Periodic polling effects on the opinion dynamics in the noisy voter model

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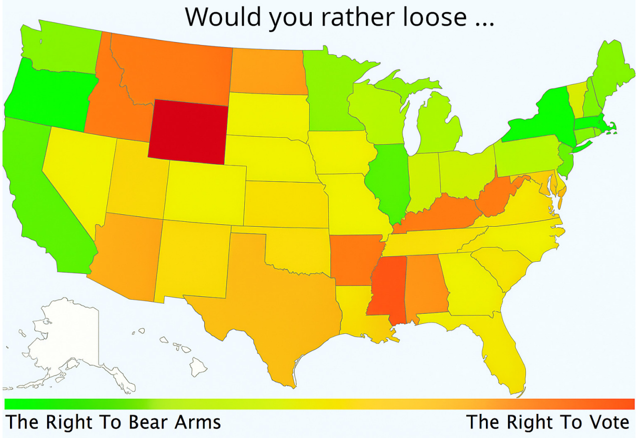
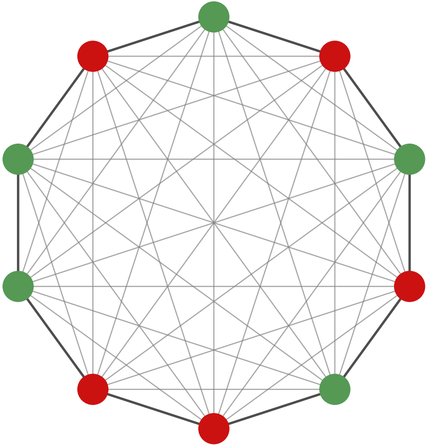
Do polls help discover truth (in real life)?



Source: theblog.okcupid.com (edited)

What about basic voter model?



Let us assume

- N identical voters
- binary opinions
- independent “exploration”
- face-to-face peer-pressure
- arbitrary social network

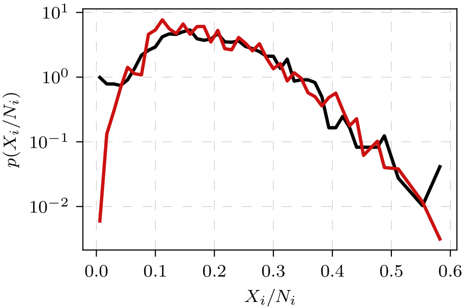
Then **polls could help**.

“Non-extensive” noisy voter model

- X – number of “1” voters
- ε – independent transition rate
- unit interaction rate

Converges to distribution:

$$X \sim \text{BetaBin}(\varepsilon, \varepsilon, N) \, .$$



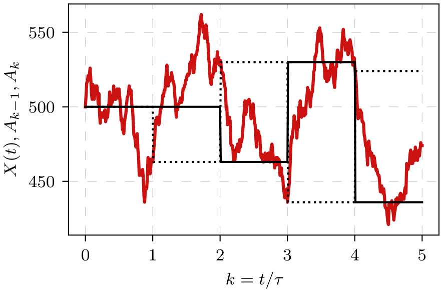
(red) Simulated PDF against (black) spatial empirical data.

Replicating election data: [Kononovicius (Complexity, 2017)], [Marmani *et al.* (Entropy, 2020)]

Peer-pressure through periodic polling

Original model rates:

$$\lambda^+(t) = (N - X(t)) \cdot [\varepsilon + X(t)]\,,$$
$$\lambda^-(t) = X(t) \cdot [\varepsilon + (N - X(t))]\,.$$



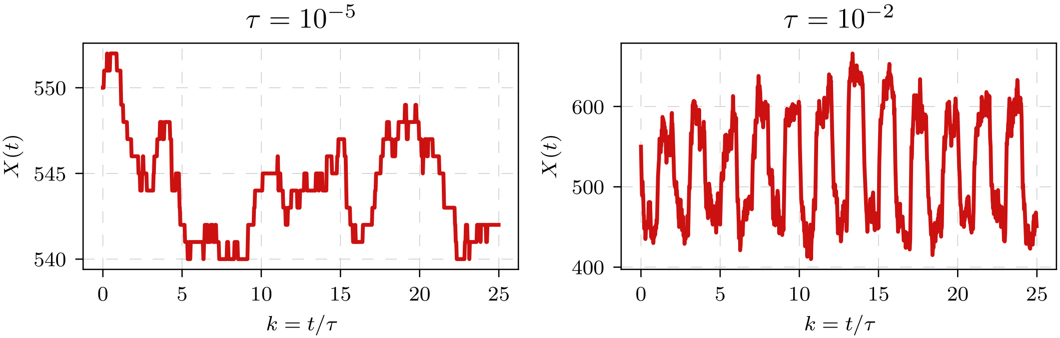
Voters do not interact with their "peers", **but know** the outcome of k-th poll,

$$A_k = X(k\tau)\,.$$

The modified model rates:

$$\lambda^+(t) = (N - X(t)) \cdot \left[\varepsilon + A_{\lfloor \frac{t}{\tau} \rfloor - 1}\right],$$
$$\lambda^-(t) = X(t) \cdot \left[\varepsilon + \left(N - A_{\lfloor \frac{t}{\tau} \rfloor - 1}\right)\right].$$

A quick look at time series



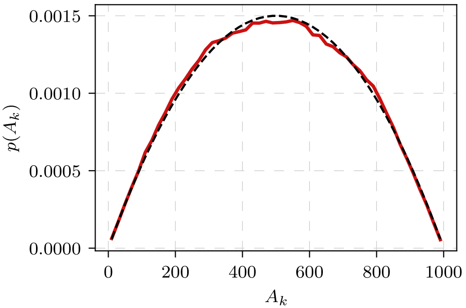
Short polling period limit

$\tau \ll N^{-2}$ limit is boring, because

- A_k updates are more common ($= \tau$)
- than X updates ($\gtrsim \tau$).

Known information traces the true state rather well,

$$A_{k-1} \approx X(t).$$



(red) Simulated PDF and (dashed) BetaBin $(\varepsilon, \varepsilon, N)$.

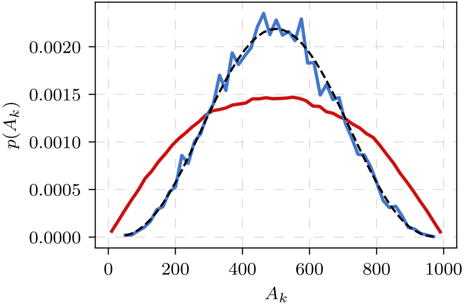
Long polling period limit

$\tau \gg \frac{1}{N+2\varepsilon}$ limit is a bit more interesting:

- A_k updates are much less common
- in comparison to X updates ($\ll \tau$).

True state incorporates known information rather well,

$$X(t) \approx A_{k-1}.$$



(red) PDF for small τ, (blue) PDF for large τ, (dashed) BetaBin $(2\varepsilon, 2\varepsilon, N)$

Understanding the poll model

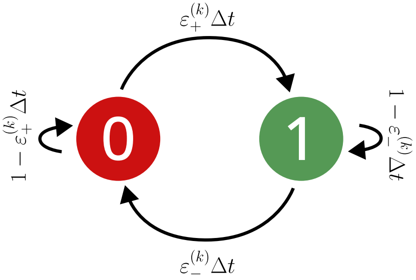
Original model rates:

$$\lambda^{\pm} = a_0^{\pm} + a_1^{\pm} X + a_2^{\pm} X^2,$$

but not for the poll model. Because between polls:

$$\varepsilon + A_{k-1} = \text{const} = \varepsilon_+^{(k)},$$

$$\varepsilon + (N - A_{k-1}) = \text{const} = \varepsilon_-^{(k)}.$$



Single agent Markov chain.

State "1" occupation probability in continuous time limit:

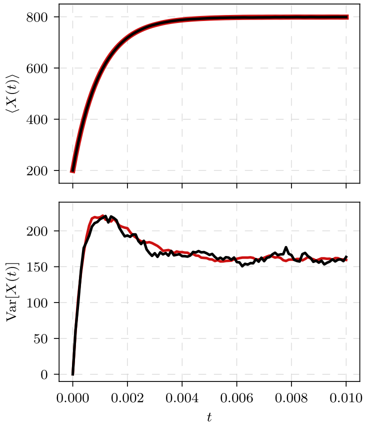
$$\boxed{P_1\left(\theta | P_1(0)\right) = P_1(\infty) + \left[P_1(0) - P_1(\infty)\right] \exp\left[-\left(2\varepsilon + N\right)\theta\right],} \quad P_1(\infty) = \frac{\varepsilon + A_{k-1}}{2\varepsilon + N}.$$

Macroscopic simulation method

We can generate two Binomial rvs, $B[N,p]$,

$$X\left(t+\theta\right)=B\left[X\left(t\right),P_1\left(\theta|1\right)\right]+B\left[N-X\left(t\right),P_1\left(\theta|0\right)\right],$$

instead of running microscopic simulation.



($\Longrightarrow$) Ensemble mean and variance: micro (black) and macro (red) simulations. Parameters: $\varepsilon = 2$, $\tau = \infty$, $A_{-1} = 800$, $X(0) = 200$, $N = 10^3$.

Equivalent AR(2) process

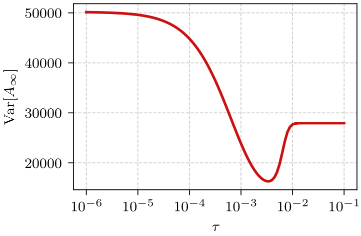
Based on the macroscopic simulation method:

$$A_{k+1} = \varphi_1 A_k + (1-\varphi_1)\,\varphi_2\,(\varepsilon_1 + A_{k-1}) + \xi_{k+1},$$

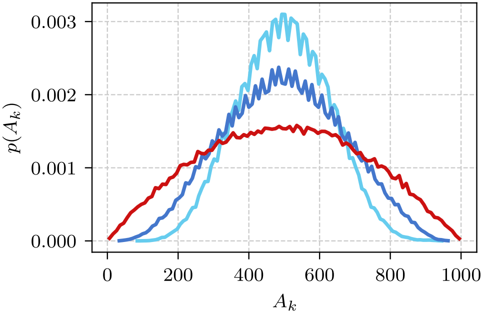
with ξ_k being white noise,

$$\varphi_1 = \exp\left[-\left(2\varepsilon + N\right)\tau\right], \quad \varphi_2 = \frac{N}{2\varepsilon + N}.$$

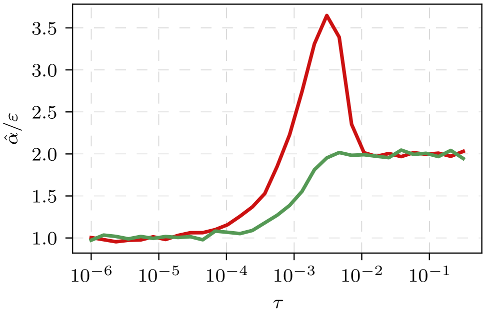
- Obtaining $\langle A_\infty \rangle$ is trivial.
- Yule–Walker equations tell us $\operatorname{Var}[A_\infty]$.



Intermediate polling periods

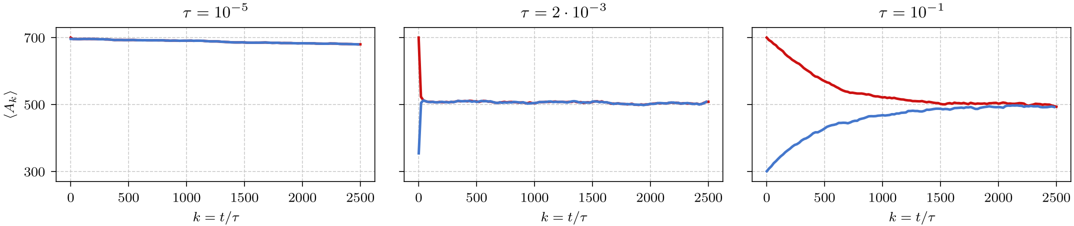


PDF for (red) small τ, (blue) large τ, (cyan) intermediate τ.



$\hat{\alpha}$ of BetaBin$(\hat{\alpha}, \hat{\alpha}, N)$: (red) with delay, (green) without delay.

The intuition



Ensemble mean: (red) even and (blue) odd polls.

(left) small τ limit: $\varphi_1 \approx 1$ and

$$A_{k+1} \approx A_k + \xi_{k+1}.$$

Single $\mathrm{AR}(1)$ process.

(right) large τ limit: $\varphi_1 \approx 0$ and

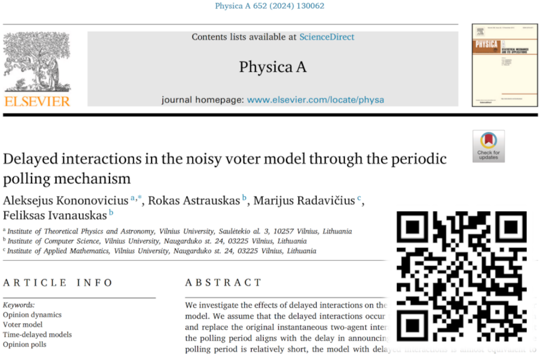
$$A_{k+1} \approx \varphi_2 \left(\varepsilon + A_{k-1}\right) + \xi_{k+1}.$$

Two independent processes.

Thank you!

Key points:

- Novel delay mechanism
- Nontrivial scaling
- ARCH-like opinion models



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Delayed interactions in the noisy voter model through the periodic polling mechanism

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ABSTRACT

We investigate the effects of delayed interactions on the
model. We assume that the delayed interactions occur
and replace the original instantaneous two-agent inter
the polling period aligns with the delay in announcin
polling period is relatively short, the model with delay