Riba $\lim_{x\to 0} \frac{\cos x - 1}{x} = 0$

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Šią riba galima rasti įvairiais būdais. Pavyzdžiui, jau žinome, kad:

$$\lim_{x\to 0} \frac{\sin x}{x} = 1. \tag{1}$$

Tada

$$\lim_{x\to 0} \frac{\cos x - 1}{x} = \lim_{x\to 0} \frac{(\cos x - 1)(\cos x + 1)}{x(\cos x + 1)} = \lim_{x\to 0} \frac{\cos^2 x - 1}{x(\cos x + 1)} =$$
$$= \lim_{x\to 0} \frac{-\sin^2 x}{x(\cos x + 1)} = \lim_{x\to 0} \frac{-\sin x}{x} \lim_{x\to 0} \frac{\sin x}{\cos x + 1} = -1 \cdot \frac{\sin 0}{\cos 0 + 1} = 0.$$

Žinoma, jei jau mokate Lopitalio taisyklę:

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}, \tag{2}$$

tai viskas dar paprasčiau:

$$\lim_{x\to 0} \frac{\cos x - 1}{x} = \lim_{x\to 0} \frac{\frac{d}{dx}(\cos x - 1)}{\frac{d}{dx}x} = \lim_{x\to 0} \frac{-\sin x}{1} = \frac{-\sin 0}{1} = 0.$$

Dabar pritaikius šį faktą, lengva išspręsti:

$$\lim_{x\to 0} \frac{\sin(a + 2x) - 2\sin(a + x) + \sin a}{x^2} =$$
$$= \lim_{x\to 0} \frac{\sin a \cos 2x + \sin 2x \cos a - 2\sin a \cos x - 2\sin x \cos a + \sin a}{x^2} =$$
$$= \lim_{x\to 0} \frac{\sin a(\cos^2 x - \sin^2 x) + 2\sin x \cos x \cos a - 2\sin a \cos x - 2\sin x \cos a + \sin a}{x^2} =$$
$$= \lim_{x\to 0} \frac{\sin a(\cos^2 x - 2\cos x + 1) - \sin a \sin^2 x + 2\sin x \cos a(\cos x - 1)}{x^2} =$$
$$= \sin a \lim_{x\to 0} \frac{(\cos x - 1)^2}{x^2} - \sin a \lim_{x\to 0} \frac{\sin^2 x}{x^2} + 2\cos a \lim_{x\to 0} \frac{\sin x}{x} \lim_{x\to 0} \frac{\cos x - 1}{x} = -\sin a.$$