Generalizing binary choice agent-based herding model

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Why am I generalizing ABM?

In the Stochastic Models session?

The macroscopic models

are mostly phenomenological (ODE, SDE, ...). We “take” the data and “make” the model.

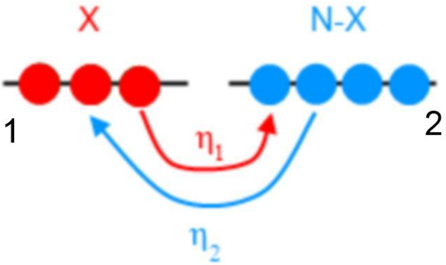
Agent-based models

emphasize microscopic interactions. The assumptions “make” the model.

Bridging between these two concepts

could potentially enable better policy making tools.

Elementary two state system



If the transition rates, η_i, are deterministic and constant,

the system will reach equilibrium:

$$X\eta_1 = (N - X)\eta_2 \quad \Rightarrow \quad \frac{X}{N} \approx x = \frac{\eta_2}{\eta_1 + \eta_2}.$$

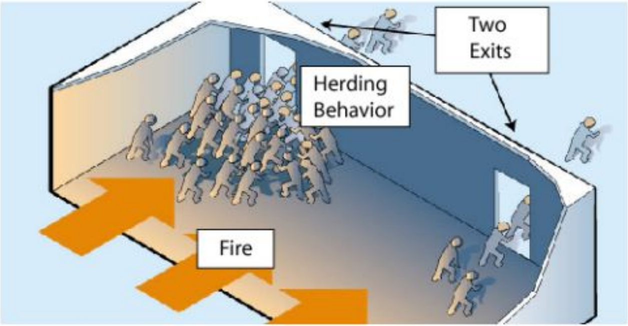
But what if $\eta_1 = f(X, N)$ and $\eta_2 = g(X, N)$? What if the transitions are probabilistic?

What can these two states represent?

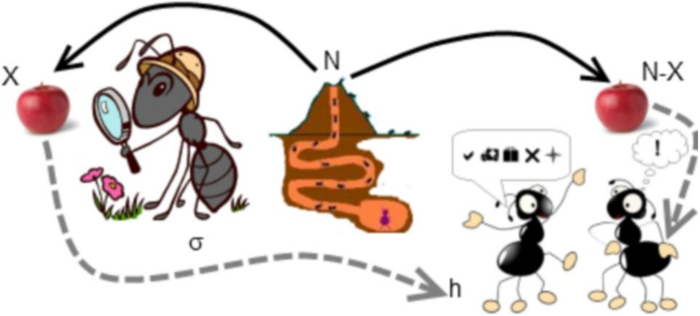
It depends on what system we want to model.

Ant colony? Two routes to the food source, two food sources.
Goods market? To buy or not to buy.
Financial market? Distinct trading strategies.
Election? Candidates.
Para/dia/fero magnetic? Spin up or down.

The social behavior in two state system



Kirman's agent-based herding model



Each agent (=ant) during each time step

may change its state (=switch the used food source):

$$\eta_2 = \sigma_1 + hX, \quad \eta_1 = \sigma_2 + h(N - X).$$

One-step formalism I

During small time step, Δt,

$$P(X \to X+1) = (N-X)\eta_2 \Delta t = (N-X)(\sigma_1 + hX)\Delta t = N^2 \pi^+ \Delta t,$$

$$P(X \to X-1) = X\eta_1 \Delta t = X[\sigma - 2 + h(N-X)]\Delta t = N^2 \pi^- \Delta t,$$

$$P(X \to X \pm \Delta X) \approx 0, \quad \forall \Delta X > 1.$$

Master equation, for $x = X/N$,

$$\partial_t \omega(x,t) = N^2 \left\{ (\boldsymbol{E}^+ - 1)[\pi^-(x)\omega(x,t)] + (\boldsymbol{E}^- - 1)[\pi^+(x)\omega(x,t)] \right\}.$$

One-step formalism II

The one-step operators:

$$\boldsymbol{E}^{\pm}[f(x)] = f(x \pm \Delta x) \approx f(x) \pm \Delta x \partial_x f(x) + \frac{\Delta x^2}{2} \partial_x^2 f(x),$$

Then the master equation leads

to Fokker-Planck equation, which leads to Langevin equation:

$$\mathrm{d}x = \left[(1-x)\eta_2 - x\eta_1\right] \mathrm{d}t + \sqrt{\frac{(1-x)\eta_2 + x\eta_1}{N}} \mathrm{d}W.$$

And in the aforementioned case:

$$\mathrm{d}x = \left[(1-x)\sigma_1 - x\sigma_2\right] \mathrm{d}t + \sqrt{2hx(1-x)} \mathrm{d}W.$$

What can be borrowed from the Lotka-Volterra model?

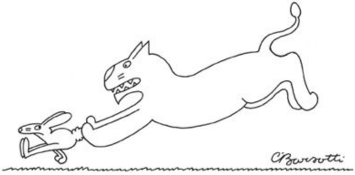
L-V model is a prey-predator model. So it has:

- birth and death rates (agents can be created and destructed),
- asymmetric herding (predators have an upper hand).

Macroscopic model:

$$\mathrm{d}x = \left[\sigma_1 (n - x) - \sigma_2 x + c x (n - x) + T_1(x)\right] \mathrm{d}t + \sqrt{2 h x (n - x)} \mathrm{d}W,$$

$$\mathrm{d}n = \left[T_1(x) + T_2(x)\right] \mathrm{d}t.$$



"What are you complaining about? It's a level playing field."

The implications of global and local interactions

Two distinct formulations of the model exist:

$$\eta_2 = \sigma_1 + hX, \quad \eta_1 = \sigma_2 + h(N - X), \qquad \text{(red)}$$

$$\eta_2 = \sigma_1 + h\frac{X}{N}, \quad \eta_1 = \sigma_2 + h\frac{N - X}{N}. \qquad \text{(blue)}$$

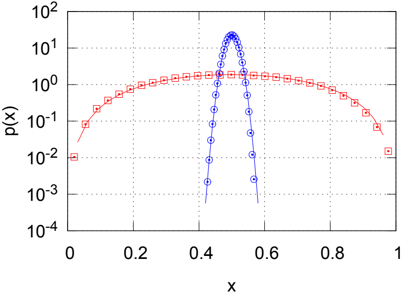


Figure: Global interaction (red) and local interaction (blue) case PDFs.

Where are the local interactions observed?

The Bass diffusion model:

$$\eta_2 = \sigma + h\frac{X}{N}, \quad \eta_1 = 0, \quad \Rightarrow \quad \mathrm{d}X = (N - X)\left(\sigma + h\frac{X}{N}\right)\mathrm{d}t.$$

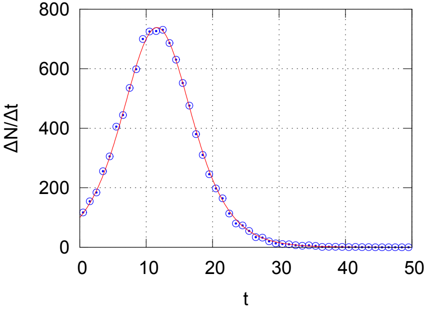


Figure: ABhM (circles) and analytical model (the curve).

Control of the social communities I

Let us now assume that we have M controlled agents:

$$\eta_2 = \sigma_1 + h(M + X), \quad \eta_1 = \sigma_2 + h(N - X).$$

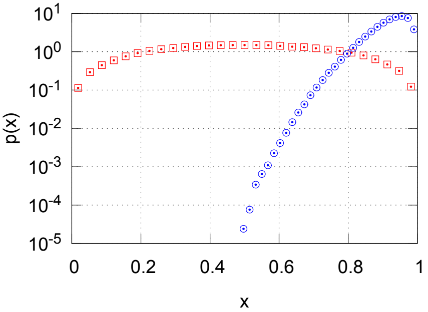


Figure: System of $N = 1000$ agents: without controlled agents, $M = 0$, (red) and with a small fraction of controlled agents, $M = 20$, (blue).

Control of the social communities II

Let us now assume that we have M controlled agents:

$$\eta_2 = \sigma_1 + h\left(M + \frac{X}{N}\right), \quad \eta_1 = \sigma_2 + h\frac{N-X}{N}.$$

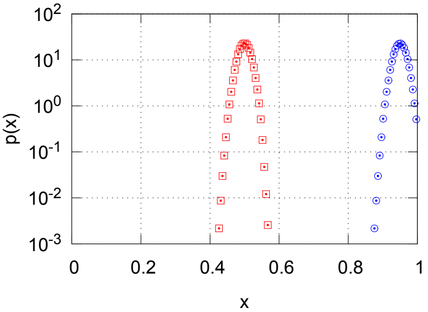


Figure: System of $N = 1000$ agents: without controlled agents, $M = 0$, (red) and with a small fraction of controlled agents, $M = 20$, (blue).

A model for the absolute modulating return, $y = x/(1-x)$:

$$dy = \left[\varepsilon_1 + (2-\varepsilon_2)y^{1+\alpha}\right](1+y)dt_s + \sqrt{2y^{1+\alpha}}(1+y)dW_s.$$

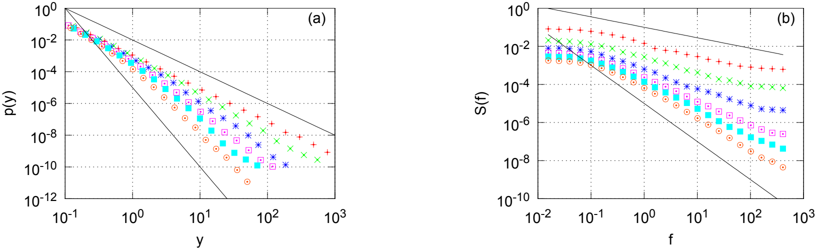


Figure: The variety of reproducible PDF (a) and PSD (b) power laws: $2 < \lambda < 5$, $0.5 < \beta < 2$.

The previous SDE can be simplified to

$$\mathrm{d}y = \left(\eta - \frac{\lambda}{2}\right) y^{2\eta-1}\mathrm{d}t_s + y^{\eta}\mathrm{d}W_s,$$

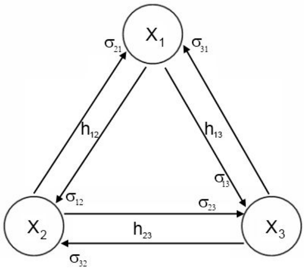
$$\eta = \frac{3+\alpha}{2}, \quad \lambda = \varepsilon_2 + \alpha + 1.$$

This model incorporates:

- Bessel process ($\eta = 0$),
- Squared Bessel process ($\eta = 0.5$),
- CEV and CIR processes (if diffusion limiting is introduced).

Financial modeling III

Complex model I



Simplifications:

$$x_1 = n_f, \quad x_2 = n_p, \quad x_3 = n_o,$$
$$\sigma_{23} = \sigma_{32} = \sigma_{cc}, \quad \sigma_{12} = \sigma_{13} = \sigma_{fc}/2,$$
$$\sigma_{21} = \sigma_{31} = \sigma_{cf}, \quad h_{12} = h_{13} = h_1.$$

Finally we arrive at:

$$\mathrm{d}n_f = \left[\frac{(1-n_f)\varepsilon_{cf}}{\tau(n_f,\xi)} - n_f \varepsilon_{fc} \right] \mathrm{d}t_s + \sqrt{\frac{2n_f(1-n_f)}{\tau(n_f,\xi)}} \mathrm{d}W_{s,1},$$
$$\mathrm{d}\xi = -\frac{2H\varepsilon_{cc}\xi}{\tau(n_f,\xi)} \mathrm{d}t + \sqrt{\frac{2H(1-\xi^2)}{\tau(n_f,\xi)}} \mathrm{d}W_{s,2}, \quad \tau(n_f,\xi) = \left[1 + \left| \frac{1-n_f}{n_f} \xi \right|^{\alpha} \right]^{-1}.$$

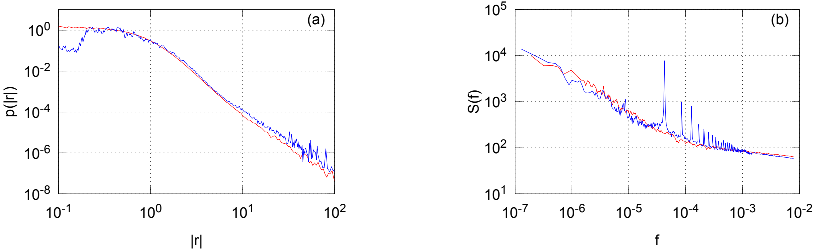


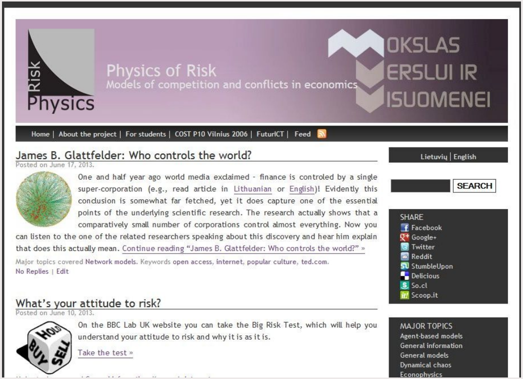
Figure: Empirical (blue) and model (red) statistical features of absolute return: PDF (a) and PSD (b).

More...

Articles:

"Agent-based and macroscopic modeling of the complex socio-economic systems" (to appear)

Our website "Physics of Risk":



http://mokslasplius.lt/rizikos-fizika/en/

Thank you for attention!