Empirical analysis of Vilnius Stock Exchange absolute return time series

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Abstract

Universal statistical properties observed in various financial markets around the world helped to establish so-called stylized facts [1, 2]. Though in [1, 2] and other scientific literature one usually finds analysis of larger financial markets (such as New York Stock Exchange (further NYSE)), while smaller markets tend to be overlooked. Therefore it is interesting to know how statistical properties scale with decreasing market size.

We have analyzed tick by tick trades of 4 stocks traded on NASDAQ OMX Vilnius Stock Exchange (further VSE), comparatively small emerging financial market, for 50 months since May 2005. Empirical data was provided by VSE. We have also extended our previous analysis [3, 4] of NYSE empirical data – 24 stocks' tick by tick trades on NYSE traded for 27 months since January 2005. As should be expected differing market sizes cause evident difference in mean inter-trade times – $362\,\mathrm{s}$ in VSE, $3.02\,\mathrm{s}$ in NYSE. The influence of market trading activity on scaling of statistical properties of absolute return seems less obvious.

We have obtained perfect match of distributions by ignoring zero return values, probability of which dramatically rises if mean inter-trade times are not significantly smaller than time scale of return. Absolute return distributions from both financial markets are well approximated by q-Gaussian distribution (see [5]) with $\lambda \approx 4$. While power spectral densities, obviously, can't be matched by using previous technique. Nevertheless, power spectral density of VSE tends to converge towards power spectral density of NYSE at larger time scales.

Thus we can conclude that small market size doesn't significantly contribute towards observed statistical properties as essential features of statistical properties are preserved.

Comparison of trading activity in NYSE and VSE

We have analyzed tick by tick trades of 4 stocks, APG1L, PTR1L, SRS1L, UKB1L, traded on VSE for 50 months since May, 2005. We have also extended and put to use results of our previous analysis [3, 4] of 24 stocks, ABT, ADM, BMY, C, CVX, DOW, FNM, GE, GM, HD, IBM, JNJ, JPM, KO, LLY, MMM, MO, MOT, MRK, SLE, PFE, T, WMT, XOM, traded on NYSE for 27 months from January, 2005.

We have started with the analysis of trading activity through the concept of mean inter-trade time,

$$\bar{\tau} = \frac{1}{N} \sum_{k=1}^{N} (t_{k+1} - t_k) \, , \tag{1}$$

here $\{t_i\}$ is set of trade event times, which we have evaluated for each stock in each stock exchange separately (see Table 1). Note that, although stocks have differing mean inter-trade times, stocks from same stock exchanges exhibit similar trading activities. We have anticipated to find this difference as compared financial markets differ by size.

Table 1: Mean inter-trade times, $\bar{\tau}$, evaluated for stocks trade on VSE and NYSE

Stock	$\bar{\tau}$, s	Stock	$\bar{\tau}$, s	Stock	$\bar{\tau}$, s	Stock	$\bar{\tau}$, s
APG1L	337	PTR1L	565	SRS1L	381	UKB1L	164
						VSE mean	362
ABT	4.09	ADM	4.22	BMY	3.27	C	1.79
CVX	2.34	DOW	3.9	FNM	5.4	GE	1.44
GM	2.34	HD	2.09	IBM	3.03	JNJ	2.64
JPM	2.41	KO	3.31	LLY	4.73	MMM	4.92
MO	3	MOT	1.66	MRK	2.47	PFE	1.24
SLE	6.58	T	2.34	WMT	1.84	XOM	1.44
						NYSE mean	3.02

Statistical properties of absolute return time series

In previous section we have shown the differences between 4 VSE and 24 NYSE stocks in terms of trading activity defined as mean inter-trade time. In this section we will show the effect of the difference in trading activity on high frequency returns. Let us define return as

$$r(t, T) = \ln(\pi(t+T)) - \ln(\pi(t)) \, , \tag{2}$$

here $\pi(t)$ is price function of time and T is return time scale, and start with one minute case ($T = 60\,\mathrm{s}$).

In high-frequency case differing mean inter-trade times of analyzed stock exchanges play major role as NYSE mean inter-trade time is significantly lower than return time scale, while VSE mean inter-trade time is comparable with return time scale. Thus it would natural to expect more probable zero return values in VSE than NYSE as due to the difference in inter-trade times probability to find deal in time interval $(t, t+T]$ is significantly smaller in VSE than NYSE.

Table 2: Zero return probabilities, $p(0)$, evaluated for stocks trade on VSE and NYSE

Stock	$p(0)$, %	Stock	$p(0)$, %	Stock	$p(0)$, %	Stock	$p(0)$, %
APG1L	95.08	PTR1L	96.86	SRS1L	96.56	UKB1L	92.1
						VSE mean	95.15
ABT	22.89	ADM	28.03	BMY	33.6	C	22.63
CVX	13.77	DOW	22.3	FNM	20.7	GE	27.01
GM	23.9	HD	20.1	IBM	14.33	JNJ	20.81
JPM	24.47	KO	27.1	LLY	21.28	MMM	17.1
MO	17.41	MOT	28.74	MRK	25.32	PFE	27.67
SLE	43.51	T	32.94	WMT	20.11	XOM	13.24
						NYSE mean	23.71

Interestingly despite different probability spikes at zero return value (see Table 2) distributions of absolute return for non-zero return values are very similar. As we see in Figure 1 (a) average, for different stock exchanges, probability density functions have similar q-Gaussian shapes [5], which overlaps if we renormalize distributions for non-zero return interval. Thus we see that the only major difference in absolute return distributions is caused by the market size itself, while power law behavior remains approximately the same.

As expected due to aforementioned differences between stock exchanges we obtain power spectral densities with some evident quantitative differences, though as we see in Figure 1 (b) those disagreements are not as major as in the probability distribution case - qualitative double power law behavior is retained. Obviously we can't ignore zero return values in the return time series in order to obtain better overlapping of power spectral densities, though one can expect to improve overlapping by increasing return time scale, T. In Figure 1 (d) and (f), correspondingly $T = 600\,\mathrm{s}$ case and $T = 1800\,\mathrm{s}$ case, we see improving agreement between power spectral densities of two different stock exchanges.

Increasing return time scale, T, should also solve problem of overly high zero return probabilities, though we still needed to ignore zero return probabilities in order to obtain matches seen in Figure 1 (c) and (e). Thus one can expect to find natural agreement of probability density functions at even larger return time scales, T. Though analyzing empirical data at those time scales, $T > 7200\,\mathrm{s}$, might prove to be meaningless as interesting statistical properties would be lost, due to their non-stationary nature, or become immeasurable, due to Nyquist sampling theorem.

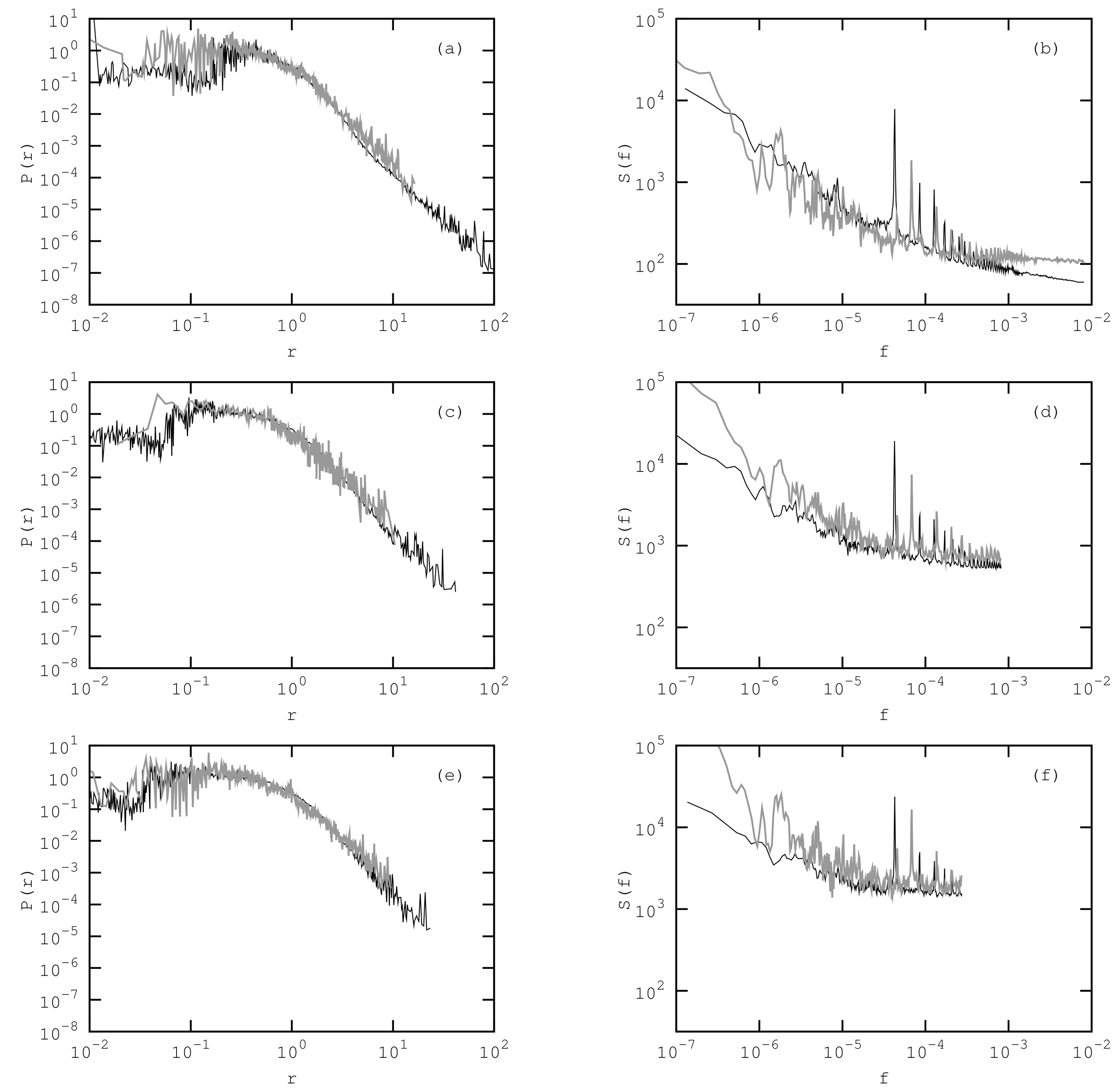

Figure 1: Comparison of empirical statistical properties of absolute returns time series of stocks traded on the NYSE (black thin lines) and VSE (gray lines). Probability density function of normalized absolute returns is given on (a),(c),(e) and powers spectral density on (b),(d),(f). (a) and (b) represents $T = 60s$ return time scale case; (c) and (d) $T = 600s$; (e) and (f) $T = 1800s$. Empirical statistical properties from NYSE was averaged over 24 stocks and empirical data from VSE was averaged over 4 stocks.

Conclusions

We have analyzed statistical properties of marginally different financial markets. Despite large difference in trading activity, expressed in our work as very different mean inter-trade times, both financial markets exhibit qualitatively same statistical behavior in terms of absolute returns. By applying few simple techniques, discussed above, one can also obtain quite good quantitative agreement.

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