Rules and Tips for Writing Mathematics

MATH 213, Discrete Mathematics

Adapted from: *Writing in Mathematics* by Annalisa Crannell

Why Should You Have To Write Papers In A Math Class?

For most of your life so far, the only kind of writing you've done in math classes has been on homeworks and tests, and for most of your life you've explained your work to people that know more mathematics than you do (that is, to your teachers). But soon, this will change. Now that you are taking Discrete, you know far more mathematics than the average American has ever learned - indeed, you know more mathematics than most college graduates remember. With each additional mathematics course you take, you further distance yourself from the average person on the street. You may feel like the mathematics you can do is simple and obvious (doesn't everybody know what a function is?), but you can be sure that other people find it bewilderingly complex. It becomes increasingly important, therefore, that you can explain what you're doing to others that might be interested: your parents, your boss, the media.

Nor are mathematics and writing far-removed from one another. Professional mathematicians spend most of their time writing: communicating with colleagues, applying for grants, publishing papers, writing memos and syllabi. Writing well is extremely important to mathematicians, since poor writers have a hard time getting published, getting attention from the Deans, and obtaining funding. It is ironic but true that most mathematicians spend more time writing than they spend doing math.

But most of all, one of the simplest reasons for writing in a math class is that writing helps you to learn mathematics better. By explaining a difficult concept to other people, you end up explaining it to yourself.

How is Mathematical Writing Different from What You've Done So Far?

A good mathematical essay has a fairly standard format. We tend to start solving a problem by first explaining what the problem is, often trying to convince others that it's an interesting or worthwhile problem to solve. On your homeworks, you've usually just said,"Problem 9(a" and then plunged ahead; but in your formal writing, you'll have to take much greater pains.

After stating what the problem is, we usually then state the answer, even before we show how we got it. Sometimes we even state the answer right along with the problem. It's uncommon, although not so uncommon as to be exceptional, to read a math paper in which the answer is left for the very end. Explaining the solution and then the answer is usually reserved for cases where the solution technique is even more interesting than the answer, or when the writers want to leave the readers in suspense. But if the solution is messy or boring, then it's typically best to hook the readers with the answer before they get bogged down in details.

Math is difficult enough that the writing around it should be simple. 'Beautiful' math papers are the ones that are the easiest to read: clear explanations, uncluttered expositions on the page, well-organized presentation. For that reason, mathematical writing is not a creative endeavor the same way that, say, poetry is: you shouldn't be spending a lot of time looking for the perfect word, but rather should be developing the most clear exposition. Unlike humanities students, mathematicians don't have to worry about over-using 'trite' phrases in mathematics. In fact, at the end of this booklet are a list of trite but useful phrases that you may want to use in your papers, either in this class or in the future.

This guide, together with the checklist, should serve as a reference while you write and will also be referred to when I comment on the drafts of the problems for your writing portfolio. If you can master these basic areas, your writing may not be spectacular, but it should be clear and easy to read - which is the goal of mathematical writing, after all.

Checklist:

1. **Clearly restate the problem to be solved.**

   > "Do not assume that the reader knows what you're talking about. (The person you're writing to might be out on vacation, for example, or have a weak memory). You don't have to restate every detail, but you should explain enough so that someone who's never seen the assignment can read your paper and understand what's going on, without any further explanation from you. Outline the problem carefully."

   In the proof portfolio you can generally just have the statement of the problem as it was given in the assignment, and then begin your proof below. You don't need to rephrase or restate the problem beyond that.

2. **State the answer in a complete sentence which stands on its own.** If you can avoid variables in your answer, do so; otherwise, remind the reader what they stand for. If your answer is at the end of the paper and you've made any significant assumptions, restate them, too. Do not assume that the reader has actually read every word and remembers it all (do you?).

3. **The burden of communication lies on you, not on your reader.** It is your job to explain your thoughts; it is not your reader's job to guess them from a few hints. You are trying to convince a skeptical reader who doesn't believe you, so you need to argue with airtight logic in crystal clear language; otherwise he/she will continue to doubt. If you didn't write something on the paper, then (a) you didn't communicate it, (b) the reader didn't learn it, and (c) the grader has to assume you didn't know it in the first place.

4. **Clearly state the assumptions which underlie the formulas.** For example, what physical assumptions do you have to make? (No friction, no air resistance? That something is lying on its side, or far away from everything else?) Sometimes things are so straightforward that there are no assumptions, but not often.

5. **Use diagrams, tables, or graphs, to help explain the math and clearly label them (if these are used).** In math, even more than in literature, a picture is worth a thousand words, especially if it's well labeled. Label all axes, with words, if you use a graph. Give diagrams a title describing what they represent. It should be clear from the picture what any variables in the diagram should represent. The whole idea is to make everything as clear and self-explanatory as possible.

6. **Define all variables used.** (a) Even if you label your diagram (and you should), you should still explain in words what your variables are. (b) If there's a quantity you use only a few times, see if you can get away with not assigning it a variable.

7. **Explain how each formula is derived, or where it can be found.** Don't pull formulas out of a hat, and don't use variables which you don't define. Either derive the formula yourself in the paper, or explain exactly where you found it, so other people can find it, too. Put important or long formulas on a line of their own, and then center them; it makes them much easier to read.

8. **In this paper, are the spelling, grammar, and punctuation correct?** It may surprise you that it is on spelling and grammar that people tend to lose most of their points on their mathematics papers. Please spell-check and proofread your work for grammar mistakes.

9. **Use English words.** Although there will usually be equations or mathematical statements in your proofs, use English sentences to connect them and display their logical relationships. If you look in your textbook, you'll see that each proof consists mostly of English words. In general, you should not use logic symbols such as $\forall$, $\implies$, $\exists$, $\wedge$, etc, unless you are writing about a problem in symbolic logic (which is not the case in your portfolio problems.)

10. **Write in complete sentences and paragraphs.** If you wrote a history essay in sentence fragments, the reader would not understand what you meant; likewise in mathematics you must use complete

sentences, with verbs to convey your logical train of thought. Some complete sentences can be written purely in mathematical symbols, such as equations (like $a^3 = b^{-1}$), inequalities (like $o(a) < 5$), and other relations (like $5 \mid 10$ or $7 \in \mathbb{Z}$). These statements usually express a relationship between two mathematical *objects*, like numbers (e.g., 7), matrices, or sets (e.g., $\mathbb{R}$).

11. **Show the logical connections among your sentences.** Use phrases like "Therefore" or "because" or "if... then..." or "if and only if" to connect your sentences.

12. **Use whitespace.** Don't cram your answer into a few lines of the paper, filled from left margin to right margin. Let your writing breathe! When you start a new thought, start a new line. Use indentation to organize your sentences. This helps the reader understand your thought much better, and it also encourages you to be more clear.

13. **Use scratch paper.** Finding your proof will be a long, messy process, full of false starts and dead ends. Do all that on scratch paper until you find a real proof, and only then break out your clean paper to write your final proof carefully. *Do not hand in your scratch work!*

    Only sentences that actually contribute to your proof should be part of the proof. Do not just perform a "brain dump," throwing everything you know onto the paper before showing the logical steps that prove the conclusion. *That is what scratch paper is for.*

14. **"=" means equals.** Don't write $A = B$ unless you mean that A actually equals B. This rule seems obvious, but there is a great temptation to be sloppy. For example in Calculus a student might write something like this:
    $$f(x) = x^2 = 2x$$
    when they really mean that the derivative of the function is $2x$.

15. **Avoid weasel words.** There are some notorious phrases that advertise that you don't really understand the logic you need. Be wary of phrases like "clearly," "obviously," and "the only way this can happen is." (There is a time and a place for all of these phrases, but most likely that time and place will not be in your proofs in this class!)

16. **In this paper, is the mathematics correct?** This is self-explanatory.

17. **In this paper, did the writer solve the question that was originally asked?**
    So is this.

18. **Do not use headers for different cases. Do start a new paragraph for a new case.**

19. **Indent the beginning of your paragraphs – don't just use a line break** And I often tell people to get rid of line breaks between paragraphs and only use the indent, particularly if there are any places where you will have displayed math at the end of a previous paragraph.

20. **Make math look "mathy."** This means that all variable names, equations, and anything else "mathy" should be in a math/equation mode/style. In LaTeX that means putting it in either inline `$ $` or displayed `\[ \]` math mode. In Word that means using an equation environment (shortcut Alt+=).

21. **Do not start a sentence with a mathematical expression.** All sentences should begin with English words, not mathematical symbols or expressions. You should revise and rewrite the sentence if this is not the case.

22. **Math is part of the sentence.** Read your sentence, saying all of the mathematical expressions in words as you go. Is the grammar and punctuation correct?

23. **Math should not break at the end of a sentence.** If a mathematical expression wraps at the end of a line, then you should rewrite the proof or possibly put longer or more important expressions in a displayed environment.

24. **Important expressions should be in a displayed environment.** "Displayed" means centered on its own line. In LaTeX the standard displayed math environment is enclosed with `\[ \]` or inside a displayed environment such as `align*`. In MS Office if you start a math environment in a new line it will default to display, otherwise click the drop-down box on an inline math environment and click the appropriate option.

25. **Death to commas.** When thinking through a proof and writing it up initially many people (myself included) tend to add unnecessary commas, perhaps due to the fact that the thought process that led to the proof had many gaps and pauses in it initially. But as you refine the proof it should become more concise, easier to read, and have a better flow to it, which means fewer commas will be needed. You should still follow standard grammatical conventions for places that require commas (for instance, ←here, ←here, ←and here) but otherwise cut the comma. For the comma-lovers out there: you may still use the Oxford comma whenever applicable.

26.

Good Phrases to Use in Math Papers:

- Therefore (thus, so, hence, accordingly, it follows that, we see that, then)
- We assume that (assuming, where, M stands for)
- show (demonstrate, prove, explain why, find)
- if (whenever, provided that, when)
- notice that (note that, notice, recall)
- since (because)

Summary of Rules/Tips for Writing Mathematics

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2. State the answer in a complete sentence which stands on its own.
3. The burden of communication lies on you, not on your reader.
4. Clearly state the assumptions which underlie the formulas.
5. Use diagrams, tables, or graphs, to help explain the math and clearly label them (if these are used).
6. Define all variables used.
7. Explain how each formula is derived, or where it can be found.
8. In this paper, are the spelling, grammar, and punctuation correct?
9. Use English words.
10. Write in complete sentences and paragraphs.
11. Show the logical connections among your sentences.
12. Use whitespace.
13. Use scratch paper.
14. "=" means equals.
15. Avoid weasel words.
16. In this paper, is the mathematics correct?
17. In this paper, did the writer solve the question that was originally asked?
18. Do not use headers for different cases. Do start a new paragraph for a new case.
19. Indent the beginning of your paragraphs – don't just use a line break
20. Make math look "mathy."
21. Do not start a sentence with a mathematical expression.
22. Math is part of the sentence.
23. Math should not break at the end of a sentence.
24. Important expressions should be in a displayed environment.
25. Death to commas.
26.