Generalizing binary choice agent-based herding model

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2013-06-21
Why am I generalizing ABM? In the Stochastic Models session?

The macroscopic models are mostly phenomenological (ODE, SDE, ...). We “take” the data and “make” the model.

Agent-based models emphasize microscopic interactions. The assumptions “make” the model.

Bridging between these two concepts could potentially enable better policy making tools.
Elementary two state system

If the transition rates, $\eta_i$, are deterministic and constant, the system will reach equilibrium:

$$X \eta_1 = (N - X) \eta_2 \implies \frac{X}{N} \approx x = \frac{\eta_2}{\eta_1 + \eta_2}.$$ 

But what if $\eta_1 = f(X, N)$ and $\eta_2 = g(X, N)$? What if the transitions are probabilistic?
What can these two states represent?

It depends on what system we want to model.

**Ant colony?** Two routes to the food source, two food sources.
**Goods market?** To buy or not to buy.
**Financial market?** Distinct trading strategies.
**Election?** Candidates.
**Para/dia/fero magnetic?** Spin up or down.
The social behavior in two state system
Kirman’s agent-based herding model

Each agent (=ant) during each time step may change its state (=switch the used food source):

$$\eta_2 = \sigma_1 + hX, \quad \eta_1 = \sigma_2 + h(N - X).$$
During small time step, $\Delta t$,

\[
P(X \to X + 1) = (N - X)\eta_2\Delta t = (N - X)(\sigma_1 + hX)\Delta t = N^2\pi^+\Delta t,
\]

\[
P(X \to X - 1) = X\eta_1\Delta t = X[\sigma - 2 + h(N - X)]\Delta t = N^2\pi^-\Delta t,
\]

\[
P(X \to X \pm \Delta X) \approx 0, \quad \forall \Delta X > 1.
\]

Master equation, for $x = X/N$,

\[
\partial_t \omega(x, t) = N^2 \left\{ (E^+ - 1)[\pi^-(x)\omega(x, t)] + (E^- - 1)[\pi^+(x)\omega(x, t)] \right\}.
\]
The one-step operators:

\[ E^\pm[f(x)] = f(x \pm \Delta x) \approx f(x) \pm \Delta x \partial_x f(x) + \frac{\Delta x^2}{2} \partial_x^2 f(x), \]

Then the master equation leads to Fokker-Planck equation, which leads to Langevin equation:

\[ dx = [(1 - x)\eta_2 - x\eta_1] \, dt + \sqrt{\frac{(1 - x)\eta_2 + x\eta_1}{N}} \, dW. \]

And in the aforementioned case:

\[ dx = [(1 - x)\sigma_1 - x\sigma_2] \, dt + \sqrt{2hx(1 - x)} \, dW. \]
What can be borrowed from the Lotka-Volterra model?

L-V model is a prey-predator model. So it has:
- birth and death rates (agents can be created and destructed),
- asymmetric herding (predators have an upper hand).

Macroscopic model:

\[
\begin{align*}
\frac{dx}{dt} &= \left[\sigma_1(n-x) - \sigma_2 x + cx(n-x) + T_1(x)\right] \, dt + \sqrt{2hx(n-x)} \, dW, \\
\frac{dn}{dt} &= \left[T_1(x) + T_2(x)\right] \, dt.
\end{align*}
\]
Two distinct formulations of the model exist:

$$\eta_2 = \sigma_1 + hX, \quad \eta_1 = \sigma_2 + h(N - X),$$  \quad \text{(red)}

$$\eta_2 = \sigma_1 + h\frac{X}{N}, \quad \eta_1 = \sigma_2 + h\frac{N - X}{N}.$$  \quad \text{(blue)}

Figure: Global interaction (red) and local interaction (blue) case PDFs.
Where are the local interactions observed?

**The Bass diffusion model:**

\[
\eta_2 = \sigma + h \frac{X}{N}, \quad \eta_1 = 0, \quad \Rightarrow \quad dX = (N - X) \left( \sigma + h \frac{X}{N} \right) dt.
\]

**Figure:** ABhM (circles) and analytical model (the curve).
Let us now assume that we have $M$ controlled agents:

$$\eta_2 = \sigma_1 + h(M + X), \quad \eta_1 = \sigma_2 + h(N - X).$$

Figure: System of $N = 1000$ agents: without controlled agents, $M = 0$, (red) and with a small fraction of controlled agents, $M = 20$, (blue).
Let us now assume that we have $M$ controlled agents:

$$
\eta_2 = \sigma_1 + h\left(M + \frac{X}{N}\right), \quad \eta_1 = \sigma_2 + h\frac{N - X}{N}.
$$

Figure: System of $N = 1000$ agents: without controlled agents, $M = 0$, (red) and with a small fraction of controlled agents, $M = 20$, (blue).
Financial market modeling I
A simple model I

A model for the absolute modulating return, \( y = x/(1 - x) \):

\[
dy = \left[ \varepsilon_1 + (2 - \varepsilon_2)y^{1+\alpha} \right] (1 + y)dt_s + \sqrt{2y^{1+\alpha}}(1 + y)dW_s.
\]

Figure: The variety of reproducible PDF (a) and PSD (b) power laws: \( 2 < \lambda < 5, 0.5 < \beta < 2 \).
The previous SDE can be simplified to

\[ dy = \left( \eta - \frac{\lambda}{2} \right) y^{2\eta-1} dt_s + y^{\eta} dW_s, \]

\[ \eta = \frac{3 + \alpha}{2}, \quad \lambda = \varepsilon_2 + \alpha + 1. \]

This model incorporates:

- Bessel process \((\eta = 0)\),
- Squared Bessel process \((\eta = 0.5)\),
- CEV and CIR processes (if diffusion limiting is introduced).
Financial modeling III
Complex model I

Simplifications:

\[ x_1 = n_f, \quad x_2 = n_p, \quad x_3 = n_o, \]
\[ \sigma_{23} = \sigma_{32} = \sigma_{cc}, \quad \sigma_{12} = \sigma_{13} = \sigma_{fc}/2, \]
\[ \sigma_{21} = \sigma_{31} = \sigma_{cf}, \quad h_{12} = h_{13} = h_1. \]

Finally we arrive at:

\[ \begin{align*}
    d n_f &= \left[ \frac{(1 - n_f)\varepsilon_{cf}}{\tau(n_f, \xi)} - n_f \varepsilon_{fc} \right] dt_s + \sqrt{\frac{2n_f(1 - n_f)}{\tau(n_f, \xi)}} dW_{s,1}, \\
    d \xi &= -\frac{2H\varepsilon_{cc}\xi}{\tau(n_f, \xi)} dt + \sqrt{\frac{2H(1 - \xi^2)}{\tau(n_f, \xi)}} dW_{s,2}, \quad \tau(n_f, \xi) = \left[ 1 + \left| \frac{1 - n_f}{n_f} \xi \right|^\alpha \right]^{-1}.
\end{align*} \]
Figure: Empirical (blue) and model (red) statistical features of absolute return: PDF (a) and PSD (b).
“Agent-based and macroscopic modeling of the complex socio-economic systems” (to appear)

http://mokslasplius.lt/rizikos-fizika/en/
Thank you for attention!