Cutting Planes Cannot Approximate Some Integer Programs☆

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Abstract

For every positive integer l, we consider a zero-one linear program describing the following optimization problem: maximize the number of nodes in a clique of an n-vertex graph whose chromatic number does not exceed l. Although l is a trivial solution for this problem, we show that any cutting-plane proof certifying that no such graph can have a clique on more than rl vertices must generate an exponential in $\min\{l, n/rl\}^{1/4}$ number of inequalities. We allow Gomory–Chvátal cuts and even the more powerful split cuts. This extends the results of Pudlák [J. Symb. Log. 62:3 (1997) 981–998] and Dash [Math. of Operations Research 30:3 (2005) 678–700; Oper. Res. Lett. 38:2 (2010), 109–114] who proved exponential lower bounds for the case when $l = n^{2/3}$ and $r = 1$.

Key words: Cutting planes, monotone circuits, interpolation, clique problem

1. Introduction

The study of inapproximability of combinatorial optimization problems has two strands. One of them shows the inapproximability by *any* algorithm working in polynomial time. But the results here rely on some unproven hypotheses in computational complexity, like $P \neq NP$. The other strand—"concrete inapproximability"—deals with restricted classes of algorithms (like backtracking, dynamic programming, cutting planes, etc.) and tries to prove *unconditional* inapproximability results. In this note we follow this last direction.

An integer linear optimization problem (ILP) is specified by an integer $m \times n$ matrix A and two integer vectors $b \in \mathbb{Z}^m$ and $e \in \mathbb{Z}^n$. This gives us a rational polyhedron $P = \{x \in \mathbb{R}^n \colon Ax \leq b\}$. The problem itself is to find an *integer* solution $x \in \mathbb{Z}^n$ maximizing a given linear target function $e^T x$ subject to constraints $Ax \leq b$. A solution $y \in P \cap \mathbb{Z}^n$ is *r-approximative* ($r \geq 1$) if $e^T y \geq l/r$ holds, where $l = \max\{e^T x \colon x \in P \cap \mathbb{Z}^n\}$ is the otpimal value of an integer solution.

The main idea of so-called cutting plane algorithms is to eliminate the necessity of integrity constraints by introducing more and more new inequalities (cutting planes). One of the most popular ways to produce cutting planes are Gomory–Chvátal cuts. These are obtained by taking rounded linear combinations of already obtained inequalities. Namely, one takes a non-negative vector λ such that $\lambda^T A$ is an integer vector, and adds to $Ax \leq b$ the inequality $\lambda^T Ax \leq \lfloor \lambda^T b \rfloor$. Since the vector $\lambda^T A$ is integral, every *integer* solution of $Ax \leq b$ is also a solution of $\lambda^T A \leq \lfloor \lambda^T b \rfloor$. Moreover, by Carathéodory's theorem, one may assume that λ has at most $n+1$ nonzero positions. Thus, each new inequality in such an algorithm is derived from only at most $n+1$ previous inequalities.

Suppose that a cutting plane algorithm outputs some integral solution $y \in P \cap \mathbb{Z}^n$. We are not interested in *how* it does this at the end. But if the algorithm "claims" it is r-approximative, then the value $e^T y$ of this solution must be at least l/r, implying that the inequality $e^T x \leq r \cdot (e^T y)$, and hence, also the inequality $e^T x \leq r \cdot l$ must hold in $P \cap \mathbb{Z}^n$. Thus, the sequence of inequalities produced by such an algorithm is, in fact, a cutting plane *derivation* of the inequality $e^T x \leq rl$ from the initial system of inequalities $Ax \leq b$. By adding the inequality $-e^T x \leq -rl-1$ to the derived inequality $e^T x \leq rl$ we obtain a cutting plane derivation of the inequality $0 \leq -1$ from the system $Ax \leq b$ augmented by the inequality $e^T x \geq rl + 1$, that is, we obtain a cutting plane *proof* of integer infeasibility of this augmented system. The *length* of such a proof is the number of produced inequalities.

More generally, for any cutting-plane proof of $e^T x \leq d$, there is always a cutting-plane proof of the same length showing that no integral solutions satisfy $e^T x \geq d+1$, whereas the converse is not true (see Cook et. al. [7] for a discussion of indirect cutting-plane proofs). In this sense cutting-plane proofs of integer infeasibility are even more general than cutting-plane proofs of optimality.

Chvátal, Cook and Hartmann [6] proved the first exponential lower bounds on the number of cutting planes gen-

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erated by Gomory–Chvátal cutting-plane algorithms for the traveling salesman problem. Their bounds are, however, exponential in the number of variables but not in the number of constraints (which is also exponential). Lower bounds exponential in the number of constraints were proved by Bonet, Pitassi and Raz [3] under a restriction that cuts must have polynomially bounded coefficients. Finally, Pudlák [13] was able to prove such a bound without any restriction on coefficients. Pudlák's result was later extended by Dash [8] to branch-and-cut proofs that use 0-1 branching and lift-and-project cuts, and to proofs using so-called split cuts [9] including Gomory–Chvátal cuts and lift-and-project cuts as special cases.

Note. In what follows, by a *cutting-plane proof* (CP proof) of an integer infeasibility $Ax \leq b$ we will mean a branch-and-cut proof where only branchings on variables ($x_i \leq 0$ and $x_i \geq 1$) are allowed but arbitrary split cuts (including Gomory–Chvátal cuts and lift-and-project cuts) can be used (see [8] for details).

1.1. *Our result*

The optimization problem considered in [13, 8, 9] is

$$\text{maximize } \sum_{i=1}^n x_i \quad \text{subject to } F_l(x,y,z) \text{ and } x,y,z \text{ integral,} \tag{1}$$

where $F_l(x,y,z)$ is the system of inequalities in $n + \binom{n}{2} + nl$ variables: for all $1 \leq i \neq j \leq n$ and $1 \leq c \leq l$ the system F_l contains inequalities $0 \leq x_i, y_{i,j}, z_{i,c} \leq 1$ together with:

$$x_i + x_j - y_{i,j} \leq 1 \tag{2}$$

$$z_{i,c} + z_{j,c} + y_{i,j} \leq 2 \tag{3}$$

$$z_{i,1} + \cdots + z_{i,l} = 1 \tag{4}$$

If treated as boolean variables, their interpretation is the following. The variables $y_{i,j}$ correspond to edges of a graph G_y on n vertices, where vertices i and j are adjacent iff $y_{i,j} = 1$. The x-variables describe a subset of vertices $S_x = \{i\colon x_i = 1\}$, and z-variables describe a coloring of vertices by l colors: $z_{i,c} = 1$ iff vertex i receives color c. Intuitively, inequalities (2) ensure that, for every 0-1 solution (x,y,z) for this system, the set of vertices $S_x = \{i\colon x_i = 1\}$ defined by vector x forms a clique in the graph G_y defined by vector y. Inequalities (3) ensure that no two adjacent vertices $i \neq j$ of G_y receive the same color, and (4) ensure that every vertex receives a color.

Since l colors are not enough to color a clique on $l+1$ vertices, l is the value of an optimal zero-one solution of this optimization problem. Let $F_{l,r}(x,y,z)$ denote the system $F_l(x,y,z)$ augmented with the inequality

$$x_1 + \cdots + x_n \geq rl + 1 \,.$$

Hence, $F_{l,r}$ is integer infeasible (has no zero-one solutions), for any $r \geq 1$.

What the authors in [13, 8, 9] actually prove is that, for $l = n^{2/3}$ and $r = 1$, any cutting-plane proof of the integer infeasibility of $F_{l,r}$ must generate an exponential number of inequalities.

In this note we show that a super-polynomial number of inequalities is also necessary to approximate the optimal solution of (1) within a factor $r = n^{1-\Omega(1)}$.

Theorem 1. *For every positive integers r and l such that $rl \leq n$, the number of inequalities produced by any cutting-plane proof of integer infeasibility of $F_{l,r}(x,y,z)$ is exponential in* $\min\{l, n/rl\}^{1/4}$.

In particular, if $l = (\log n)^8$ and $r \leq n/l^2$ then $n^{\Omega(\log n)}$ inequalities are necessary. That is, although the inequality $\sum_i x_i \leq l$ holds for every 0-1 solution (x,y,z) of $F_l(x,y,z)$, no cutting-plane proof of polynomial length can prove that even the weaker inequality $\sum_i x_i \leq rl$ holds for every such solution.

2. Tools we use

The proof of the theorem itself is a combination of two known results, one from circuit complexity and the other from proof complexity. So, the contribution of this note is just the observation that these two results in computational complexity do in fact imply that cutting-plane algorithms not only fail to efficiently find an optimal solution of some integer optimization problems—they cannot even approximate it within any non-trivial factor.

The result from the circuit complexity we use concerns so-called *monotone real circuits*. Such a circuit can use any real valued functions $f : \mathbb{R}^2 \to \mathbb{R}$ as gates. The only requirement is that f must be monotone: If $x_1 \leq x_2$ and $y_1 \leq y_2$ then $f(x_1,y_1) \leq f(x_2,y_2)$. A circuit for a boolean function $g(x_1,\ldots,x_n)$ is a sequence $g_1, g_2, \ldots, g_t$ of monotone real functions, where $g_i = x_i$ for $i = 1,\ldots,n$, each g_i for $i > n$ is of the form $g_i = f(g_j, g_k)$ with $j,k < i$ and f a monotone real function of two variables, and $g_t = g$. The size of such a circuit is the total number t of gates in it.

An n-vertex graph is a *k-clique* if it consists of a clique (complete graph) on k vertices and $n-k$ isolated vertices. A *k-coclique* is a graph whose complement consist of $\leq k$ vertex-disjoint cliques. An (a,b)-*clique function* ($3 \leq a < b \leq n$) is a monotone boolean function whose inputs are n-vertex graphs encoded by $\binom{n}{2}$ boolean variables x_e, each for one potential edge e. It accepts every b-clique and rejects every a-coclique; on other graphs, the function can take arbitrary values—monotonicity of the function is the only restriction.

In his proof that the optimal solution of (1) cannot be found by polynomial-time cutting-plane algorithms, Pudlák [13] used the fact (proved in the same paper) that any monotone real circuit computing an (a,b)-clique function with $a = n^{2/3}$ and $b = a+1$ requires exponentially many gates. In the case of *approximation* algorithms we need a similar result in the case when the gap $b-a$ is large. For monotone *boolean* circuits such a result was proved by Alon and Boppana [1] based on earlier result of Razborov [14]. For monotone *real-valued* circuits such a result can be proved using a general lower bounds criterion from [11].

Theorem 2. *The number of gates in any monotone real circuit computing an (a,b)-clique function on graphs with n vertices is exponential in* $\min\{a, n/b\}^{1/4}$.

We give the proof in the Appendix.

The result from proof complexity we use relates the length of cutting plane proofs with the size of monotone circuits. Suppose that our system of inequalities has the form $F(x,y,z) = F_1(x,y) \wedge F_2(y,z)$, where the inequalities in $F_1(x,y)$ do not have z-variables, and those in $F_2(y,z)$ do not have x-variables. Suppose that $F(x,y,z)$ is unsatisfiable, that is, has no zero-one solution. Then, for any truth assignment α to the y-variables, at least one of systems $F_1(x,\alpha)$ and $F_2(\alpha,z)$ must be unsatisfiable, for otherwise $F(x,\alpha,z)$ would be satisfiable. A so-called "interpolant" just tells us which of these two system is unsatisfiable.

An *interpolant* of F is a boolean function $I(y)$ (on the common variables y) such that for any truth assignment α to the y-variables:

- if $I(\alpha) = 0$ then $F_1(x,\alpha)$ has no 0-1 solution x;
- if $I(\alpha) = 1$ then $F_2(\alpha,z)$ has no 0-1 solution z.

Say that a system $F_1(x,y) \wedge F_2(y,z)$ of linear inequalities is *separated* if in at least one of the systems F_1 and F_2, all y-variables appear in all inequalities of this system with only non-negative coefficients, or all appear with non-positive coefficients.

The following theorem was proved Pudlák [13] based on earlier ideas of Krajicek [12] and Bonet, Pitassi and Raz [3].

Theorem 3 (Pudlák [13])**.** *If an unsatisfiable system F of linear inequalities is separated then it has an interpolating monotone real circuit of size polynomial in the minimal length of a cutting plane proof of F.*

Pudlák [13] proved this result for proofs using Gomory–Chvátal cuts. Dash [8, 9] extended it to proofs using arbitrary split cuts. As observed by Dash [9], Theorem 3 is quite general: it holds for any proof system whose derivation rules, producing in one step a new inequality (a "cut") $c^\top x \le d$ from a system $Ax \le b$ of already derived inequalities, satisfy the following three conditions:

1. If $g^T x + h^T y \le d$ is a cut for $Ax + By \le c$, then for any 0-1 assignment α to the y-variables, $g^T x \le d - h^T \alpha$ is a cut for $Ax \le c - B\alpha$.
2. If $g^T x + h^T y \le d$ is a cut for $Ax \le e, By \le f$, then there are numbers r and s such that $g^T x \le r$ is a cut for $Ax \le e$, and $h^T y \le s$ is a cut for $By \le f$, and $r + s \le d$.
3. The numbers r can be computed from A, e or the numbers s can be computed from B, f with polynomially many monotone operations.

3. Proof of Theorem 1

Let T be the minimum number of inequalities in a cutting-plane derivation of $0 \le -1$ from the system $F_{l,r}(x,y,z)$. The system itself consists of two systems Clique(x,y) and Color(y,z), where Clique(x,y) consists of all inequalities

$$\begin{aligned} &x_i + x_j - y_{i,j} \le 1 \\ &-x_1 - \cdots - x_n \le -rl - 1 \\ &0 \le x_i \le 1 \\ &-y_{i,j} \le 0 \end{aligned}$$

stating that the graph G_y has a clique on $rl + 1$ vertices, whereas Color(y,z) consists of all inequalities

$$\begin{aligned} &z_{i,c} + z_{j,c} + y_{i,j} \le 2 \\ &z_{i,1} + \cdots + z_{i,l} = 1 \\ &0 \le z_{i,c} \le 1 \\ &y_{i,j} \le 1 \end{aligned}$$

stating that the graph G_y is l-colorable. Since all y-variables appear in Clique(x,y) with non-positive coefficients (as well as in Color(y,z) with non-negative coefficients), the system $F_{l,r}$ is separated. We can therefore apply Theorem 3 and obtain a monotone real circuit $I(y)$ of size $S \le T^{O(1)}$ computing an interpolant of $F_{l,r}$.

Claim 4. *The circuit $I(y)$ computes an $(l, rl+1)$-clique function.*

Proof. Every assignment $\alpha \in \{0,1\}^{\binom{n}{2}}$ to y-variables describes a graph G_α. We have only to show that $I(\alpha) = 1$ if G_α is a clique on $rl + 1$ vertices, and $I(\alpha) = 0$ if G_α is complete l-partite graph.

If G_α is a complete l-partite graph, then it can be colored by l colors, implying that the system Color(α,z) *has* a 0-1 solution z (an l-coloring of G_α). By the definition of the interpolant, we have that $I(\alpha) = 0$ in this case.

If G_α is a clique S on $|S| = rl + 1$ vertices, then the incidence 0-1 vector x of S is a solution of Clique(x,α). By the definition of the interpolant, we have that $I(\alpha) = 1$ in this case. ∎

From Claim 4 and Theorem 2 we obtain that the circuit $I(y)$ must have size S exponential in $\min\{l, n/(rl)\}^{1/4}$. Since, by Theorem 3, S is polynomial in the total number T of inequalities produced by the algorithm, this implies that T must be exponential in $\min\{l, n/(rl)\}^{1/4}$ as well. ∎

4. Conclusion

Pudlák [13] exhibited a linear program $l = \max\{e^T x : Ax \le b, x \in \mathbb{Z}^n\}$ such that any Chvátal–Gomory cutting planes proof that $e^T x \le l$ is valid for all integer solutions x must generate an exponential (in the number of variables) number of inequalities. Dash [8, 9] has shown that the result remains also valid if one allows more powerful cutting rules. We show that cutting plane proofs of polynomial length cannot even prove a much weaker inequality $e^T x \le rl$, as long as $l \ge \log^8 n$ and $r \le n/l^2$.

A disadvantage of these results is that they say nothing about the complexity of cutting plane proofs for natural optimization problems, like the *maximum independent set* (MIS) problem. Given a graph G on n vertices $[n] = \{1, \ldots, n\}$, we can associate a variable x_i to each its vertex i and consider the system consisting of inequalities $\sum_{i \in S} x_i \leq 1$ for all cliques $S \subseteq [n]$ in G, and $x_i \geq 0$ for all vertices $i \in [n]$. Every integral solution x for this system is a 0-1 vector which corresponds to an independent set $I_x = \{i \colon x_i = 1\}$ in G, and the incidence 0-1 vector of every independent set is a valid solution. The problem is to maximize the sum $\sum_{i=1}^n x_i$ over all integral solutions of this system. We are interested in cutting-plane derivation of $\sum_{i=1}^n x_i \leq \alpha(G)$ from this system, where $\alpha(G)$ is the maximum cardinality of an independent set in G. An upper bound $\binom{n}{\alpha(G)}$ on the length of such derivations was proved by Chvátal, Cook and Hartmann [6, Theorem 6.2]. But to our best knowledge, no non-trivial *lower* bound is known.

The best known polynomial time approximation algorithm of Boppana and Halldórsson [5] for the MIS problem achieves an approximation ratio $n/\log^2 n$. (Note that n is a trivial approximation ratio for n-vertex graphs.) Hastad [10] has proved that the MIS problem cannot be approximated by any polynomial time algorithm within a factor of $n^{1-\epsilon}$, unless every problem in NP can be solved in probabilistic polynomial time. This is a very strong result (it captures *all* possible algorithms running in polynomial time). A disadvantage, however, is that it relies on an unproven conjecture concerning complexity classes.

Thus, a super-polynomial lower bound on the length of cutting-plane derivations of $\sum_{i=1}^n x_i \leq \alpha(G)$ from the system of independent set inequalities of G would be interesting, even if it only concerns a particular class of algorithms—cutting plane algorithms. Such a lower bound in the case of resolution proofs was already given by Beame, Impagliazzo and Sabharwal in [2]. But resolution is also known to be exponentially weaker than cutting plane proofs on some instances, like systems of inequalities describing the pigeonhole principle.

5. Appendix: Proof of Theorem 2

The proof is almost the same as that of Theorem 3.4 in [11]. Fix a set V of $|V| = n$ vertices, and an (a, b)-clique function $f : 2^{\binom{V}{2}} \to \{0, 1\}$ for graphs $G \subseteq \binom{V}{2}$ on these vertices; we look at graphs as sets of their edges. An a-coloring is a mapping $h : V \to \{1, \ldots, a\}$. Each such mapping defines the graph $G_h = (V, E)$ where $\{u, v\} \in E$ iff $h(u) = h(v)$. Note that complements of such graphs are a-cocliques, and hence, must be rejected by f. In the proof of Theorem 3.4 in [11] a measure $\nu(G) - \kappa(G)$ for rejected graphs was used, where $\nu(G)$ is the set of vertices incident with at least one edge of G, and $\kappa(G)$ is the number of connected components in G. Since every tree on m vertices has $m-1$ edges, $\nu(G) - \kappa(G)$ is just the maximum number $|F|$ of edges in a spanning forest $F \subseteq G$. With this proviso, when adopted to this special function f, the lower bounds criterion in [11, Theorem 2.2] states the following.

If an (a, b)-clique function can be computed by a monotone real-valued circuit of size t then, for all integers $1 \leq r, s \leq n-1$ there exist a set E of $|E| \leq s^2$ edges, a family $\mathscr{Q}$ of $|\mathscr{Q}| \leq t(2s)^{4r}$ r-cliques, and a family $\mathscr{F}$ of $|\mathscr{F}| \leq t(2r)^{4s}$ forests with $|F| \geq s$ edges in each $F \in \mathscr{F}$ such that at least one of the following two conditions hold:

(i) Every b-clique either intersects the set E or contains at least one clique $Q \in \mathscr{Q}$.
(ii) For every a-coloring h, the graph G_h must contain at least one forest $F \in \mathscr{F}$.

To show that the size t of a circuit must be exponential in $\min\{a, n/b\}^{1/4}$, set $r := \lfloor (a/32)^{1/4} \rfloor$ and $s := \lfloor (n/32b)^{1/4} \rfloor$. Suppose first that, for this choice of parameters, the first condition (i) holds. At least $\binom{n}{b} - s^2\binom{n-2}{b-2} \geq \frac{1}{2}\binom{n}{b}$ of b-cliques must avoid a fixed set I of $|I| \leq s^2$ edges. Each of these b-cliques must contain at least one of r-cliques Q_i. Since only $\binom{n-r}{b-r}$ of b-cliques can contain one clique Q_i, and we only have $L \leq t(2r)^{4s}$ of the Q_i, in this case we have the lower bound

$$t \geq \frac{\frac{1}{2}\binom{n}{b}}{(2s)^{4r}\binom{n-r}{b-r}} = \left(\frac{n}{16s^4 b}\right)^{\Omega(r)} = 2^{\Omega(a^{1/4})}.$$

Suppose now that the second condition (ii) holds. Fix one forest $F \in \mathscr{F}$, and let $T_1, \ldots, T_d$ be all its connected components (trees). By the definition, a graph G_h contains F iff $h(u) = h(v)$ for all edges $\{u, v\}$ of F. Thus, all vertices in each of these trees must receive the same color. Since each tree T_i has $|T_i| + 1$ vertices, the total number of vertices in the forest F is $v = \sum_{i=1}^d (|T_i| + 1) = |F| + d \geq s + d$. There are a^d ways for the coloring h to color the trees T_i, and at most $a^{n-v} \leq a^{n-(s+d}$ ways to color the remaining $n - v$ vertices. Thus, the number of graphs G_h containing one fixed forest $F \in \mathscr{F}$ does not exceed $a^d a^{n-(s+d)} = a^{n-s}$. Since we only have $|\mathscr{F}| \leq t(2r)^{4s}$ forests in $\mathscr{F}$, in this case we have the lower bound

$$t \geq \frac{a^n}{(2r)^{4s} a^{n-s}} = \left(\frac{a}{16r^4}\right)^s = 2^{(n/b)^{1/4}}.$$

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