Light-induced Abelian and non-Abelian gauge potentials for cold atoms

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1 Motivation

2 Some aspects of adiabatic approximation

3 Abelian effective potentials

4 Non-Abelian effective potentials for tripod coupling scheme
   - Rashba-type Hamiltonian with spin 1/2

5 Non-Abelian fields in $N$-pod schemes
   - Rashba-type Hamiltonian with spin 1
Outline

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Why effective magnetic field for atoms?

Atomic physics $\iff$ Solid state physics:
- Degenerate Fermi gas $\iff$ Electrons in solids
- Atoms in optical lattices

Advantages and disadvantages of cold atoms

- **Advantage:** Freedom in changing experimental parameters that are often inaccessible in standard solid state physics
- **Disadvantage:** Trapped atoms are electrically neutral particles. Direct analogy with magnetic properties of solids is not necessarily straightforward
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Analogies with the elementary particle physics

Cold atomic gases are an analog not only to the solid state physics. Creation of the effective gauge potentials allows for the motion of cold atoms to be described by equations that usually appear in the elementary particle physics.

- Non-Abelian gauge potentials
- Magnetic monopole
- Ultrarelativistic Dirac fermions
- Zitterbewegung
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Ways to create effective magnetic field for cold atoms

- **Rotation** — usual method to create effective magnetic field
  - Constant effective magnetic field $B_{\text{eff}} \sim \Omega$
  - Trapping frequency $\omega_{\text{eff}} = \omega - \Omega$
  - Effective magnetic field acts on atoms in the same way

- **Optical lattices** having asymmetry in the atomic transitions between the lattice sites.
  - **Abelian** effective gauge potentials
  - **Non-Abelian** effective gauge potentials in optical lattices
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Non-Abelian gauge fields:
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Adiabatic Approximation

The full atomic Hamiltonian

\[ \hat{H} = \frac{\hat{p}^2}{2M} + \hat{V}(\mathbf{r}) + \hat{H}_0(\mathbf{r}, t). \]

- \( \hat{H}_0(\mathbf{r}, t) \) — the Hamiltonian for the electronic (fast) degrees of freedom,
- \( \hat{p}^2/2M + \hat{V}(\mathbf{r}) \) — the Hamiltonian for center of mass (slow) degrees of freedom.
- \( \hat{V}(\mathbf{r}) \) — the external trapping potential.
- \( \hat{H}_0(\mathbf{r}, t) \) has eigenfunctions \(|\chi_n(\mathbf{r}, t)\rangle\) with eigenvalues \(\varepsilon(\mathbf{r}, t)\).
- Full atomic wave function

\[ |\Phi\rangle = \sum_n \psi_n(\mathbf{r}, t)|\chi_n(\mathbf{r}, t)\rangle. \]
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- Full atomic wave function
  \[ |\Phi\rangle = \sum_n \psi_n(r, t)|\chi_n(r, t)\rangle. \]
Substituting into the Schrödinger equation \( i\hbar \partial / \partial t \langle \Phi \rangle = \hat{H} \langle \Phi \rangle \) one can write the equation for the coefficients \( \Psi_n(r, t) \) in the form

\[
i\hbar \frac{\partial}{\partial t} \Psi = \left[ \frac{1}{2M}(-i\hbar \nabla - A)^2 + V + \beta \right] \Psi,
\]

where

\[
\Psi = \begin{pmatrix} 
\psi_1 \\
\vdots \\
\psi_n 
\end{pmatrix},
\]

\[
A_{n,n'} = i\hbar \langle \chi_n(r, t) | \nabla \chi_{n'}(r, t) \rangle,
\]

\[
V_{n,n'} = \varepsilon(r, t) \delta_{n,n'} + \langle \chi_n(r, t) | \hat{V}(r) | \chi_{n'}(r, t) \rangle,
\]

\[
\beta_{n,n'} = -i\hbar \langle \chi_n(r, t) | \frac{\partial}{\partial t} \chi_{n'}(r, t) \rangle.
\]
Adiabatic Approximation

Non-degenerate states

The first state is well separated from the rest. Off-diagonal terms are neglected.

\[
i\hbar \frac{\partial}{\partial t} \psi_1 = \left[ \frac{1}{2M} (\hbar \nabla - A)^2 + V + \phi + \beta \right] \psi_1,
\]

where

\[
A = A_{1,1},
\]

\[
V = V_{1,1},
\]

\[
\phi = \frac{1}{2M} \sum_{n \neq 1} A_{1,n} \cdot A_{n,1}.
\]
Degenerate states

The first $q$ dressed states are degenerate and these levels are well separated from the remaining $N - q$

$$i\hbar \frac{\partial}{\partial t} \tilde{\Psi} = \left[ \frac{1}{2M} (-i\hbar \nabla - \mathbf{A})^2 + V + \phi + \beta \right] \tilde{\Psi},$$

where $A$ and $\phi$ are truncated $q \times q$ matrices,

$$\phi_{n,n'} = \frac{1}{2M} \sum_{m=q+1}^{N} A_{n,m} \cdot A_{m,n'}.$$

The effective vector potential $\mathbf{A}$ is called the Mead-Berry connection. The effective scalar potential $\phi$ is called the Born-Huang potential.
Non-degenerate states

We have freedom of choosing the phase of the adiabatic states

\[ |\chi_n(r, t)\rangle \rightarrow e^{-\frac{i}{\hbar}u_n(r, t)}|\chi_n(r, t)\rangle. \]

The transformation of the potentials

\[ A \rightarrow A + \nabla u_1, \]
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Degenerate states

The adiabatic basis can be changed by a local unitary transformation $U(r, t)$

$$\tilde{\Psi} \rightarrow U(r, t)\tilde{\Psi}.$$ 

The transformation of the potentials

$$A \rightarrow UAU^\dagger - i\hbar(\nabla U)U^\dagger,$$

$$\phi \rightarrow U\phi U^\dagger + i\hbar \frac{\partial U}{\partial t} U^\dagger.$$

The Berry connection $A$ is related to a curvature $B$ as

$$B_i = \frac{1}{2} \epsilon_{ikl} F_{kl},$$

$$F_{kl} = \partial_k A_l - \partial_l A_k - i\hbar [A_k, A_l].$$
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Λ-type Atoms

Probe beam: $\Omega_p = \mu_{13} E_p$
Control beam: $\Omega_c = \mu_{23} E_c$

Dark state

$|D\rangle \sim \Omega_c |1\rangle - \Omega_p |2\rangle$

Destructive interference, cancellation of absorption

— EIT
\( \Lambda \)-type Atoms

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Destructive interference, cancellation of absorption — EIT
Effective Magnetic Field

\[ \mathbf{A} = -\hbar \frac{|\zeta|^2}{1 + |\zeta|^2} \nabla S, \quad \mathbf{B} = \hbar \frac{\nabla S \times \nabla |\zeta|^2}{(1 + |\zeta|^2)^2}, \]

\[ \phi = \frac{\hbar^2}{2M} \frac{(\nabla |\zeta|)^2 + |\zeta|^2 (\nabla S)^2}{(1 + |\zeta|^2)^2}, \]

where

\[ \zeta = \frac{\Omega_p}{\Omega_c} = |\zeta| e^{iS}. \]

- Light beams with relative OAM can introduce an effective magnetic field which acts on the electrically neutral atoms.
- The vector potential \( \mathbf{A} \) is determined by:
  - the gradient of phase difference between the probe and control beams,
  - the ratio between the intensities of the control and probe beams.
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Light beams with OAM: Light Vortices

Light vortex

Light vortex — light beam with phase

\[ e^{ikz+il\varphi}, \]

where \( \varphi \) is azimuthal angle, \( l \) — winding number.

Light vortices have orbital angular momentum (OAM) along the propagation axis \( M_z = \hbar l \).


The relative phase $S = (k_p + k_c)y$

Effective magnetic field $B_{\text{eff}}$ and effective trapping potential $V_{\text{eff}} = V + \phi$ produced by counter-propagating Gaussian beams.
Effective magnetic field induced by position-dependent detuning

Alternative method

Effective gauge potentials also can be created using position-dependent detuning.

- The Hamiltonian for the electronic degrees of freedom $\hat{H}_0(r)$ includes position-dependent detuning $\delta(r)$.
- Using adiabatic approximation the same general expressions for the geometric potentials apply.
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Experimental realization

(a) Experimental layout

(b) Level diagram

- Counterpropagating $\sigma_+$ and $\pi$ laser beams
- Atom in a real magnetic field ($F=1$)
- Raman coupling between the ground states $m_F = \pm 1$ and $m_F = 0$.

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Experimental realization

An alternative description to the one used by Lin et al..

The Hamiltonian for the electronic degrees of freedom

\[ H_0 = \hbar \begin{pmatrix} -\delta & \Omega_R^* & 0 \\ \Omega_R & 0 & \Omega_R^* \\ 0 & \Omega_R & \delta \end{pmatrix} \]

with two-photon coupling \( \Omega_R = |\Omega|e^{ik_d x} \).

Atom stays in the lowest-energy eigenstate

\[ |\chi_-\rangle = e^{-ik_d x} \cos^2(\theta/2) | -1\rangle - 1/\sqrt{2} \sin \theta |0\rangle + e^{ik_d x} \sin^2(\theta/2) |1\rangle \]

where \( \theta \equiv \arctan(\sqrt{2}|\Omega|/\delta) \).

The effective vector potential

\[ A = \hbar k_d \cos \theta \mathbf{e}_x \approx \hbar k_d \delta/(\sqrt{2}|\Omega|) \mathbf{e}_x \]
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The effective vector potential

\[ \mathbf{A} = \hbar k_d \cos \theta \mathbf{e}_x \approx \hbar k_d \delta / (\sqrt{2}|\Omega|) \mathbf{e}_x \]
Experimental realization


Julius Ruseckas (Lithuania)
Adiabatic motion of many-level cold atoms in the laser fields varying in space creates effective non-Abelian gauge fields. It is possible to simulate motion of the particles in the non-Abelian fields using cold atomic gases.
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Tripod Coupling Scheme

\[ \Omega_1 \quad \Omega_2 \quad \Omega_3 \]

Ultracold atoms
Tripod Coupling Scheme

- Two degenerate dark states
- Non-Abelian gauge potentials


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Two degenerate dark states:

\[ |D_1\rangle = \sin \phi e^{iS_{31}} |1\rangle - \cos \phi e^{iS_{32}} |2\rangle, \]
\[ |D_2\rangle = \cos \theta \cos \phi e^{iS_{31}} |1\rangle + \cos \theta \sin \phi e^{iS_{32}} |2\rangle - \sin \theta |3\rangle, \]

where

\[ \Omega_1 = \Omega \sin \theta \cos \phi e^{iS_1}, \quad \Omega_2 = \Omega \sin \theta \sin \phi e^{iS_2}, \quad \Omega_3 = \Omega \cos \theta e^{iS_3}. \]

Vector gauge potential:

\[ A_{11} = \hbar \left( \cos^2 \phi \nabla S_{23} + \sin^2 \phi \nabla S_{13} \right), \]
\[ A_{12} = \hbar \cos \theta \left( \frac{1}{2} \sin(2\phi) \nabla S_{12} - i \nabla \phi \right), \]
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Laser fields:

\[ \Omega_{1,2} = \Omega_0 \frac{\rho}{R} e^{i(kz + \varphi)}, \quad \Omega_3 = \Omega_0 \frac{Z}{R} e^{ik'x}. \]

The effective magnetic field

\[ B = \frac{\hbar}{r^2} e_r \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + \cdots. \]

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\Omega_1 = \Omega \sin \theta e^{-i\kappa x}/\sqrt{2}, \quad \Omega_2 = \Omega \sin \theta e^{i\kappa x}/\sqrt{2}, \quad \Omega_3 = \Omega \cos \theta e^{-i\kappa y}
\]

where

\[
\theta = \theta_0, \quad \cos \theta_0 = \sqrt{2} - 1
\]
The Hamiltonian

\[ H_k = \frac{\hbar^2}{2m} (k + \kappa' \sigma_\perp)^2 + V_1 \]

with

\[ \kappa' = \kappa \cos \theta_0, \quad \sigma_\perp = e_x \sigma_x + e_y \sigma_y \]

For small wave vectors \( k \ll \kappa' \), the atomic Hamiltonian reduces to the Hamiltonian for the 2D relativistic motion of a two-component massless particle of the Dirac type known also as the Weyl equation

\[ H_k = \hbar v_0 k \cdot \sigma_\perp + V_1 + m v_0^2 \]

where the velocity \( v_0 = \hbar \kappa' / m \) corresponds to the velocity of light. For cold atoms this velocity is of the order 1 cm/s.
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Ultrarelativistic Dirac fermions

The Hamiltonian $H_k$ commutes with the 2D chirality operator

$$\sigma_k = k \cdot \sigma_{\perp} / k$$

The dispersion

$$\hbar \omega_k^\pm = \hbar v_0 (k^2 / 2 \kappa' \pm k) + V_1 + m v_0^2$$

For small wave vectors

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The Hamiltonian for small momenta with an additional scalar potential:

\[ H = v_0 \sigma_\perp \cdot p + V \sigma_z \]

The velocity operator

\[ v \equiv \dot{r} = \frac{1}{i\hbar} [r, H] = v_0 \sigma_\perp \]

The eigenfunctions of the Hamiltonian do not have a definite velocity. **Consequence:** oscillations in the movement of the wave packet.

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Angle of the negative reflection

\[ \alpha_2 = \arcsin \left( \frac{k}{k_2} \sin \alpha \right) \]

where \( k_2 = 2\kappa - k \). Reflection coefficients

\[ r_1 = \frac{e^{i\alpha} - e^{i\alpha_2}}{e^{-i\alpha} + e^{i\alpha_2}}, \quad r_2 = -1 - r_1. \]

The corresponding reflection probabilities

\[ P_1 = |r_1|^2, \quad P_2 = \frac{\cos \alpha_2}{\cos \alpha} |r_2|^2 \]
Negative reflection

Reflection probabilities.

- $P_1(\kappa=0.5)$
- $P_2(\kappa=0.5)$
- $P_1(\kappa=1.1)$
- $P_2(\kappa=1.1)$
Spin field effect transistor with ultracold atoms

Tetrapod scheme with counter-propagating beams

Lambda-type scheme, no Raman coupling to the $F = 2$ levels

Tetrapod scheme with counter-propagating beams

Lambda-type scheme, no Raman coupling to the $F = 2$ levels

Tetrapod scheme with counter-propagating beams
Tetrapod scheme with counter-propagating beams

\[ \begin{align*}
A & \quad A' \\
B & \quad B' \\
B'' & \quad B'''
\end{align*} \]
Tetrapod scheme with counter-propagating beams

Spin-1 Rashba-type Hamiltonian

$$\hat{H} = \frac{1}{2m}(\hat{p} + \hbar \kappa \hat{J}_\perp)^2 + V$$

where $\hat{J}_\perp$ is the projection of spin-1 operator onto the $xy$ plane.
Tetrapod scheme with counter-propagating beams

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Comparison of transmission probabilities for spin-1/2 and spin-1 systems

\[ T(\alpha) \]

\( \alpha \)

Julius Ruseckas (Lithuania)

Gauge potentials for cold atoms

August 26, 2010 41 / 43
Summary

Light beams with relative orbital angular momentum can introduce Abelian and non-Abelian effective gauge potentials acting on the electrically neutral atoms.

Non-Abelian fields can be formed for cold atoms using the plane-wave setups. This was not possible for the Abelian fields.

Atomic motion in non-Abelian fields exhibits a number of non-trivial features, such as their quasirelativistic behavior or the negative refraction and reflection.

The plane wave setups can lead to the spin 1/2 or the spin 1 Rashba-type Hamiltonian for cold atoms.
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Thank you!