Physics of socio-economic phenomena

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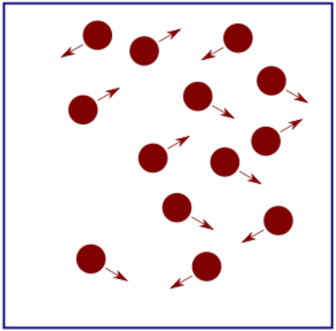
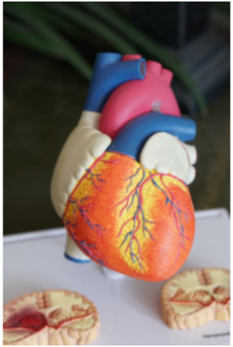
Faculty of Physics

Complex Physical and Social Systems group





Areas of interest: nonlinear dynamics, synchronization, long-range memory, stochastic processes, physics of socio-economic systems.

unsplash.com, KAL's cartoons

Physics of Risk blog

Physics of Risk   About   Students   Search using Google

Adaptive strategy in Colonel Blotto game

February 25, 2025 · Aleksejus Kononovicius · #interactive #game theory #conflicts #Colonel Blotto game

Last week I have shared a story about Colonel Blotto tournament I held back when I still used to teach Matlab. This tournament is interesting from the perspective of Physics of Risk, because it was designed to encourage adaptive strategy. Admittedly, only few students actually did that, but...

Anyway, let us build an adaptive strategy for one of the variations of the Colonel Blotto game we have explored recently. Let us revisit Colonel Blotto game with varied castles!

The game

As in an earlier post:

ops.

ags raised above its battlements. These flags represent the value of the castle.

th 9 points (flags). Thus, to win the war (game) 5 flags (points) are sufficient.

puter-controlled. "CPU" warlord will use simple fixed or random strategies (you will be

y). "ADA" warlord will use an adaptive strategy (i.e., this strategy allocates troops based

llocations).

strategy work?

n the aforementioned tournament against my students, aims to discover the value

Physics of risk, complexity and socio-economic systems.

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- Vilnius University
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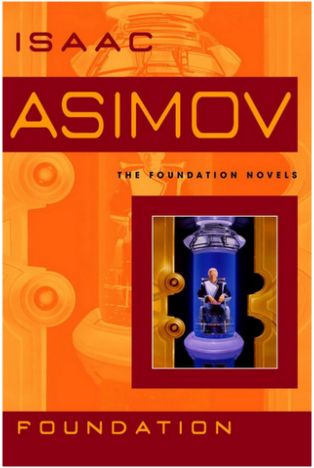
https://rf.mokslasplius.lt

The plan

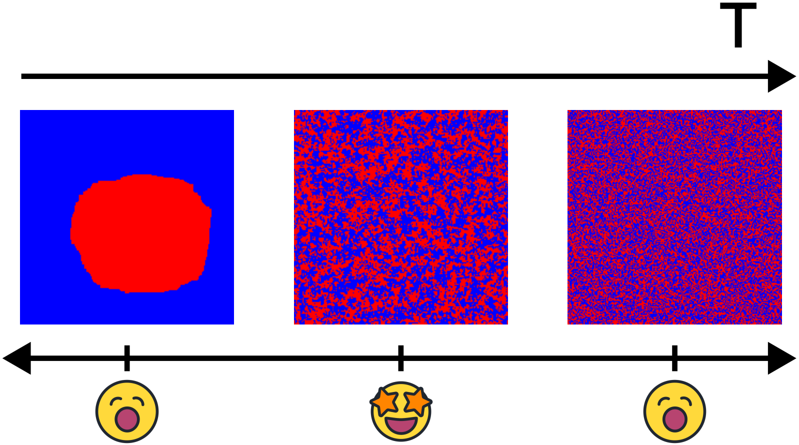
1. Society as a complex matter
2. Wealth and ideal gasses
3. Rational agents and game theory
4. Network science
5. Opinion dynamics
6. Financial markets



goodreads.com

Society as a complex matter

Complex matters



$$m \sim \left| \frac{T - T_c}{T_c} \right|^{\beta},$$

$$\xi \sim \left| \frac{T - T_c}{T_c} \right|^{-\nu}.$$

☺ flaticon.com; Pun intended: [Ball (2012)]; Ising model app: Physics of Risk

Sense of scale

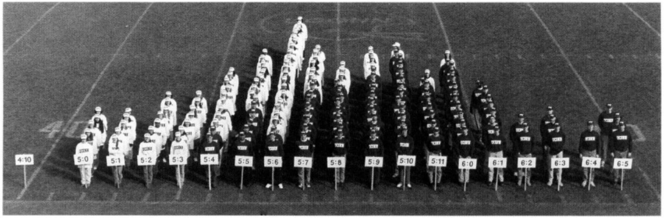
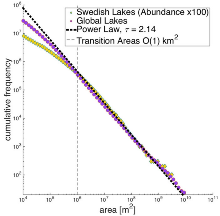


Figure 7. Living histogram of 143 student heights at University of Connecticut.

Figure 1: Globally, there are many small lakes, but only a small number of large lakes.

The asymmetry between lake size and abundance is clearly visible in this image of the Turtle Mountains, a lake district in North Dakota (USA). Image courtesy NASA.

Figure 2: Abundance (cumulative frequency) -area plots of global and Swedish lakes illustrate the asymmetry between lake size and abundance.



Large lakes are power-law distributed and the tail exponents for both datasets (τ=2.13, τ=2.14) are

Most people come from the “normal” distribution.

Many small lakes, while few are quite large.

[Schilling *et al.*(2012)], [Cael and Seekell (2016)]

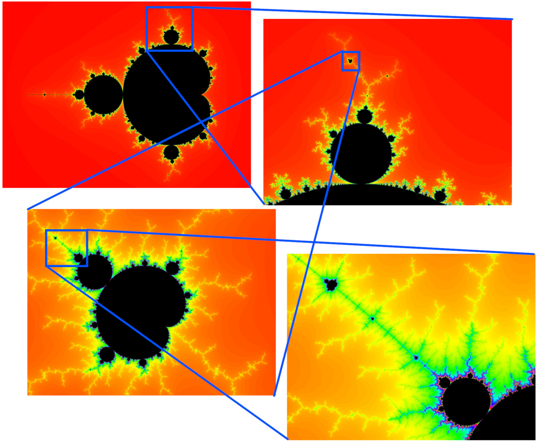
Free of scale

$f(x)$ is scale invariant if:

$$f\left(bx\right)=g\left(b\right)\cdot f\left(x\right).$$

Only solution:

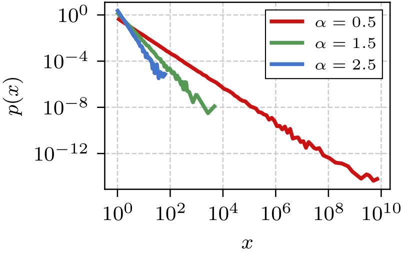
$$f\left(x\right)=Cx^{-\alpha}.$$



Zooming into the Mandelbrot set.

[Newman (2005)]; Mandelbrot set app: David J. Eck; BBC Ideas: Fractals

Scale-free distribution



Let x follow a power-law PDF:

$$p(x) = (\alpha - 1)\, x^{-\alpha},$$

with $x \geq 1$.

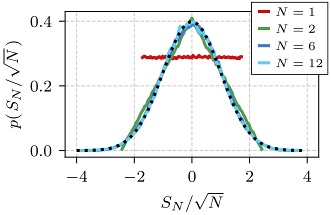
Raw moments:

$$\langle x^m \rangle = (\alpha - 1) \int_1^\infty x^{m-\alpha} dx = \left. \frac{(\alpha - 1)\, x^{m+1-\alpha}}{m + 1 - \alpha} \right|_1^\infty .$$

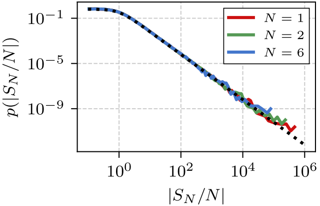
For m-th moment to be finite, we need $\alpha > 1 + m$.

Central limit theorem?

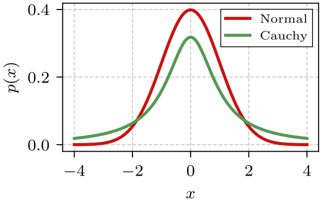
Consider $S_k = \sum_{i=1}^{k} X_i$.



$X_i \sim \mathcal{U}\left(-\sqrt{3}, \sqrt{3}\right)$



$X_i \sim$ Cauchy $(0, 1)$



Comparison

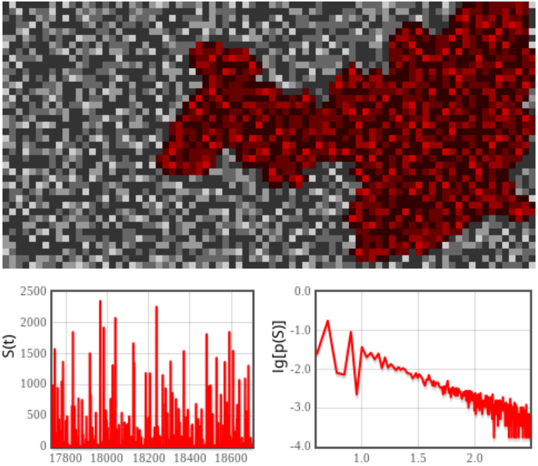
Complexity is about emergence

You can have power-law distribution without complexity! Let:

$$y \sim \text{Exp}(1),$$
$$x = \exp(y).$$

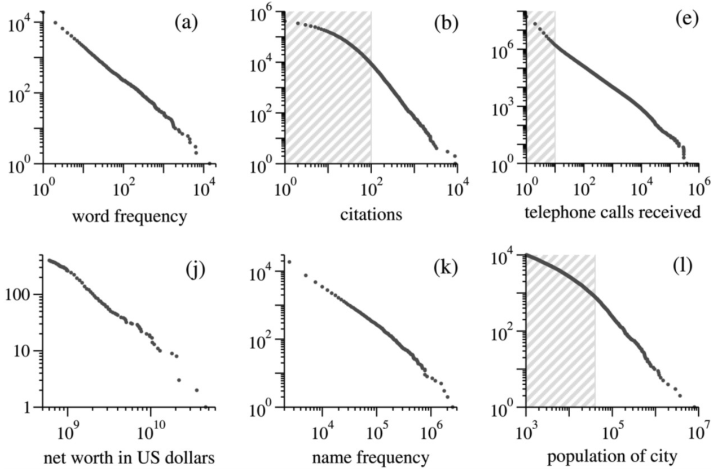
Then:

$$p_x(x) = p_y(\ln(x)) \left|\frac{dy}{dx}\right| = x^{-2}.$$



[Newman (2005)], [Bak *et al.*(1987)]; Sandpile model app: Physics of Risk

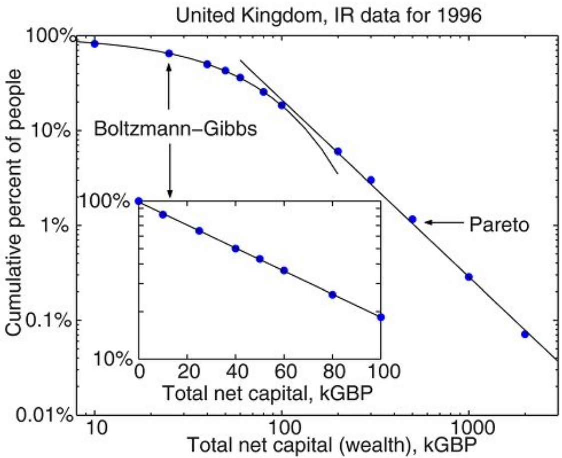
Social complexity



[Newman (2005)], Wolfgang Eckert (pixabay.com); Interactive app: Stop-and-go waves (Physics of Risk)

Wealth and ideal gasses

Empirical wealth data



[Dragulescu and Yakovenko (2001)]

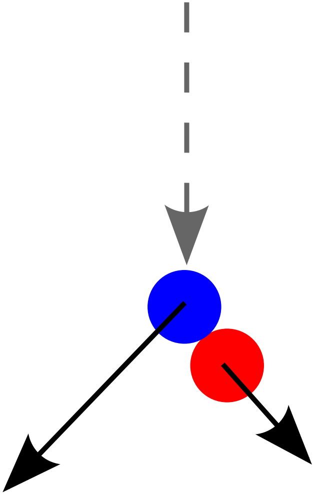
Kinetic exchange framework

1. Two particles i and j collide.
2. Δw_{ij} energy is transferred:

$$\Delta w_{ij} = r_i w_i - r_j w_j.$$

3. Update particle energies:

$$w_i\left(t+1\right) = w_i\left(t\right) - \Delta w_{ij},$$
$$w_j\left(t+1\right) = w_j\left(t\right) + \Delta w_{ij}.$$

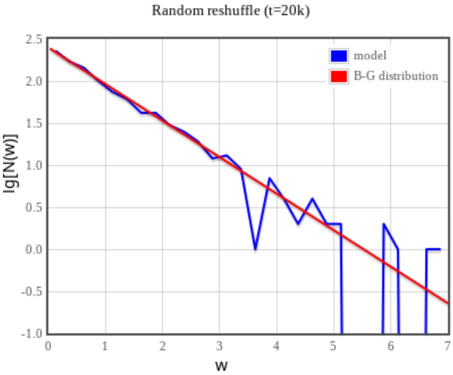


Simplest model

1. Two agents i and j meet.
2. Wealth is split randomly,

$$\Delta w_{ij} = (1 - \varepsilon)\, w_i - \varepsilon w_j,$$

   with $\varepsilon \sim \mathcal{U}(0, 1)$.
3. Update agent wealth.



[Patriarca and Chakraborti (2013)]; Interactive app: Physics of Risk

Analytical approach to the model

The master equation:

$$\frac{\partial p(w,t)}{\partial t} = \frac{\partial N^{+}(w,t)}{\partial t} - \frac{\partial N^{-}(w,t)}{\partial t}$$

We care about stationary distribution:

$$\frac{\partial p_{\infty}(w)}{\partial t} = 0 \quad \Rightarrow \quad \frac{\partial N^{-}(w,t)}{\partial t} = \frac{\partial N^{+}(w,t)}{\partial t} \quad \Rightarrow$$

$$p_{\infty}(w) = \int_{w}^{\infty} \frac{1}{U} \left[\int_{0}^{U} p_{\infty}(u_i)\, p_{\infty}(U-u_i)\, du_i \right] dU \;\Rightarrow\; p_{\infty}(w) = \frac{1}{\langle w \rangle} \exp\left(-\frac{w}{\langle w \rangle}\right).$$

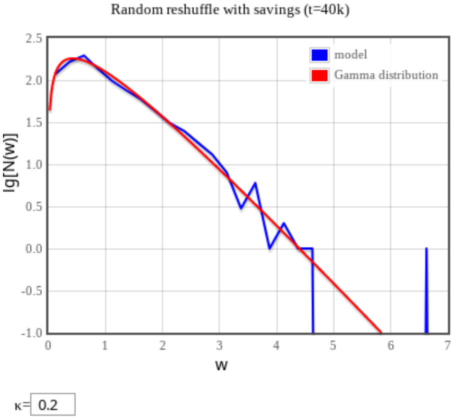
[Calbet *et al.*(2011)]

Introducing saving propensity

1. Two agents i and j meet.
2. Both reserve κ fraction of their wealth. Remaining wealth is split randomly,

$$\Delta w_{ij} = (1-\kappa)\left[(1-\varepsilon)\, w_i - \varepsilon w_j\right].$$

   with $\varepsilon \sim \mathcal{U}(0,1)$.
3. Update agent wealth.



[Patriarca and Chakraborti (2013)]; Interactive app: Physics of Risk

Deriving moments

By definition, lhs and rhs should be equal in distribution:

$$w_i\left(t+1\right) \overset{d}{=} \kappa w_i\left(t\right)+\varepsilon\left(1-\kappa\right)\left[w_i\left(t\right)+w_j\left(t\right)\right]$$

Thus, for the m-th raw moment of a stationary distribution:

$$\left\langle w^m\right\rangle = \left\langle\left\{\kappa w_i+\varepsilon\left(1-\kappa\right)\left[w_i+w_j\right]\right\}^m\right\rangle .$$

Needs to be solved recurrently:

$$\left\langle w^1\right\rangle = 1,$$

$$\left\langle w^2\right\rangle = \frac{\kappa+2}{1+2\kappa},$$

$$\left\langle w^3\right\rangle = \frac{3\left(\kappa+2\right)}{\left(1+2\kappa\right)^2},$$

$$\left\langle w^4\right\rangle = \frac{72+12\kappa-2\kappa^2+9\kappa^3-\kappa^5}{\left(1+2\kappa\right)^2\left(3+6\kappa-\kappa^2+2\kappa^3\right)}.$$

Suggested approximation

$$p\left(w\right)\sim w^{n-1}\exp\left(-nw\right),$$

with $n=1+\frac{3\kappa}{1-\kappa}$.

How to construct power-law distribution?

Start with:

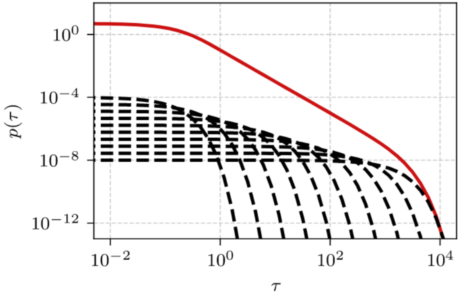
$$p(\tau|\gamma) = \gamma \exp(-\gamma\tau).$$

Assume that for $\gamma_{\text{min}} < \gamma < \gamma_{\text{max}}$:

$$p(\gamma) \propto \frac{1}{\gamma^{\alpha}}.$$

Combine:

$$p(\tau) = \int_{\gamma_{\text{min}}}^{\gamma_{\text{max}}} p(\gamma)\, p(\tau|\gamma)\, d\gamma \propto \frac{1}{\tau^{2-\alpha}}.$$



[Kononovicius and Kaulakys (2024)]; Interactive app: Physics of Risk

Note that:

$$w^{n-1}\exp\left(-nw\right)=\exp\left[\left(n-1\right)\ln\left(w\right)-nw\right]\approx$$
$$\approx\exp\left(-nw\right)$$

So assume that $\kappa\sim\mathcal{U}\left(0,1\right)$.

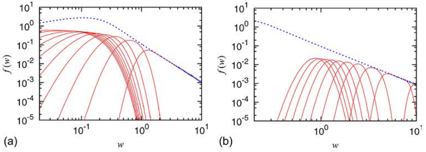
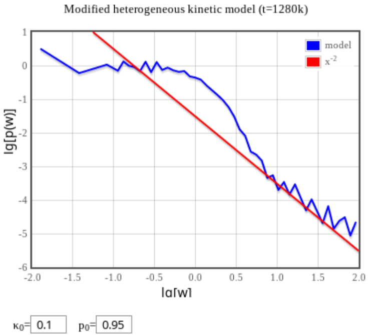


Fig. 3. (Color online) Wealth distribution $f(w)$ for uniformly distributed κ_i (or λ_i) in the interval (0,1); $f(w)$ is decomposed into partial distributions $f_i(w)$, where each $f_i(w)$ is obtained by counting the statistics of those agents with parameter λ_i in a specific sub-interval (from Ref. 36). The left panel shows the decomposition of $f(w)$ into ten partial distributions in the λ-subintervals (0, 0.1), (0.1, 0.2),..., (0.9, 1). The right panel decomposes the final partial distribution in the λ-interval (0.9, 1) into partial distributions obtained by counting the statistics of agents with λ-subintervals (0.9, 0.91), (0.91, 0.92),..., (0.99, 1). Note how the power law appears as a consequence of the superposition of the partial distributions.

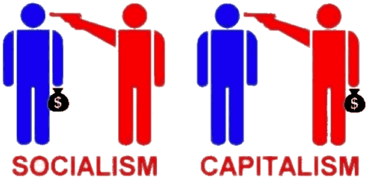


[Patriarca and Chakraborti (2013)]; Interactive app: Physics of Risk

Delving deeper

Wealth / income:

- Compatibility with Economics
- Skills and luck
- Temporal dynamics
- Realistic income mechanisms



But not only wealth / income:

- Opinion dynamics (Biswas-Chatterjee-Sen model)
- Designing ranking systems (ELO)
- Epidemiological models
- Alcohol consumption

Recommendations: [Patriarca and Chakraborti (2013)], [Toscani *et al.*(2022)]

politifake.org

Rational agents and game theory

Game theory

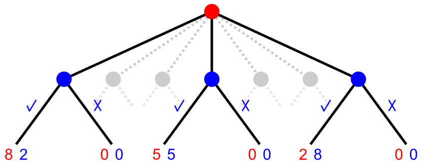
Explores interactions between rational and self-interested agents.

Games:

- cooperative or competitive
- (non-)zero sum
- (a)symmetric
- (a)synchronous
- (in)finite
- ...

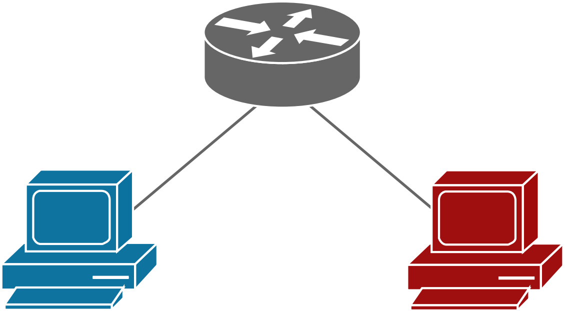
	R	P	S
Rock	0, 0	-1, 1	1, -1
Paper	1, -1	0, 0	-1, 1
Scissors	-1, 1	1, -1	0, 0

Payoff matrix for a r-p-s game



Decision tree of an ultimatum game

Pure strategies (in the TCP backoff game)



	B	C
Back-off	-1, -1	-4, 0
Continue	0, -4	-3, -3

- What is desirable?
- What do we get?

vecta.io

Some games have no pure strategy...

GK \ Taker	L	R
Left	1, 0	0, 1
Right	0, 1	1, 0

Matching pennies game

But there might be a mixed strategy. To find it you need to make your opponent not care about their action.

Wikimedia

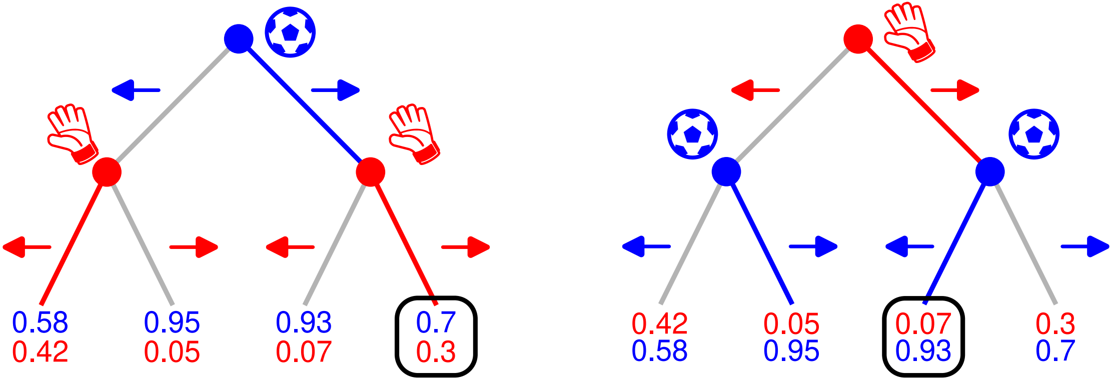
A practical problem for a football manager...

GK \ Taker	L	R
Left	0.42, 0.58	0.07, 0.93
Right	0.05, 0.95	0.3, 0.7

1. Should GK jump left? (Answer: $p_{GK} \approx 0.42$)
2. Should penalty taker shoot left? (Answer: $p_T \approx 0.38$)
3. Expected outcome? (Answer: $U_{GK} \approx 0.2$)

[Palacios-Huerta (2003)]; sportingnews.com

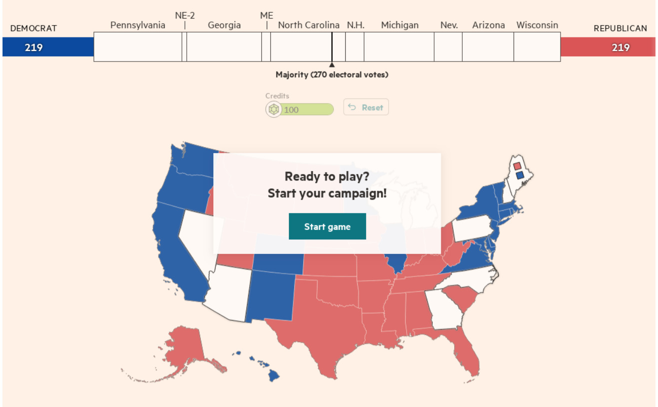
Solution: rig the game



Quick GK (left) or quick taker (right).

“Eleven: Football Manager Board Game” (Portal Games)

A really important game. . .



- 100 credits.
- Assign credits to states.
- Outspend to get votes.

US elections app: Financial Times; Colonel Blotto apps: Physics of Risk

Delving deeper

- More actions
- More players
- Consecutive games
- Random games
- Behavioral rationality

Designing:

- Auctions
- Voting
- Board games ("prof." Reiner Knizia)

Resilience:

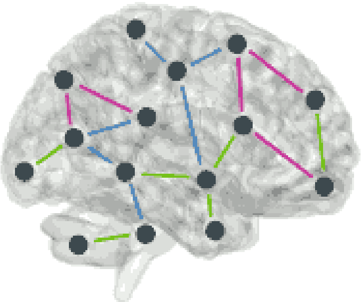
- Errors
- Manipulations

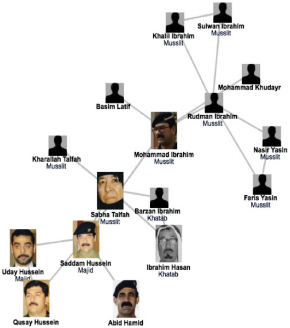
Recommendation: "Game Theory" course (Coursera and Youtube)

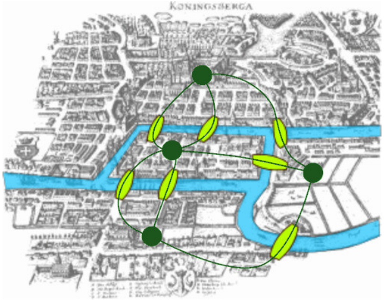
Greg Montani (pixabay.com); Veritasium: Why Democracy Is Mathematically Impossible

Network science

Connections



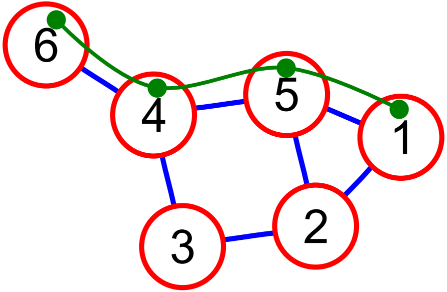




[Lynn and Basset (2019)], slate.com, Wikimedia.

Main terminology

Network is a collection of **nodes** and **links**. Mathematicians prefer terms **graph**, **vertex** and **edge**.



- **Neighboring** nodes - connected by edges.
- Node's **degree** - a number of its neighbors.
- **Path** - sequence of neighboring nodes.
- **Geodesic** - shortest $i \to j$ path.
- **Diameter** - longest geodesic in a network.
- …

Adjacency matrix

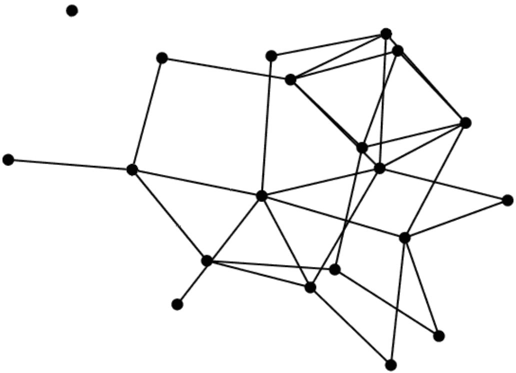
$$\mathbf{A} = \begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

- If $A_{ij} \neq 0$, then there exists an edge pointing from j to i.
- Node degree: $k_i = \sum_{j=1}^{N} \mathbf{1}_{A_{ij}\neq 0} = \sum_{j=1}^{N} \mathbf{1}_{A_{ji}\neq 0}$.
- $(\mathbf{A}^m)_{ij}$ counts all $j \to i$ paths.

Specific links can be

- **looping**, if $A_{ii} = 1$.
- **directional**, if $A_{ij} \neq A_{ji}$.
- **multiple**, if $A_{ij} \in \mathbb{N}_0$.
- **weighted**, if $A_{ij} \in \mathbb{R}$.

Erdos-Renyi (random) network



1. Start with N nodes and $L = 0$ edges.
2. Iterate over all possible pairs. Add edge with probability p.

Edges on average:

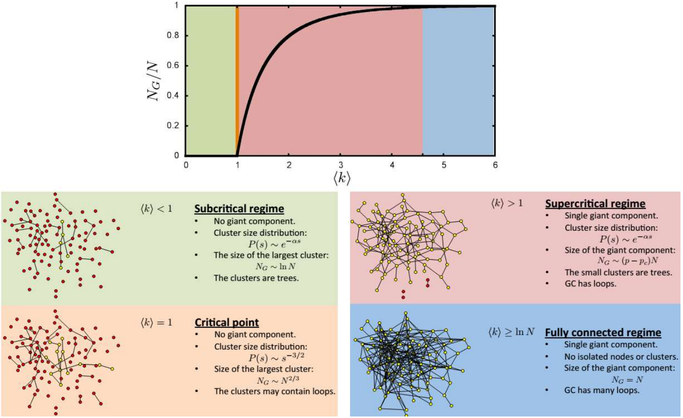
$$\langle L \rangle = pN\left(N-1\right)/2.$$

Average degree:

$$\langle k \rangle = 2L/N = p\left(N-1\right).$$

Interactive app: Physics of Risk

Phase transition in E-R network



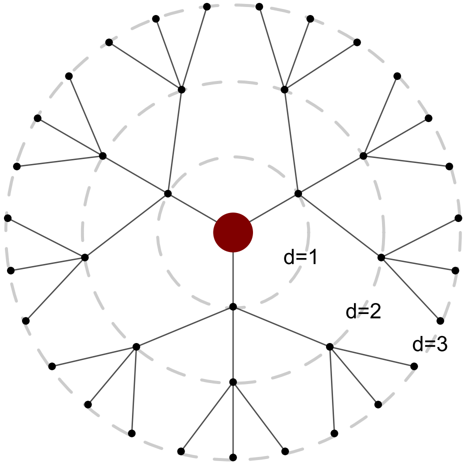
Probability to not be in g.c.:

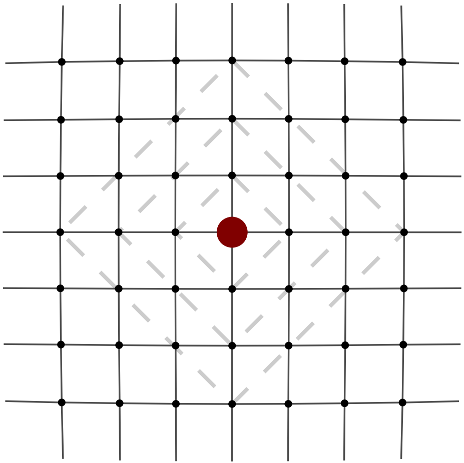
$$u = \left[(1-p) + up\right]^{N-1},$$

$$\frac{N_G}{N} = 1 - \exp\left[-\langle k \rangle \frac{N_G}{N}\right].$$

networksciencebook.com (V26 edition)

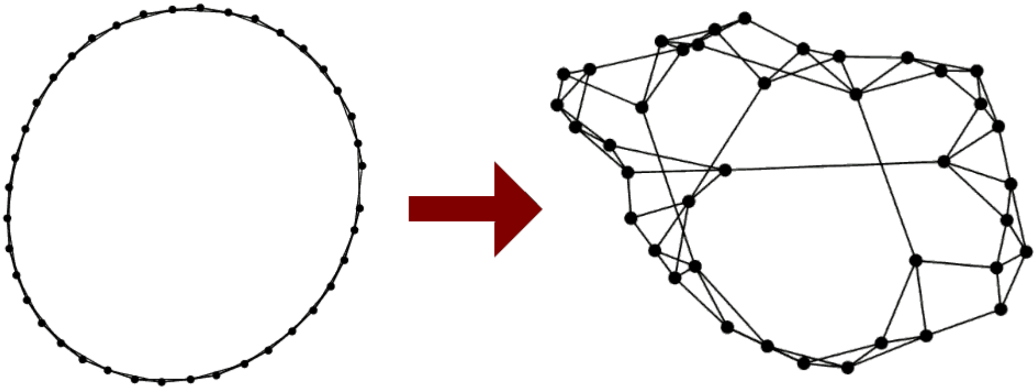
Randomness enhances reach





Nodes reached: 3^d (left) and $4d$ (right).

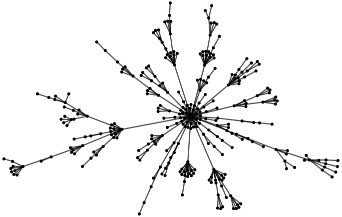
Watts-Strogatz network



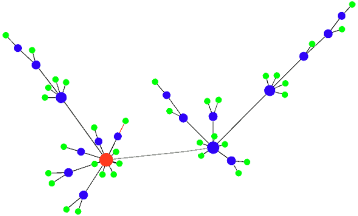
Idea: Introduce random edges into a regular structure.

Interactive app: Physics of Risk

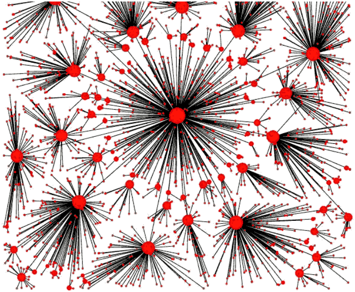
Scale-free networks



Barabasi-Albert



Edge redirection



Luck-and-reason

Interactive apps: Barabasi-Albert, Edge redirection,
Luck-and-reason (Physics of Risk)

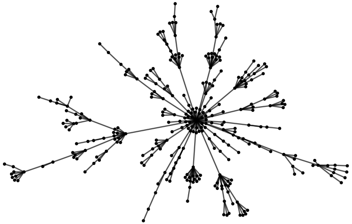
Continuum approach to B-A network

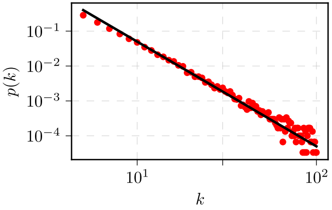
Expected degree of j-th node,

$$\frac{dk_j}{dt} = mp\left(t \to j\right) = \frac{k_j}{2t}, \quad \Rightarrow \quad k_j\left(t\right) = m\sqrt{\frac{t}{t_j}}.$$

Looking for "younger" nodes is the same as looking for lower degree nodes. Thus:

$$P\left(t_j > T\right) = P\left(k_j < k\right) \propto k^{-2},$$
$$p\left(k\right) \propto k^{-3}.$$





[Barabasi *et al.*(1999)]

Delving deeper

Further general topics:

- Degree correlations
- Clustering
- Centrality and influence
- Strategic network formation

Recent research directions:

- Community detection
- Evolving networks
- Multi-layer networks
- Hyper-graphs
- Higher-order networks
- Predicting missing edges
- Reconstructing processes

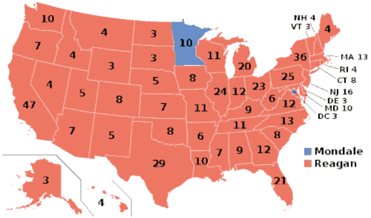
Recommendations: Barabasi "Network Science",
"Social and Economic Networks" course (Coursera and Youtube)

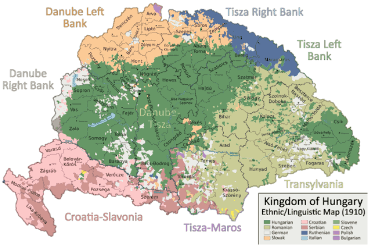
Generated by Copilot

Opinion dynamics

Diverse research direction

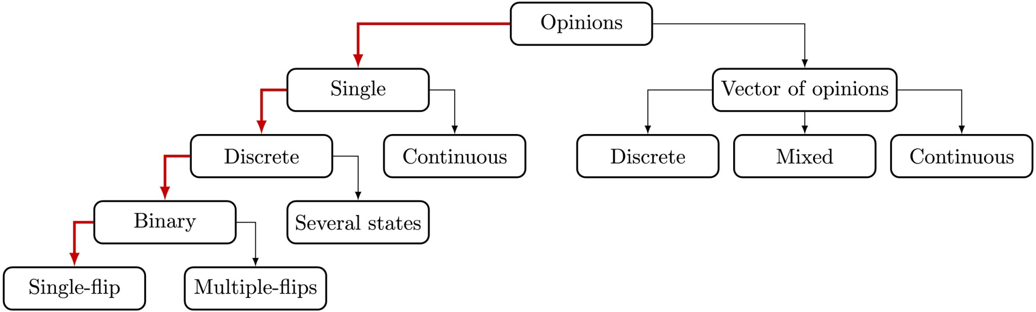
- Elections, polls, census data
- Online discussion
- Collective behavior
- Laboratory experiments





Gizmodo, Wikimedia, Wikimedia

Model taxonomy

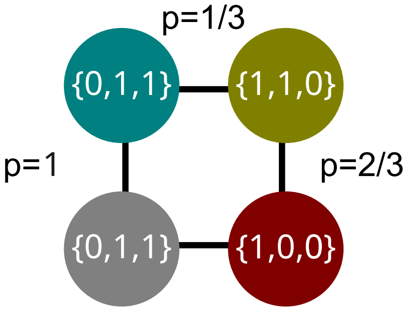


[Jedrzejewski and Sznajd-Weron (2019)]

Axelrod model

- Opinion is given by d-dimensional **vector**.
- Each component may take q distinct values.

1. Choose a random agent i.
2. Choose a random neighbor j.
3. Interaction probability is proportional to the number of shared components.
4. During interaction i copies a single component from j.



[Axelrod (1997)]; Interactive app: Physics of Risk

Bounded confidence models

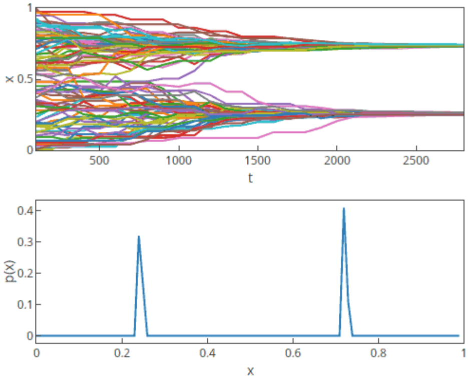
- Agents have **continuous opinion** x_i.
- There exists a "trust" threshold ε.

1. Choose random agents i and j.
2. Check if i trusts j:

$$|x_j(t) - x_i(t)| < \varepsilon.$$

3. If yes, update agent's i opinion:

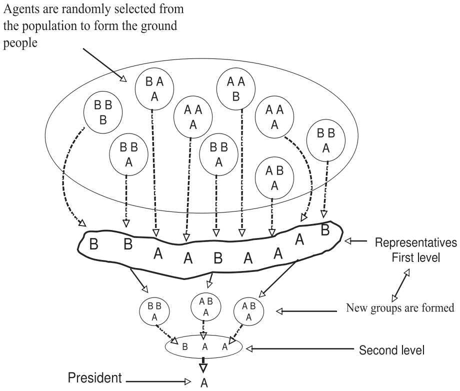
$$x_i(t+1) = x_i(t) + \mu \left[x_j(t) - x_i(t)\right].$$



[Flache *et al.*(2017)]; Interactive app: Physics of Risk

Galam models

- Each agent has **discrete opinion**.
- Interactions occur in groups imposed by hierarchy, or in randomized groups.
- Agents may elect their representative, or they may align their opinions.
- Consider *status quo* effects.

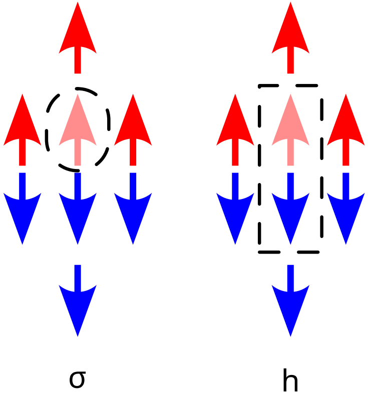


Interactive apps: Hierarchical voting model, Referendum model (Physics of Risk).

[Galam (2008)]

Noisy voter model

- Discrete binary opinions
- Agents may change their opinion independently, rate σ
- Agents may change their opinion by imitating their peers, rate $\propto h$
- Interactions may occur on a complete network or other arbitrary network



Recommendations: [Redner (2019)], [Jedrzejewski and Sznajd-Weron (2019)]
Interactive voter model apps: Physics of Risk

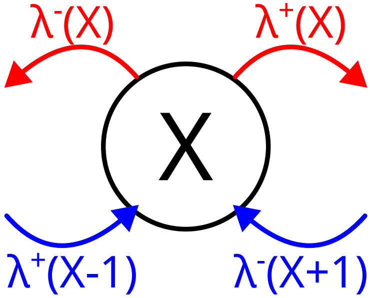
Master equation

We can formalize NVM using birth and death rates:

$$\lambda^+(X) = (N - X)\left[\sigma^+ + hX\right], \quad \lambda^-(X) = X\left[\sigma^- + h\left(N - X\right)\right].$$

Master equation:

$$\frac{\Delta p(X,t)}{\Delta t} = -\lambda^+(X)p(X,t) - \lambda^-(X)p(X,t) + \\ + \lambda^+(X-1)p(X-1,t) + \\ + \lambda^-(X+1)p(X+1,t).$$



[van Kampen (2007)], [Anderson (2007)]

Deriving stationary distribution

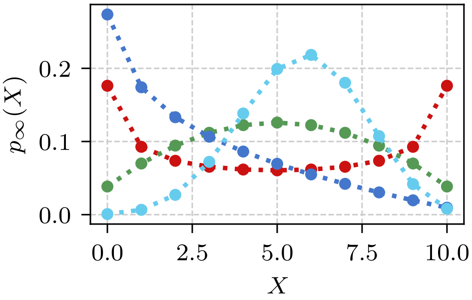
Detailed balance:

$$\lambda^-(X)p_\infty(X) = \lambda^+(X-1)p_\infty(X-1).$$

Getting rid of recursion:

$$p_\infty(X) = p_\infty(0) \cdot \frac{\prod_{i=1}^{X} \lambda^+(i-1)}{\prod_{k=1}^{X} \lambda^-(k)} =$$

$$= p_\infty(0) \cdot \frac{N!}{X!\,(N-X)!} \cdot \frac{B\left(\frac{\sigma^+}{h} + X, \frac{\sigma^-}{h} + N - X\right)}{B\left(\frac{\sigma^+}{h}, \frac{\sigma^-}{h} + N\right)}.$$



Interactive app: Detailed balance (Physics of Risk)

Continuous ($N \to \infty$) limit

Rewrite the rates:

$$\lambda_s^+(x) = N^2 \cdot (1-x)\left[\frac{\varepsilon^+}{N} + x\right], \quad \lambda_s^-(x) = N^2 \cdot x\left[\frac{\varepsilon^-}{N} + (1-x)\right].$$

Master equation:

$$\begin{aligned}\frac{\Delta p(x,t)}{\Delta t} =& -\lambda_s^+(x)p(x,t) - \lambda_s^-(x)p(x,t) \\ &+ \lambda_s^+(x-\Delta x)p(x-\Delta x,t) + \lambda_s^-(x+\Delta x)p(x+\Delta x,t) = \\ =& \left(\mathbf{E}^+ - 1\right)\left[\lambda_s^-(x)p(x,t)\right] + \left(\mathbf{E}^- - 1\right)\left[\lambda_s^+(x)p(x,t)\right].\end{aligned}$$

Here: $\mathbf{E}^\pm f(x) = f(x \pm \Delta x) \approx f(x) \pm \Delta x f'(x) + \frac{(\Delta x)^2}{2} f''(x)$.

[van Kampen (2007)]

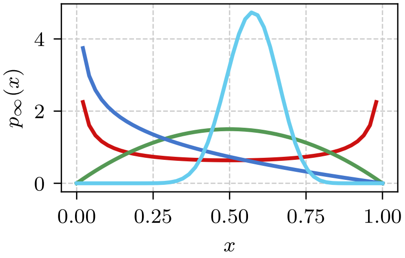
Fokker-Planck equation

$$\frac{\partial p(x,t)}{\partial t} \approx -\frac{\partial}{\partial x}\left[\frac{\lambda_s^+(x) - \lambda_s^-(x)}{N} p(x,t)\right] + \frac{1}{2}\frac{\partial^2}{\partial x^2}\left[\frac{\lambda_s^+(x) + \lambda_s^-(x)}{N^2} p(x,t)\right]$$

Stationary distribution:

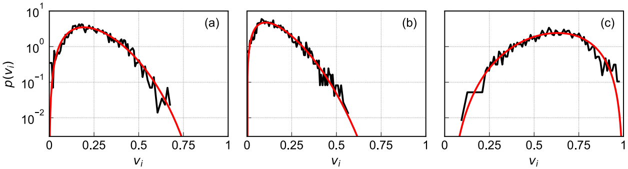
$$0 = -\left\{\varepsilon^+ (1-x) - \varepsilon^- x\right\} p_\infty(x) + \\ + \frac{d}{dx}\left[x(1-x)\, p_\infty(x)\right],$$

$$p_\infty(x) = C_N \cdot x^{\varepsilon^+ - 1} (1-x)^{\varepsilon^- - 1}.$$

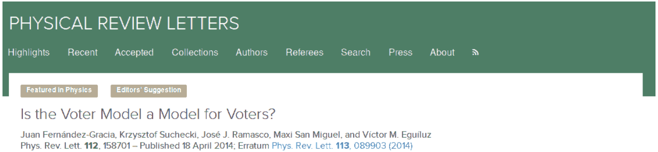


[Risken (1996)]

Empirical fitness



Party (SK (a), LKDP (b) and LDDP (c)) vote shares in Lithuanian 1992 parliamentary election.



PHYSICAL REVIEW LETTERS

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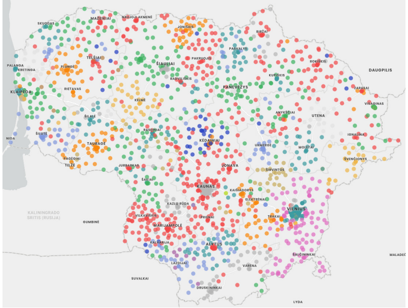
Featured in Physics Editors' Suggestion

Is the Voter Model a Model for Voters?

Juan Fernández-Gracia, Krzysztof Suchecki, José J. Ramasco, Maxi San Miguel, and Víctor M. Eguíluz

Phys. Rev. Lett. 112, 158701 – Published 18 April 2014; Erratum Phys. Rev. Lett. 113, 089903 (2014)



Lithuanian 2022 municipality election results map.

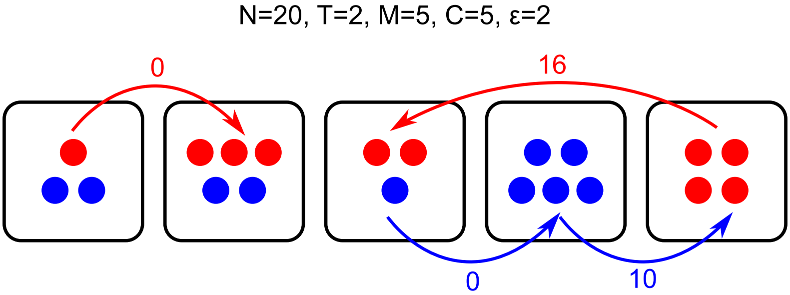
[Kononovicius (2017)], [Fernandez-Gracia *et al.*(2014)]

rinkimai.maps.lt

It might be more about exchange...

Let the exchange rate between the spatial units be:

$$\lambda_{(k)}^{i\to j} = \begin{cases} X_i^{(k)} \left(\varepsilon^{(k)} + X_j^{(k)} \right) & \text{if } i \neq j \text{ and } \sum_k X_j^{(k)} < C, \\ 0 & \text{otherwise,} \end{cases}$$



[Kononovicius (2019)]

Delving deeper

- q-voter model
- Non-Markovian dynamics
- Dynamics on networks
- Polarization (physicsworld.com)
- Detecting election fraud
- Compatibility with social sciences

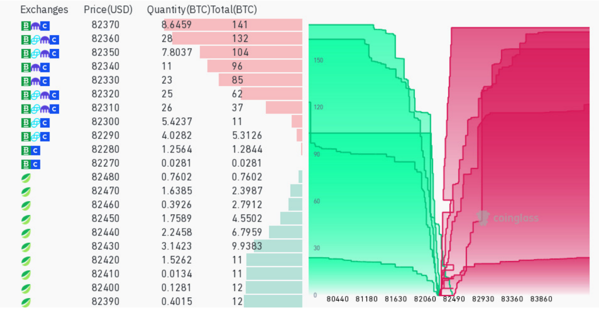
Recommendations: [Castelano *et al.*(2009)], [Flache *et al.*(2017)], [Peralta *et al.*(2023)]

Recent: [Pal *et al.*(2025)]

(unknown), “spinson”

Financial markets

The big picture



Exchanges	Price(USD)	Quantity(BTC)	Total(BTC)
	82370	8.6459	141
	82360	28	132
	82350	7.8037	104
	82340	11	96
	82330	23	85
	82320	25	62
	82310	26	37
	82300	5.4237	11
	82290	4.0282	5.3126
	82280	1.2564	1.2844
	82270	0.0281	0.0281
	82480	0.7602	0.7602
	82470	1.6385	2.3987
	82460	0.3926	2.7912
	82450	1.7589	4.5502
	82440	2.2458	6.7959
	82430	3.1423	9.9383
	82420	1.5262	11
	82410	0.0134	11
	82400	0.1281	12
	82390	0.4015	12

finance.yahoo.com, coinglass.com, imgflip.com

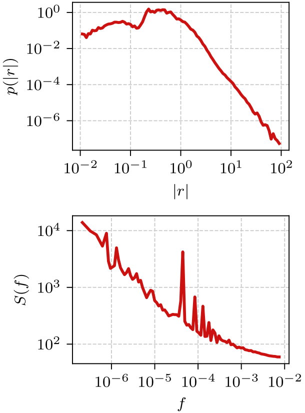
Some statistical facts

Let us introduce return:

$$r(t, \Delta t) = \ln \frac{P(t)}{P(t - \Delta t)}.$$

If we look at high frequency data ($\Delta t < 24\mathrm{h}$). We find that:

- r has power-law tails
- r is mostly not correlated
- $|r|$ is correlated



(right) PDF and PSD of $|r|$ ($\Delta t = 60$ s; NYSE 2005–2007 data)

[Cont (2001)], [Gontis *et al.*(2010)]

Just some opinion dynamics?

Let there be two type of traders:

- Chartists bet on their mood:

$$D_c(t) = -r_0 X_c(t)\,\xi(t)\,.$$

- Fundamentalists follow strategy:

$$D_f(t) = X_f(t) \ln\left[\frac{P_f}{P(t)}\right].$$

[Kononovicius and Gontis (2012)]; Jeff Parker (cagle.com)

Stochastic differential equations

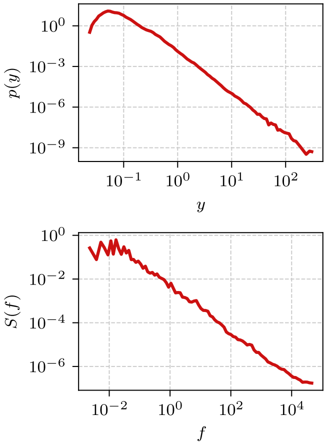
Stochastic differential equation (Ito sense) for the noisy voter model:

$$dx = \left[\varepsilon^{+}(1-x) - \varepsilon^{-}x\right]dt + \sqrt{2x(1-x)}dW.$$

For long-term return, $y = \frac{x}{1-x}$:

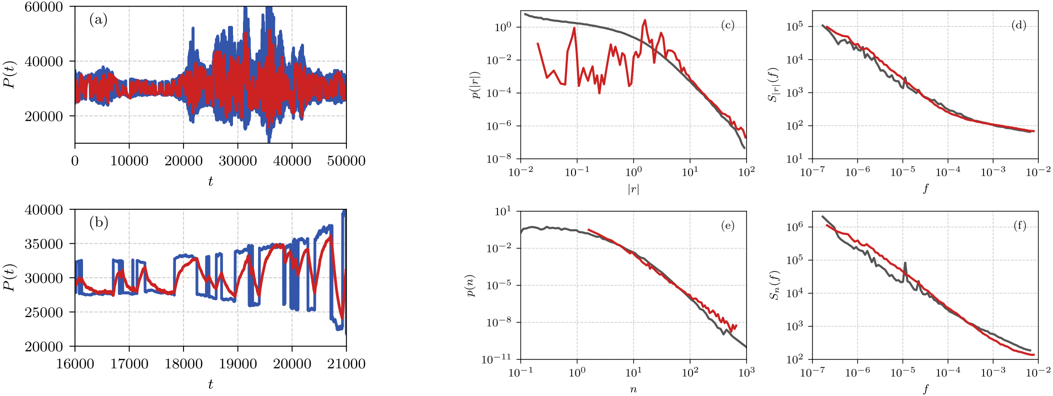
$$dy = \left[\varepsilon^{+} + (2-\varepsilon^{-})\frac{y}{\tau(y)}\right](1+y)dt + \sqrt{\frac{2y}{\tau(y)}}(1+y)dW,$$

with $\tau(y) = y^{-\alpha}$.



[Kononovicius and Gontis (2012)]; Interactive app: Physics of Risk

Order book model



Incorporation of new information through order book takes time.

[Kononovicius and Ruseckas (2019)]; Interactive app: Physics of Risk

Delving deeper

- Stock cross-correlation
- Multi-scaling behavior
- Bayesian inference
- Portfolio optimization
- Option pricing problem
- Risk estimation
- Bubble detection
- Market efficiency, maturity
- Deep forecasting

Recommendations: [Mantegna and Stanley (1999)], [Slanina (2014)]

Generated by Copilot; Veritasium: The Trillion Dollar Equation

Thank you!

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kononovicius.lt, rf.mokslasplius.lt

Faculty of Physics