

Introduction

Although all votes are equally important, the probability that one single vote will decide the outcome of the election is negligible. Thus the utility of voting in the election appears to be small, while the cost of voting is obvious. Thus it seems to be rational to abstain from voting. A few early game theory papers, e.g., [1], shown promise to resolve the voting paradox as well as explain election results. But it appears that such feat is possible only in rather limited circumstances [2]. This could just show that people are seldom rational (at least in game theoretic sense).

This paradox can be understood by considering vast knowledge outside game theory. One could explain the paradox from the behavioral science perspective [3] as well as from the statistical physics [4]. The behavioral models, while are realistic, are extremely complex, while physical models are often criticized for detaching the human nature from the social atoms. The ideal should combine the best of these two worlds – be plausible from behavioral perspective as well as be simple and tractable enough to be useful.

Another important aspect of the opinion dynamics is the analysis of the available empirical data. Most of the empirical papers are dedicated to analyzing election data in countries with established democratic tradition. Here we analyze voting data from parliamentary elections in a young democracy, Lithuania. Also most of the previous works ignored the matter of party vote share statistics, as it is often said that party vote share heavily depends on the policies the party promises to implement [5]. Here we approach voting for the parties as complex social phenomena. From empirical point of view we consider party vote share distributions over all polling stations during the same elections.

From this point of view detailed parliamentary election data was studied in very few approaches. In political science literature one can find attempts to forecast Lithuanian parliamentary elections in macro-scale [6]. Smaller scale data is also often analyzed, but with different goals in mind – to provide comment on social, economic and demographic changes in Lithuania [7, 8, 9].

Parts of the research presented here are covered in [10].

Empirical analysis

We analyze the empirical data from two perspectives – using probability density functions (PDF) and rank-size (RS) distributions. These two approaches are equivalent, but using both of them reveals a more detailed picture.

In both cases we analyze the same parameter – parties' vote shares over different polling stations.

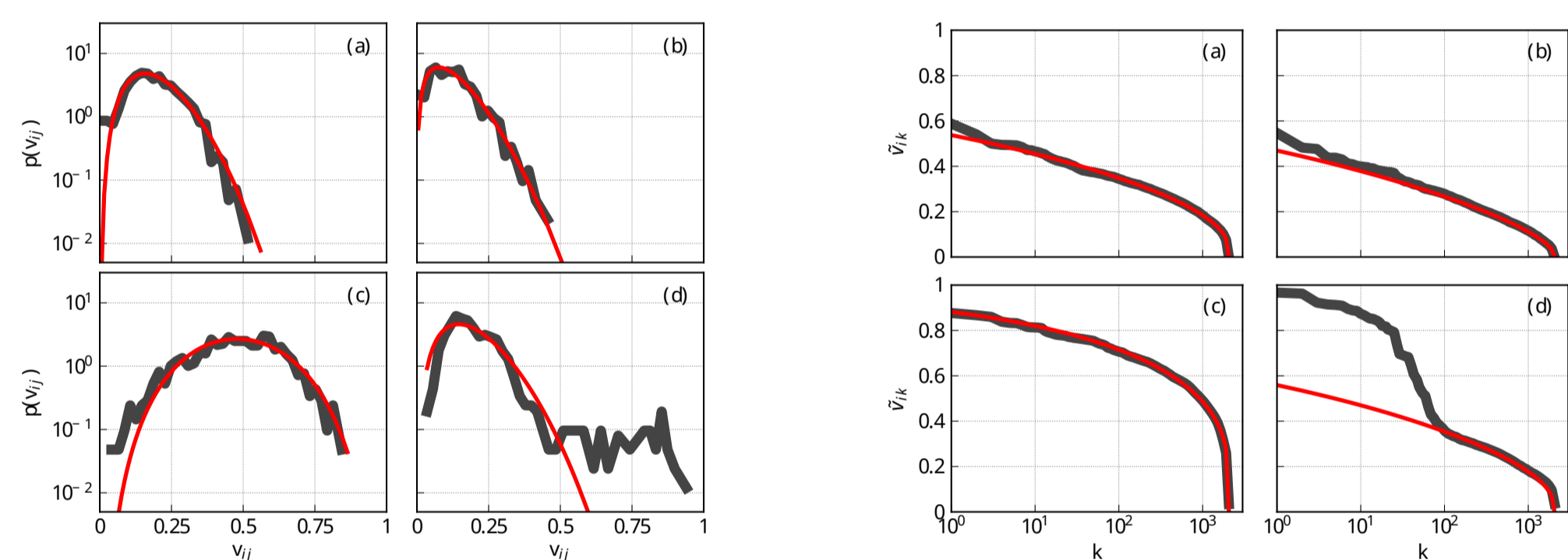


Fig 1: PDF (left) and RS (right) statistics obtained from the 1992 parliamentary election voting data. Parties considered: (a) - SK, (b) - LKDP, (c) - LSDP, (d) - others combined. Empirical curves are gray, while red curves are theoretical fits (assuming Beta distribution).

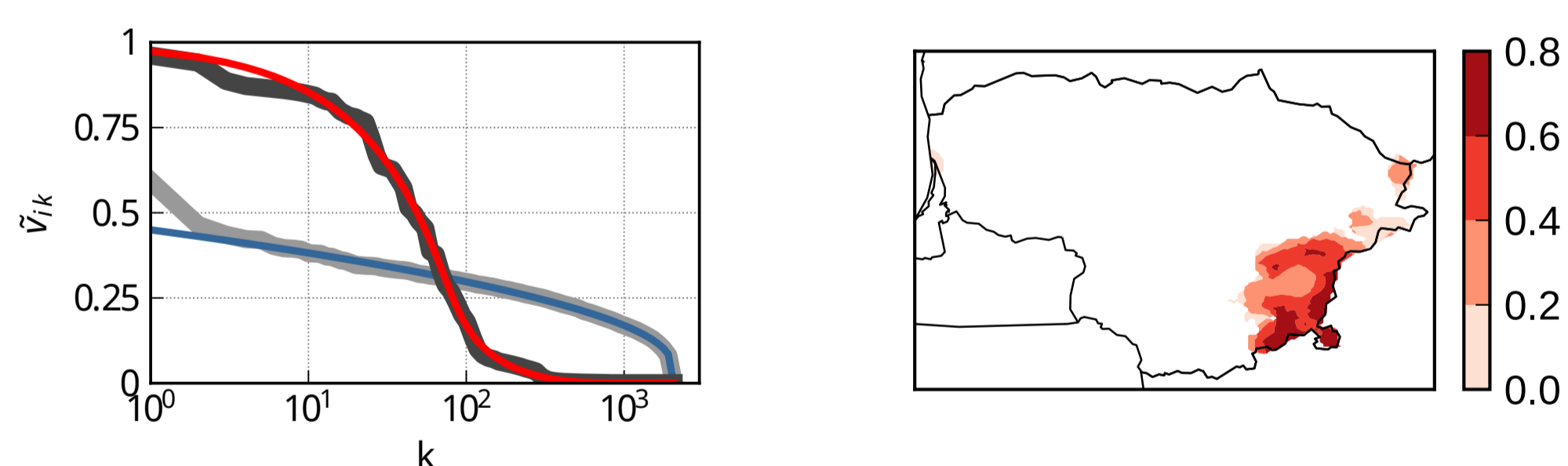


Fig 2: The deviation observed in Fig 1 (d) is observed due to minority vote segregation (observed for minority representing party). Map shown on the right. We can account for segregation by assuming that data is distributed according to double Beta distribution.

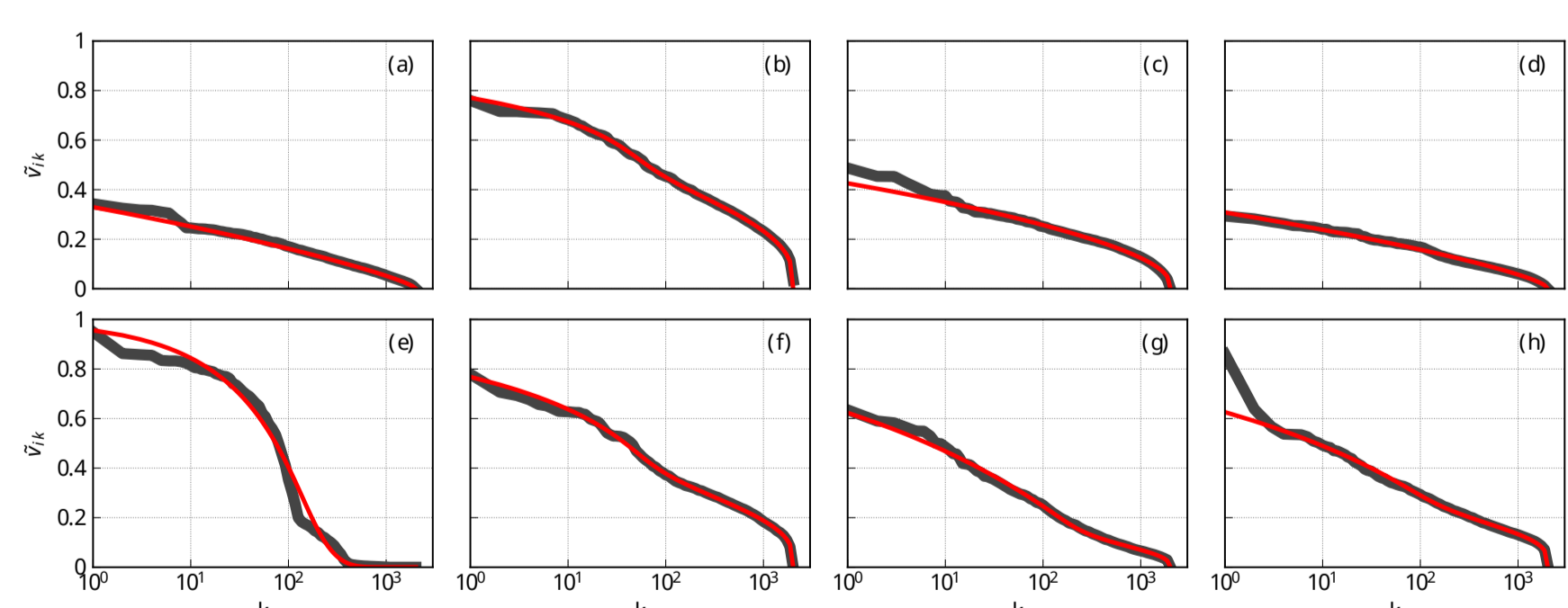


Fig 3: Segregation pattern observed in the empirical data from a later election (data of 2012 election is shown). Parties considered: LRLS (a), DP (b), TS-LKD (c), DK (d), LLRA (e), LSDP (f), TT (g) and others combined (h). Red curves are fits provided by assuming that data is distributed according to a double Beta distribution.

Agent-based model

Here we generalize a well known two state model [11] to account for switching between multiple states. In Fig 4 we illustrate the model for the free state case. This model is equivalent to the noisy voter model [12] on complete or random graph [10, 13].

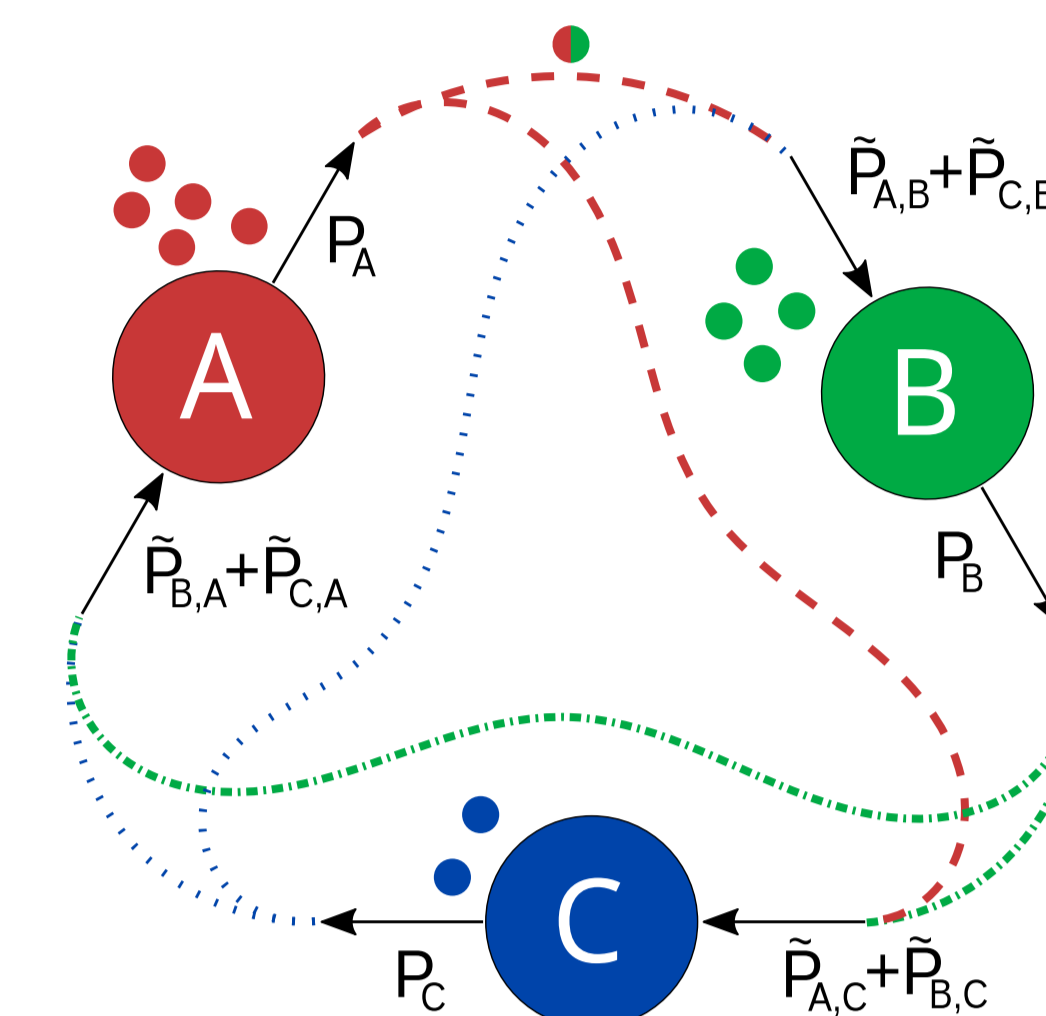


Fig 4: Model schema with the three available states. Notation explained in text.

Probability for a single agent to switch from party i to j during time interval Δt is given by:

$$\tilde{P}_{i,j} = X_i (\sigma_{i,j} + hX_j) \Delta t. \quad (1)$$

Here σ controls idiosyncratic behavior, independent discovery of the state, while there is also recruitment term which depends on parameter h .

If there are only two states, then it is to show that $x_j = X_j/N$ is distributed according to the Beta distribution, $x_j \sim \mathcal{Be}(\sigma_{i,j}, \sigma_{j,i})$. If there are more than two state, unless $\varepsilon_{i,j} = \varepsilon_j$, x_j distribution will not be the Beta distribution. Thus we simplify the model by making this assumption. As is shown in the paper this assumption must be violated if one wants to reproduce results of the less-popular parties.

Under the simplifying assumption we can write the exit probabilities (probability that voter will no longer vote for party):

$$P_i = X_i \sum_{j \neq i} (\varepsilon_j + X_j) h \Delta t. \quad (2)$$

Reproducing the results of the 1992 elections

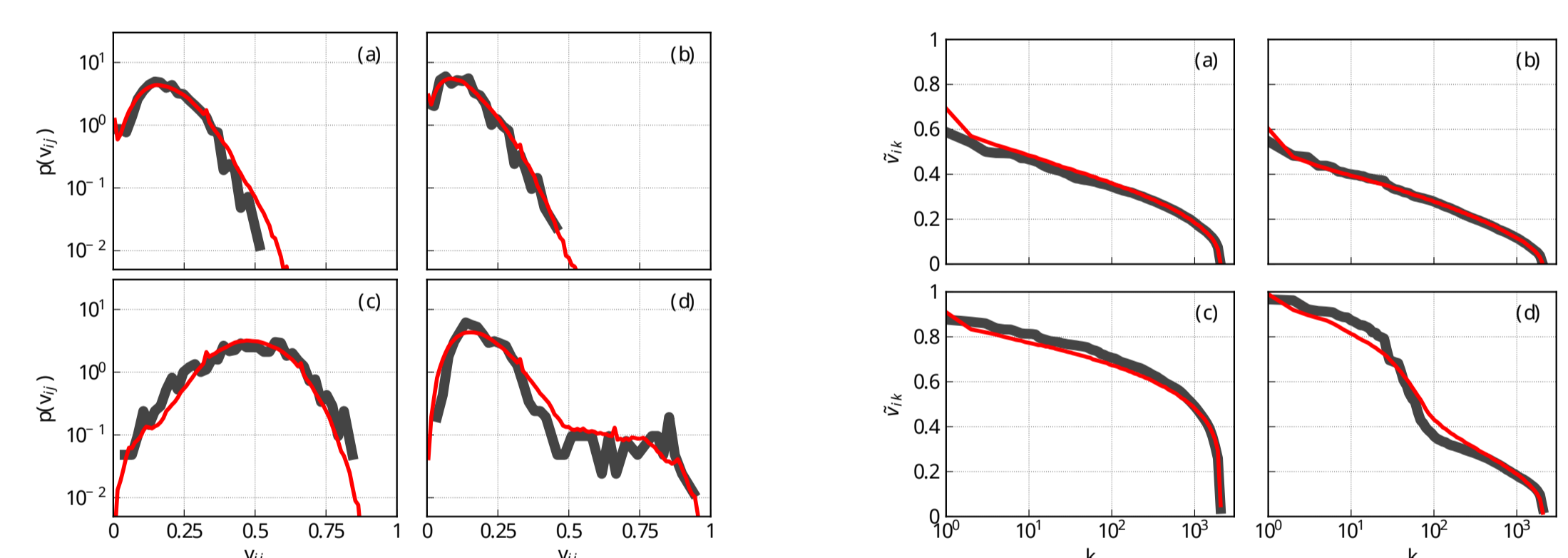


Fig 5: Comparison of PDFs (left) and RS (right) statistics obtained numerically from the model (red curve) and the empirical data (gray curves). Parties considered: (a) - SK, (b) - LKDP, (c) - LSDP, (d) - others combined. Model parameters were obtained using Bayesian inference.

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