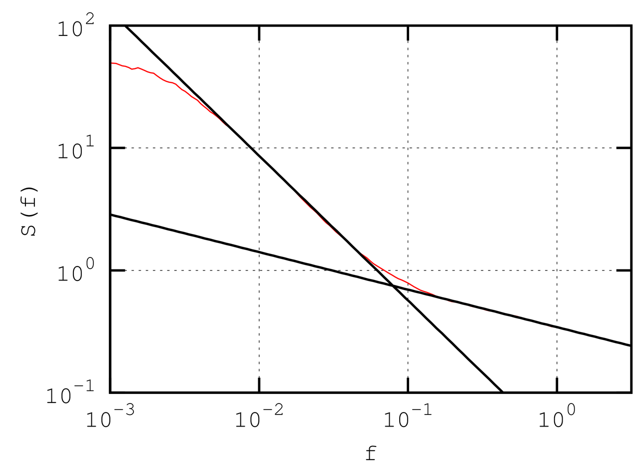


Figure 3: Slightly fractured double model's spectral density (red curve), which is fitted in low, $\beta_1 = 1.2$, and high, $\beta_2 = 0.3$, frequency ranges (black curves). Model parameters: $\sigma_1 = \sigma_2 = 0.009$, $h = 0.003$, $a_1 = a_2 = 0.9$, $b = 0.3$, $r_0 = 50$, $T = 0.5$.

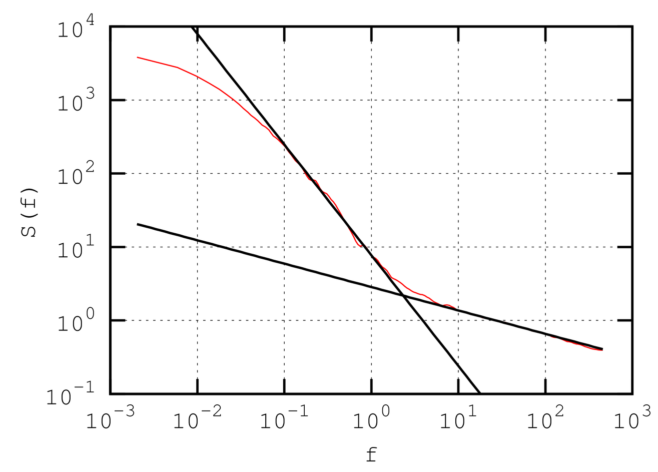


Figure 4: Fractured spectral density (red curve) obtained from the model, when $\xi$ process uses $\tau(y) = y^{-2}$ scenario. Black curves fit spectral density for low, $\beta_1 = 1.5$, and high, $\beta_2 = 0.34$, frequency ranges. Model parameters: $\sigma_1 = \sigma_2 = 0.009$, $h = 0.003$, $a_1 = a_2 = 0.9$, $b = 0.3$, $r_0 = 50$, $T = 10^{-3}$.

## Conclusions

We have modified Kirman's ant colony agent based model and achieved fractured spectral density using the two process model. This results suggests that in the financial markets there are at least two processes happening on two significantly different times scales, with one's of those processes being dependent on absolute returns. We have already used similar idea to obtain better agreement of the previous stochastic model and empirical data [1, 2].

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