Rinktiniai analizės skyriai
FDM 4 semestras
el. namų darbas VIII

2015 04 20

1 Diferencialinės lygtys atskiriamais kintamaisiais

Uždavinys 1.1.

$$
\begin{aligned}
2xydy + (1 + y^2)dx &= 0 \\
2xydy &= -(1 + y^2)dx \\
\int \left(\frac{2y}{1 + y^2} \right) dy &= -\frac{dx}{x} \\
ln(1 + y^2) &= -ln|x| + lnC \\
&\quad arba \\
|x|(1 + y^2) &= C
\end{aligned}
$$

Uždavinys 1.2.

$$
\begin{aligned}
e^{3y} \sin^2 xdx + \cos^2 xdy &= 0 \\
-\frac{\sin^2 x}{\cos^2 x}dx &= e^{-3y}dy \\
\int -\frac{1}{\cos^2 x} + 1dx &= -\frac{1}{3}e^{-3y} + C
\end{aligned}
$$

Uždavinys 1.3.

$$
\sqrt{1 + y^2}dx = xydy
$$

Uždavinys 1.4.

$$e^{-s}\left(1 + \frac{ds}{dt}\right) = 1$$

$$\frac{ds}{dt} = e^s - 1$$

$$\frac{ds}{e^s - 1} = dt$$

$$-\frac{1 - e^s + e^s}{1 - e^s} = -1 - \frac{de^s - 1}{e^s - 1}$$

Uždavinys 1.5.

$$(x^2 - 1)y' + 2xy^2 = 0, \quad y(0) = 1$$

I būdas:

$$(x^2 - 1)\frac{dy}{dx} = -2xy^2$$

$$\int \frac{dy}{y^2} = -\int \frac{2xdx}{x^2 - 1}$$

$$\frac{1}{y} = \ln|x^2 - 1| + C$$

$$y(0) = 1, \quad todėl \ 1 = \ln 1 + C$$

$$ir$$

$$C = 1$$

$$\frac{1}{y} = \ln|x^2 - 1| + 1$$

II būdas:

$$\int\limits_1^y \frac{dy}{y^2} = -\int\limits_0^x \frac{dx^2}{x^2 - 1}$$

$$-\frac{1}{y}\Big|_1^y = -\ln|x^2 - 1|\big|_0^x$$

$$-\frac{1}{y} + 1 = -\ln|x^2 - 1| + 0$$

$$\frac{1}{y} = \ln|x^2 - 1| + 1$$

Uždavinys 1.6.

$$xy' + y = y^2, \ y(1) = 0,5$$

$$xdy = (y^2 - y)dx$$

$$\frac{dy}{y^2 - y} = \frac{dx}{x}$$

$$-\frac{d(\frac{1}{y})}{1 - \frac{1}{y}} = \frac{dx}{x}$$

Uždavinys 1.7.

$$
\begin{aligned}
x^2 y' - \cos(2y) &= 1, \ \ y(\infty) = \frac{9}{4}\pi \\
x^2 \frac{dy}{dx} &= \cos 2y + 1 \\
\frac{dy}{1 + \cos 2y} &= \frac{dx}{x^2} \\
\frac{1}{2}\tan y &= -\frac{1}{x} + C \\
\frac{1}{2}\tan \frac{9}{4}\pi &= \lim\left(-\frac{1}{x}\right) + C
\end{aligned}
$$