Microscopic herding model leading to long-range processes and $1/f$ noise with application to absolute return in financial markets

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$1/f$ (1-over-$f$), or flicker, or pink, noise

It’s a type of noise with spectral density, $S(f)$, behaving as

$$S(f) \sim 1/f^\beta,$$

where $\beta \approx 1$. This type of noise is interesting as it lies in-between white noise and Brownian noise. Furthermore it is observed in a wide variety of physical, biological, economic, social systems.

Figure: Sample spectral density, $S(f) \sim 1/f$. 
Iterative equation for the inter-event time:

\[ \tau_{k+1} = \tau_k + \gamma \tau_k^{2\mu-1} + \sigma \tau_k^\mu \zeta_k, \quad S_\tau(f) \sim 1/f^{\beta}, \quad \beta = 1 + \frac{2(\mu \sigma^2 - \gamma)}{\sigma^2(2\mu - 3)}. \]

Recently used to model musical rhythm [Levitin et al., 2012 (PNAS)].
While SDE for signal intensity, $x$:

$$dx = \left( \eta - \frac{\lambda}{2} \right)x^{2\eta-1}dt_s + \sigma x^{\eta}dW_s.$$  

$$p(x) \sim x^{-\lambda}, \quad S_x(f) \sim 1/f^\beta, \quad \beta = 1 + \frac{\lambda - 3}{2(\eta - 1)}.$$  

This model can be further extended to reproduce sophisticated behavior of the financial markets (see [Gontis et al., 2010 (Sciyo)]).
Kirman’s herding model

X dynamics are determined by the one-step transition probabilities:

\[
p(X \rightarrow X + 1) = (N - X)\sigma_1 + hX(N - X),
\]
\[
p(X \rightarrow X - 1) = X\sigma_2 + hX(N - X).
\]
Experiment by Deneubourg

Taken from [Detrain & Deneubourg, 2006 (Physics of Life Reviews)].
Towards herding model for the financial markets I

If market is stabilized,

\[ D_f + D_c = 0, \]

\[ r(t) \approx r_0 \frac{X(t)}{N - X(t)} \Delta \xi(t). \]

One can assume that the two states in the population dynamics correspond to the chartist trading strategy, excess demand given by

\[ D_c = -r_0 X(t) \xi(t), \]

and fundamentalist trading strategy,

\[ D_f = [N - X(t)] \ln \frac{P_f}{P(t)}. \]
Stochastic model (explicitly derived from the previous ABM) for $y = \frac{X}{N-X}$ is given by:

$$dy = \left[ \varepsilon_1 + y \frac{2 - \varepsilon_2}{\tau(y)} \right] (1 + y) dt_s + \sqrt{\frac{2y}{\tau(y)}} (1 + y) dW_s,$$

$$\sigma_2 \rightarrow \frac{\sigma_2}{\tau(y)}, \quad h \rightarrow \frac{h}{\tau(y)},$$

$$\varepsilon_i = \frac{\sigma_i}{h}, \quad t_s = ht.$$
The SDE for $y \gg 1$ and assuming that $\tau(y) \sim y^{-\alpha}$ becomes:

$$dy = (2 - \varepsilon_2)y^{2+\alpha}dt_s + \sqrt{2}y^{\frac{3+\alpha}{2}}dW_s.$$ 

The comparison with

$$dx = \left(\eta - \frac{\lambda}{2}\right)x^{2\eta-1}dt_s + x^\eta dW_s,$$

yields: $\eta = \frac{3+\alpha}{2}$, $\lambda = \varepsilon_2 + \alpha + 1$. Thus we can expect that:

$$p(y) \sim y^{-\varepsilon_2 - \alpha - 1}, \quad S_y(f) \sim 1/f^\beta, \quad \beta = 1 + \frac{\varepsilon_2 + \alpha - 2}{1 + \alpha}.$$
Reproducing $1/f$ noise

Figure: Reproducing $1/f$ noise in three cases, $\alpha = 0$ (red squares), $\alpha = 1$ (blue circles) and $\alpha = 2$ (magenta triangles). Other model parameters were set as follows: $\varepsilon_1 = 0.1$, $\varepsilon_2 = 2 - \alpha$. All model data are fitted by: (a) $\lambda = 3$, (b) $\beta = 1$. 
The multifractal spectrum of the model

Figure: Checking for multifractality, using MF-DFA method, in the aforementioned distinct cases.

Note:
- $h(2) \approx 1$ for $\alpha = 1$ and $\alpha = 2$; $\alpha = 0$ case in the crossover region.
- model possesses multifractality.
Figure: Numerical results obtained from the CEV-like case, $\alpha = 0$ (red squares), $\alpha = 1$ (blue circles) and $\alpha = 2$ (magenta triangles). Other model parameters were set as follows: $\varepsilon_1 = \varepsilon_2 = 2$.

$$dy = \varepsilon_1 y dt_s + \sqrt{2y^{\frac{3+\alpha}{2}}} dW_s, \quad p(y) \sim y^{-3-\alpha}, \quad S_y(f) \sim 1/f^{1+\frac{\alpha}{1+\alpha}}.$$
Figure: Wide spectra of obtainable \( \lambda \) and \( \beta \) values. Model parameters were set as follows: \( \alpha = 1, \varepsilon_1 = 0.1, \varepsilon_2 = 0.1 \) (red plus), 0.5 (green cross), 1 (blue stars), 1.5 (magenta open squares), 2 (cyan filled squares) and 3 (orange open circles). Black curves correspond to the limiting cases: (a) \( \lambda_1 = 2 \) and \( \lambda_2 = 5 \), (b) \( \beta_1 = 0.5, \beta_2 = 2 \)
Key points

- Nonlinear stochastic model possessing power law spectral density, $S(f) \sim 1/f^\beta$, can be obtained from a microscopic agent based model.

- The nonlinear herding terms in the transition probabilities are essential in reproduction of $1/f$ noise.

- Introducing variability of herding interaction generalizes the model and enables more possibilities to reproduce $1/f$ noise.

For further reference see [Kononovicius & Gontis, 2012 (Physica A)], [Ruseckas, Kaulakys & Gontis, 2011 (EPL)].
http://mokslasplius.lt/rizikos-fizika/en

Thank You!