Rinktiniai analizės skyriai. el. NAMŲ DARBAS I
FDM 3 semestras

2014 spalio 14

1 Funkcijos riba

1. Uždavinys. Pagal apibrėžimą įrodykite teiginius:

(a)
$$\lim_{(x,y)\to(0,0)} \frac{x^2}{|x| + |y|} = 0$$

(b)
$$\lim_{(x,y)\to(2,-1)} \sqrt{x + y} = 1$$

(c)
$$\lim_{(x,y)\to(0,0)} \frac{xy(x + y)}{x^2 + y^2 - xy} = 0$$

2. Uždavinys. Įrodykite, kad šios ribos neegzistuoja:

(a)
$$\lim_{(x,y)\to(0,0)} \frac{x^2 + xy + y^2}{x^2 - xy + y^2}$$

(b)
$$\lim_{(x,y)\to(1,0)} \frac{ln(x + y)}{y}$$

3. Uždavinys. Apskaičiuokite kartotines ribas $\lim_{x\to x_0} \lim_{y\to y_0} f(x,y)$ ir $\lim_{y\to y_0} \lim_{x\to x_0} f(x,y)$, jei

(a)
$$f(x,y) = \frac{x^2 + xy + y^2}{x^2 - xy + y^2} \qquad x_0 = 0, \quad y_0 = 0$$

(b)
$$f(x,y) = \frac{\cos x - \cos y}{x^2 + y^2} \qquad x_0 = 0, \quad y_0 = 0$$

(c)
$$f(x, y) = x \sin \frac{1}{x^2 + y^2} \qquad x_0 = 0, \quad y_0 = 0$$

(d)
$$f(x, y) = \frac{\sin^2 x + \cos^2 y}{x^2 + y^2} \qquad x_0 = 0, \quad y_0 = +\infty$$

(e)
$$f(x, y) = \frac{1}{xy} \tan \frac{xy}{1 + xy} \qquad x_0 = 0, \quad y_0 = \infty$$

(f)
$$f(x, y) = \frac{\sqrt{x^2 + y^2}}{x + y} \qquad x_0 = 0, \quad y_0 = 0$$

4. **Uždavinys.** Raskite ribą (jei ji egzistuoja)

(a)
$$\lim_{(x,y) \to (+\infty, +\infty)} \frac{x^2 + y^2}{e^{x+y}}$$

(b)
$$\lim_{(x,y) \to (0,0)} (x^2 + y^2)^{x^2 y^2}$$

(c)
$$\lim_{(x,y) \to (0,2)} (1 + xy)^{\frac{2}{x^2 + xy}}$$

(d)
$$\lim_{(x,y) \to (1,1)} \frac{x^3 - 2xy + y^3}{x^3 - y^3}$$

(e)
$$\lim_{(x,y) \to (0,0)} \frac{a - \sqrt{a^2 - xy}}{xy} \quad a > 0$$

(f)
$$\lim_{(x,y) \to (0,\infty)} (1 + xy)^y$$

(g)
$$\lim_{(x,y) \to (0,3)} (1 + x^2 y)^{\frac{2}{x \sin x + x^2 y}}$$