Stochastic Model of Return matching to the data of financial markets with differing liquidity

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Outline

- Background.
- Concept of liquidity and difference among analyzed markets.
- Return and it's statistical properties:
  - High liquidity case (NYSE[1] time series),
  - Low liquidity case (VSE[2] time series).
- Earlier proposed Stochastic Model of Return[3] and it's universality.

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[1] – abbreviation standing for New York Stock Exchange.
[2] – abbreviation standing for NASDAQ OMX Vilnius Stock Exchange.
[3] – V. Gontis, J. Ruseckas, A. Kononovičius. *Long-range memory stochastic model of the return in financial markets*. Physica A **389**, 2010, p. 100-106.

Background

- What is Econophysics? It's approach towards financial markets, aiming for fundamental understanding of underlying phenomena.
- Why are we interested in general? Pioneering econophysicists used to say that they are not satisfied with traditional explanations.
- Why are we interested in liquidity-wise comparison? Larger financial markets are thoroughly analyzed in scientific literature, while smaller emerging markets are being overlooked.

Defining liquidity

Generally speaking inter-trade times relate to understanding of liquidity – ideally liquid stocks are easily sold or brought at any time.

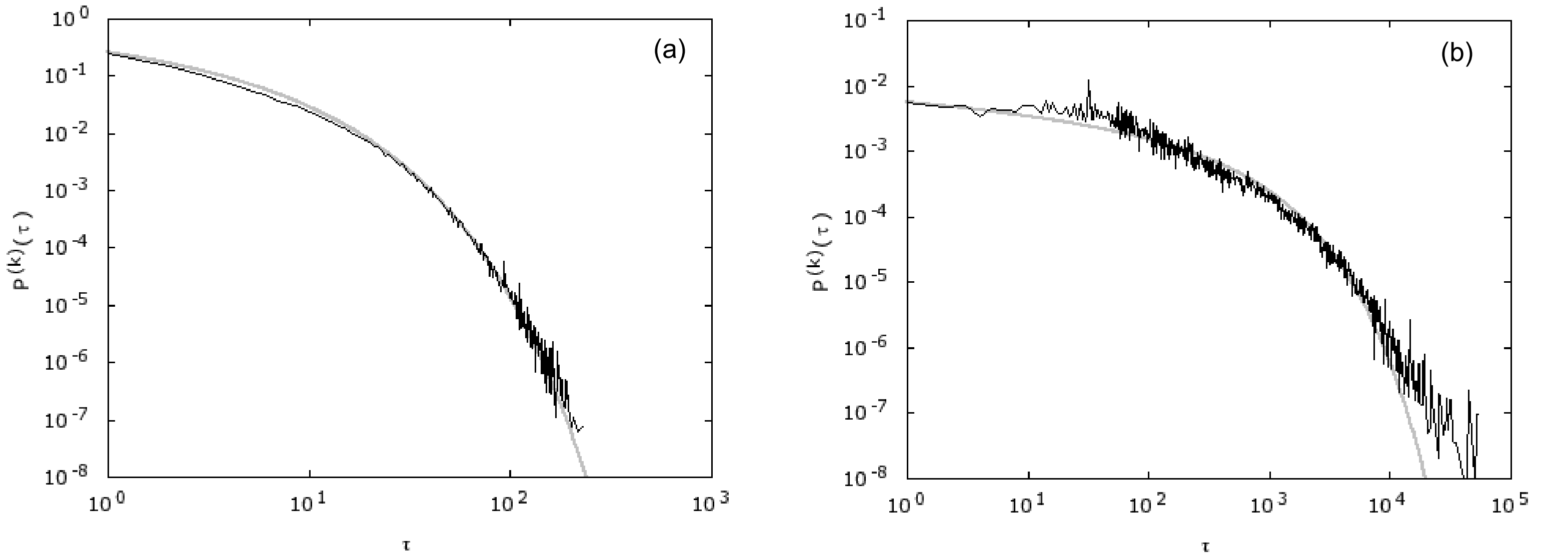

Fig 1. Distributions of inter-trade times in NYSE (a) and VSE (b).

Defining return

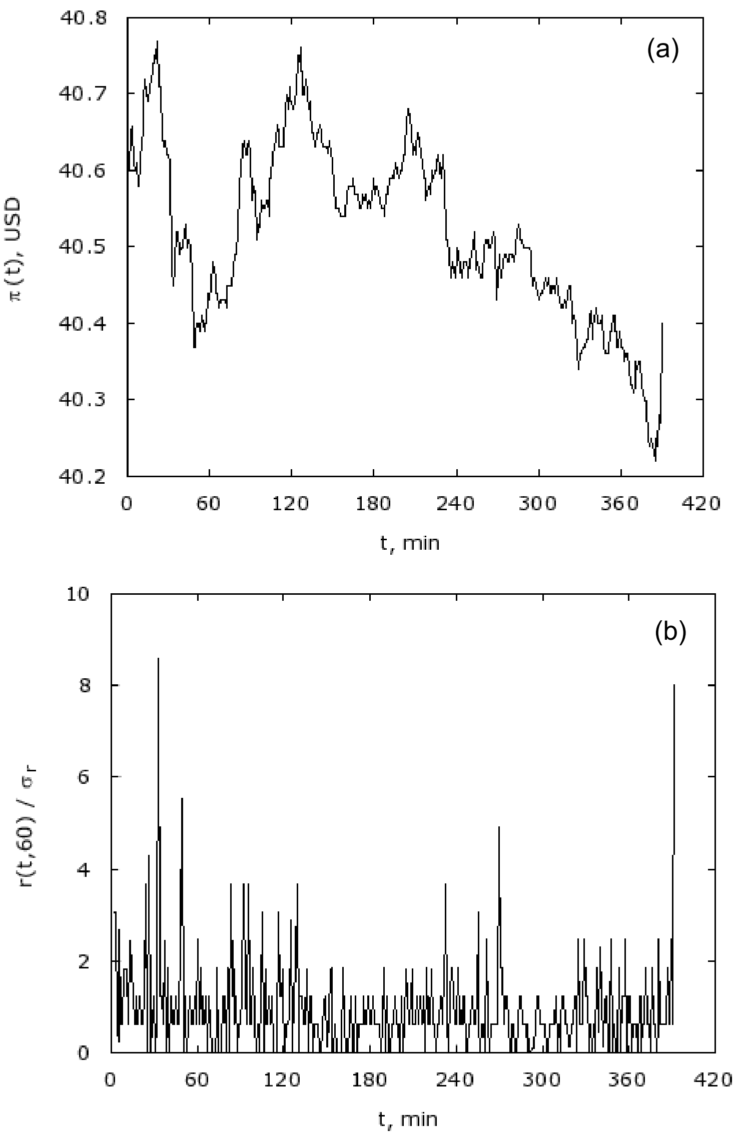

Fig 2. Price evolution (a) of randomly selected NYSE stock and one minute return evolution (b) of the same time series.

Concept of return is drawn from basic properties of price dynamics. It is defined as

$$r(t, \Delta t) = \left| \ln \left[\frac{\pi(t)}{\pi(t - \Delta t)} \right] \right|, \qquad (1)$$

here *r* is return and *π* is price function of time.

Statistical properties of return

Early researchers expected to find statistical similarities with Brownian motion, but they discovered anomalies – power law distribution and $1/f^{\beta}$, $0<\beta<1$,spectral density!

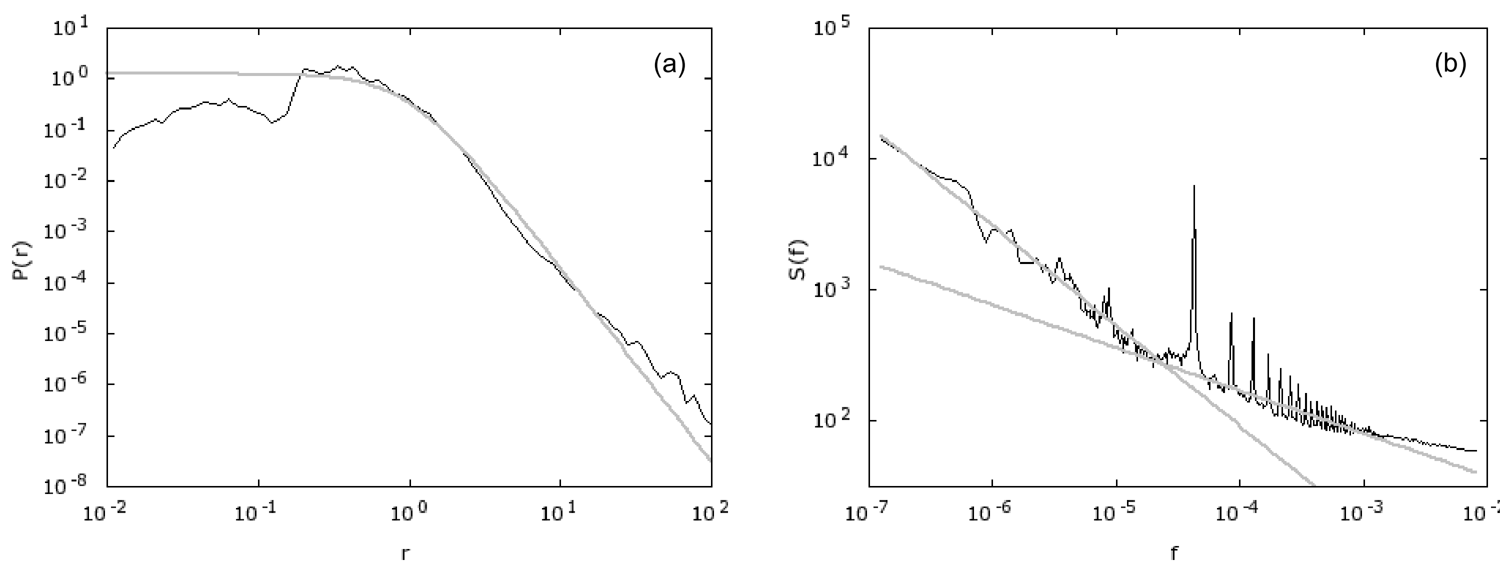

Fig 3. Statistical properties of one minute return observed in NYSE – PDF (a) and PSD (b). PDF is approximated by *q*-Gaussian with λ=3.8. PSD is approximated by two power law functions with β_1=0.8, β_2=0.3. Statistical properties averaged over 24 stocks.

Statistical properties of return

With larger mean inter-trade times probability of 0 one minute return value dramatically increases. Though by ignoring those values one can obtain similar PDF. Due to same reasons PSD at higher frequencies is distorted by white noise.

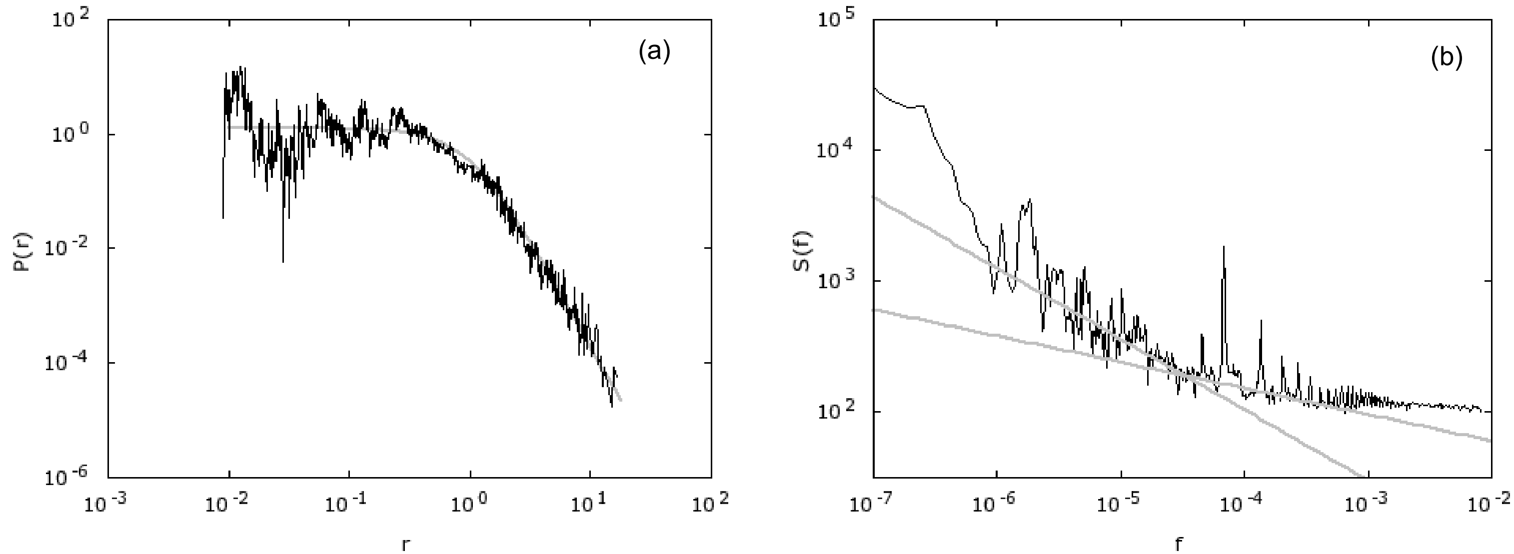

Fig 4. Statistical properties of one minute return observed in VSE – PDF (a) and PSD (b). PDF is approximated by *q*-Gaussian with λ=3.4. PSD is approximated by two power law functions with β_1=0.54, β_2=0.2. Statistical properties averaged over 4 stocks.

Stochastic Model of Return

Previously we proposed model is based on stochastic differential, Langevin, equation mathematical framework. We generate *q*-Gaussian process with double power law PSD by solving

$$\mathrm{d}x = \left[\eta - \frac{\lambda_0}{2} - \left(\frac{x}{x_{max}}\right)^2\right] \frac{\left(1+x^2\right)^{\eta-1}}{\left(1+\epsilon\sqrt{1+x^2}\right)^2} x\,\mathrm{d}t_s + \frac{\left(1+x^2\right)^{\frac{\eta}{2}}}{1+\epsilon\sqrt{1+x^2}}\mathrm{d}W_s. \qquad (2)$$

This process must be modulated by secondary *q*-Gaussian process with λ=5 and standard deviation related parameter being expressed as

$$r_0(t) = 1 + \frac{\bar{r}_0}{\Delta t}\left|\int_{t-\Delta t/2}^{t+\Delta t/2} x(a)\,\mathrm{d}a\right|. \qquad (3)$$

Those two stochastic processes underlie our model.

Universality of our model

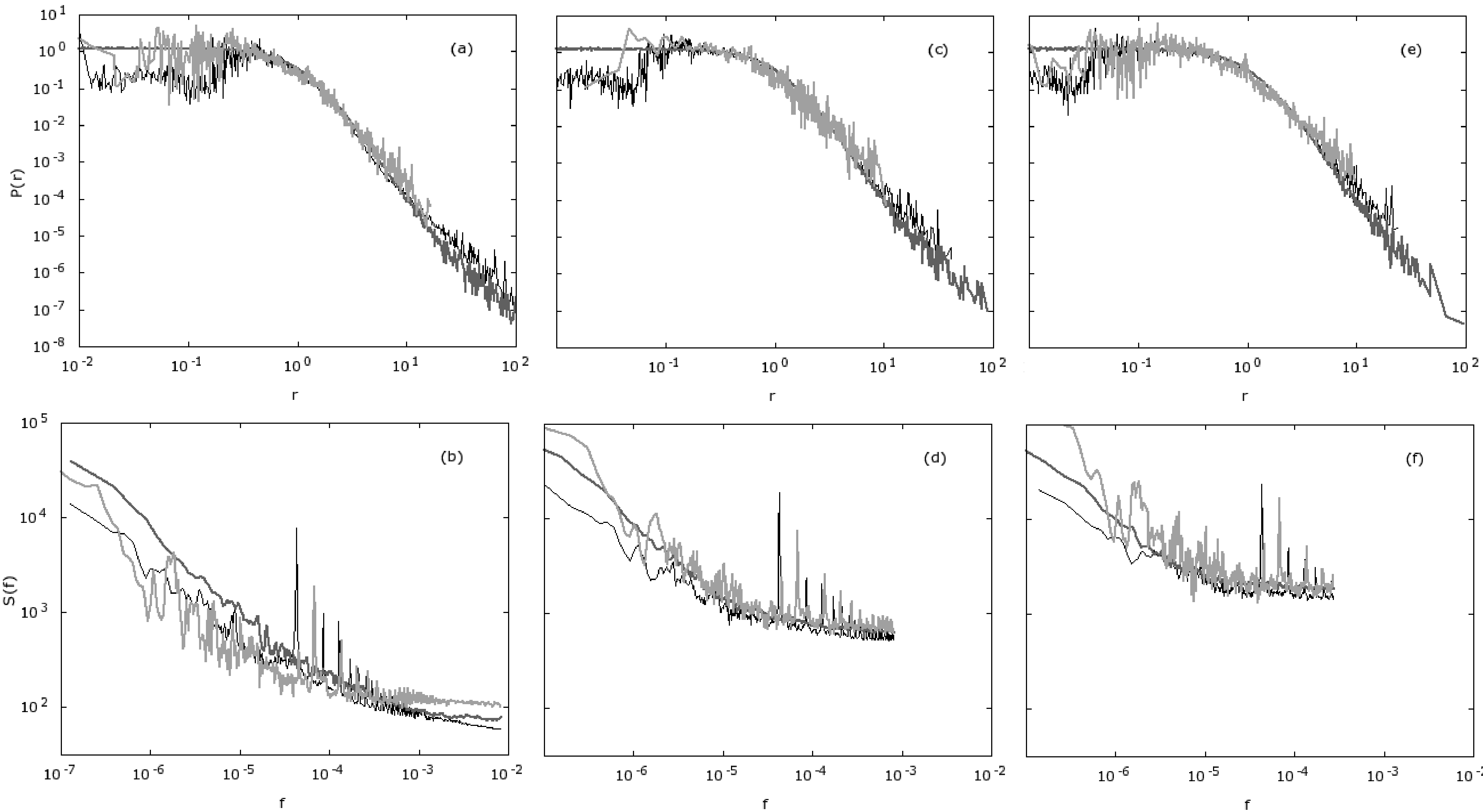

Fig 5. NYSE (black curves), VSE (light gray curves), model (dark gray curves) statistical properties – PDF (a), (c), (e) and PSD (b), (d), (e) – at various time scales, *Δt*: 1 min (a), (b), 10 min (c), (d) ir 30 min (e), (f).

Summary

- Contribution of liquidity towards statistical properties is obvious – probability of 0 return values increases distorting both PDF and PSD.
- Statistical properties of low liquidity market scale towards statistical properties of high liquidity market with increasing return time scale, Δt.
- Empirical statistical properties of return are mimicked by our earlier proposed model at various time scales using same parameters.

Thank You for Your attention!

We'd like to express our gratitude towards NASDAQ OMX Vilnius,
which provided VSE empirical data for this research.

NASDAQ OMX