

# Beselio pluoštai

Skaliarinės bangų lygties sprendinys

Beselio-Gauso pluoštų formavimo būdai:

- holografinis
- Fabri ir Pero interferometras
- Žiedinė diafragma
- Eksikonas (axicone)

# Beselio pluoštai

$$\left( \Delta - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right) E(\mathbf{r}, t) = 0$$

$$E(\mathbf{r}, t) = \exp[i(\beta z - \omega t)] J_0(\alpha \rho)$$

$$\beta^2 + \alpha^2 = \omega^2 / c^2$$

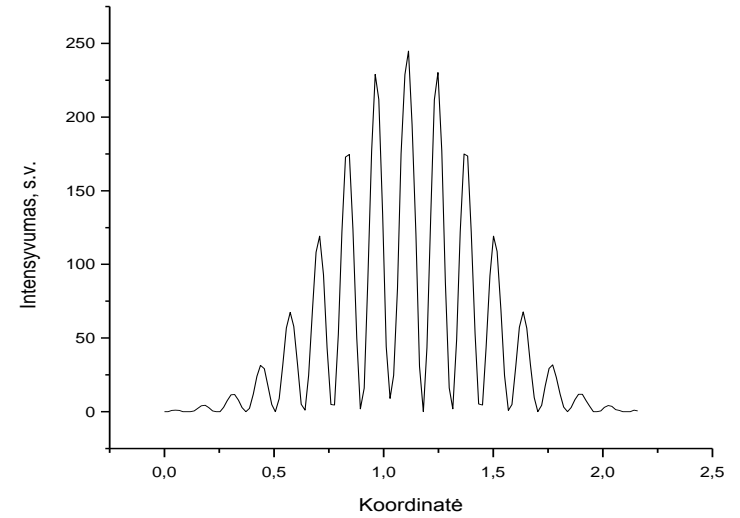
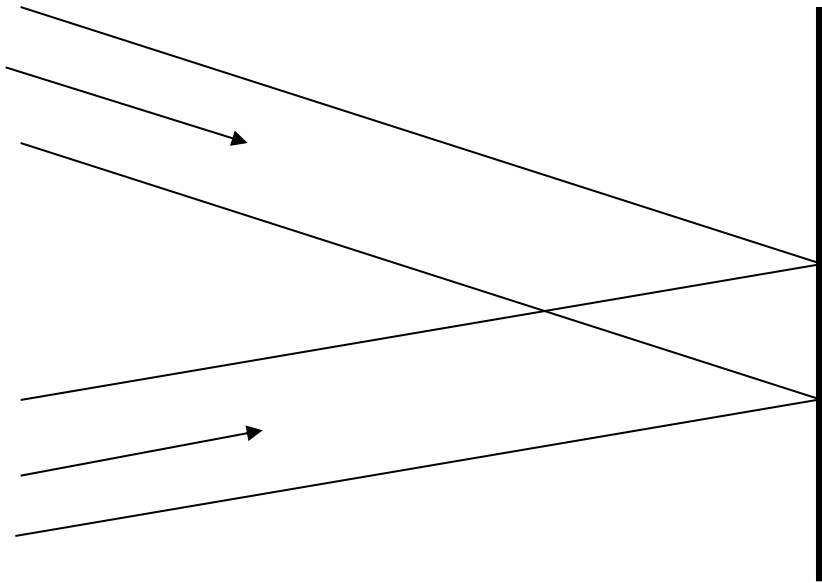
$$\rho^2 = x^2 + y^2 \quad \omega = c \sqrt{k_{\parallel}^2 + k_{\perp}^2}$$

$$\lambda = \frac{2\pi}{\sqrt{k_{\parallel}^2 + k_{\perp}^2}}$$

$$E(\mathbf{r}, t) = \exp[i(\beta z - \omega t)] \sum_n E_n \exp[in\varphi] J_n(\alpha \rho)$$

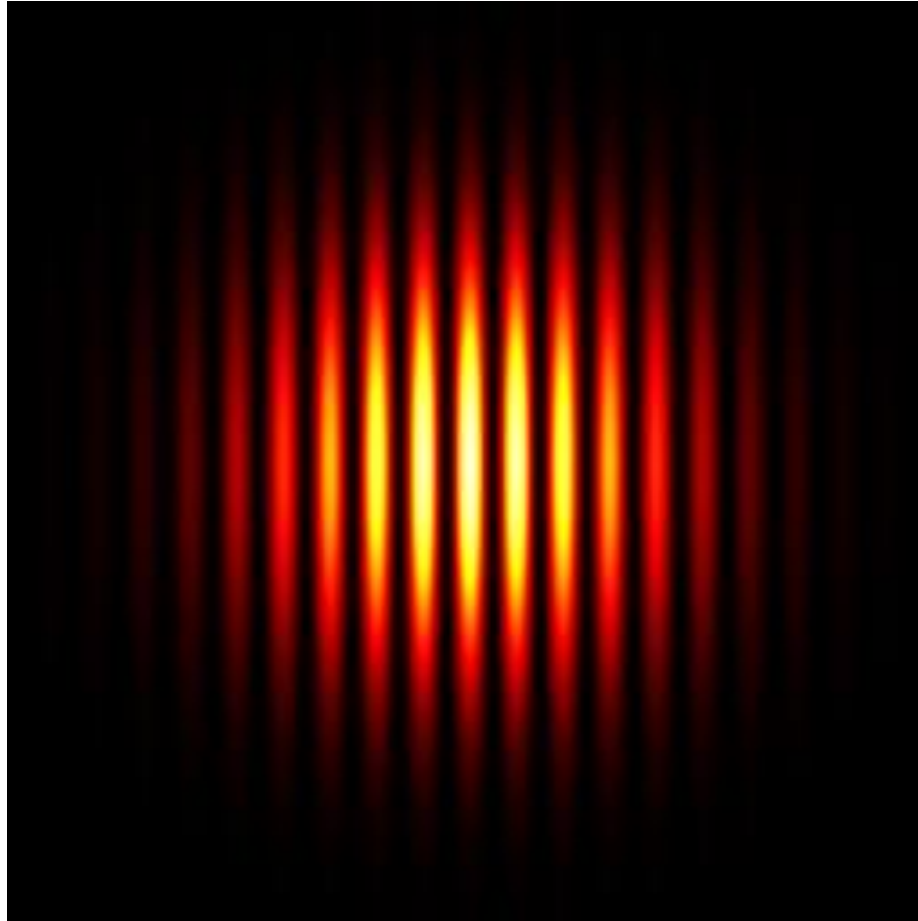
$$A_n(\mathbf{r}, \varphi) = a_n J_n(\alpha_0 \rho) \exp(in\varphi - \rho^2 / d^2)$$

# Beselio pluoštai



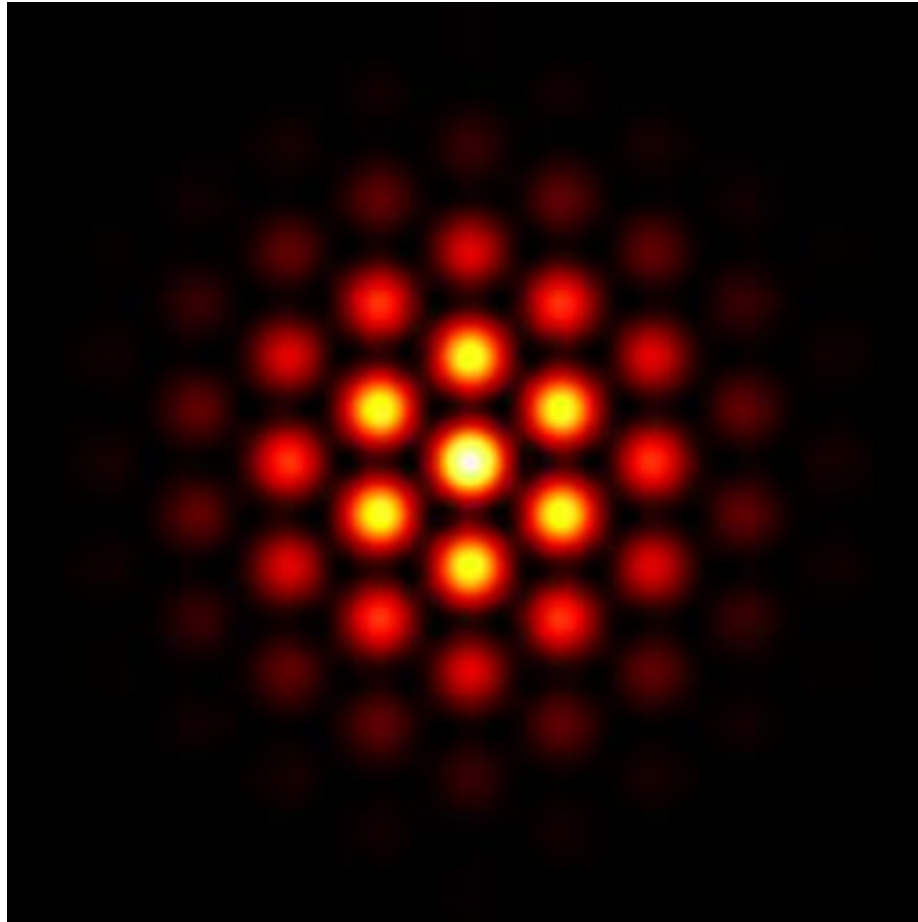
J. Durnin, "Exact solutions for nondiffracting beams. I. The scalar theory", J. Opt. Soc. Am. A, vol. 4, p.651-654 (1987).

# Beselio pluoštai



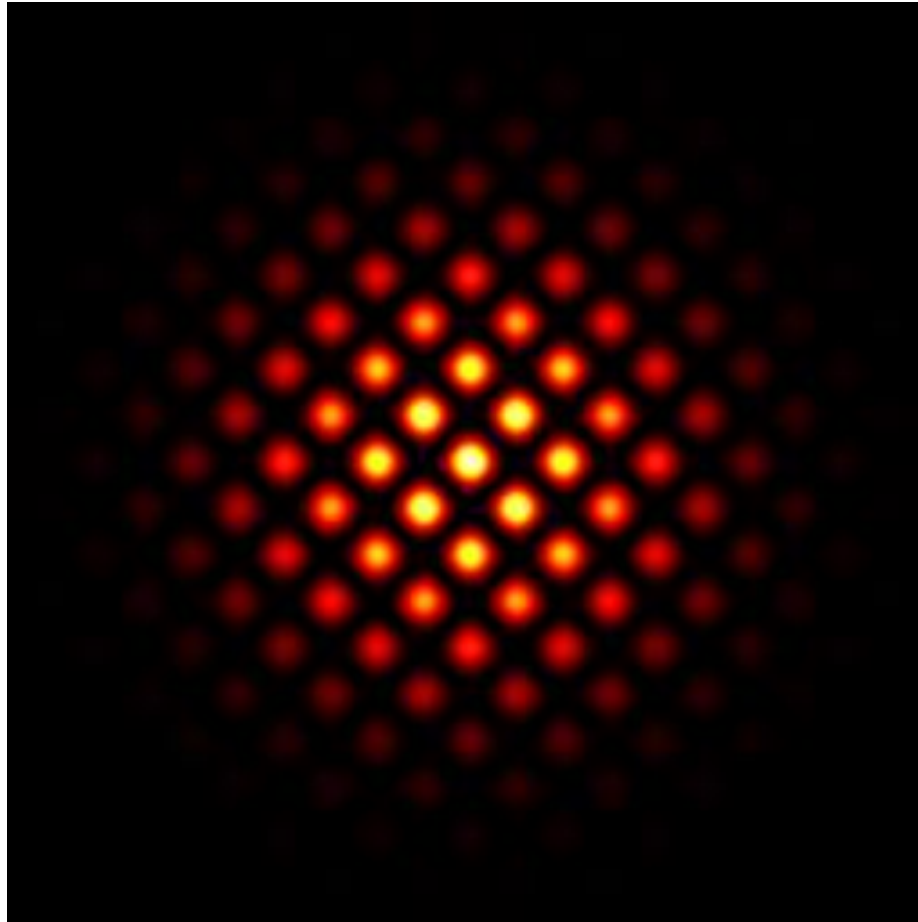
Du pluoštai:  $\vartheta = \pm 0.03^\circ$  ,  $\varphi = 180$

# Beselio pluoštai



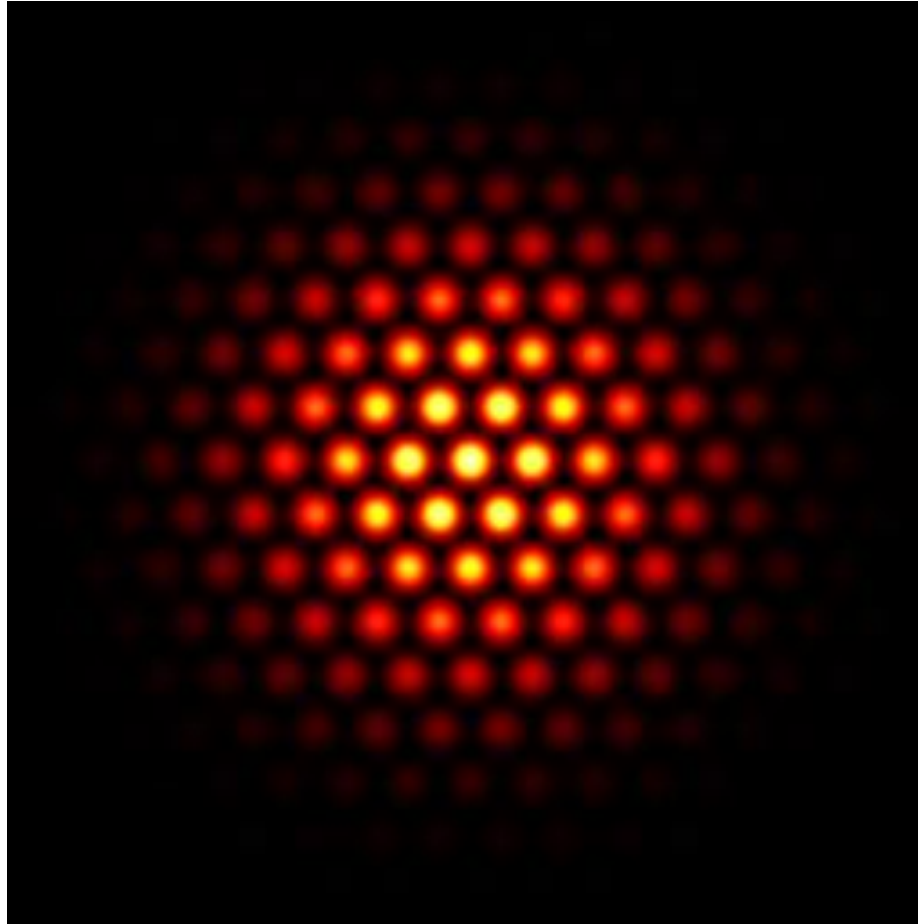
Trys pluoštai:  $\vartheta = \pm 0.03^\circ$ ,  $\Delta\varphi = 120$

# Beselio pluoštai



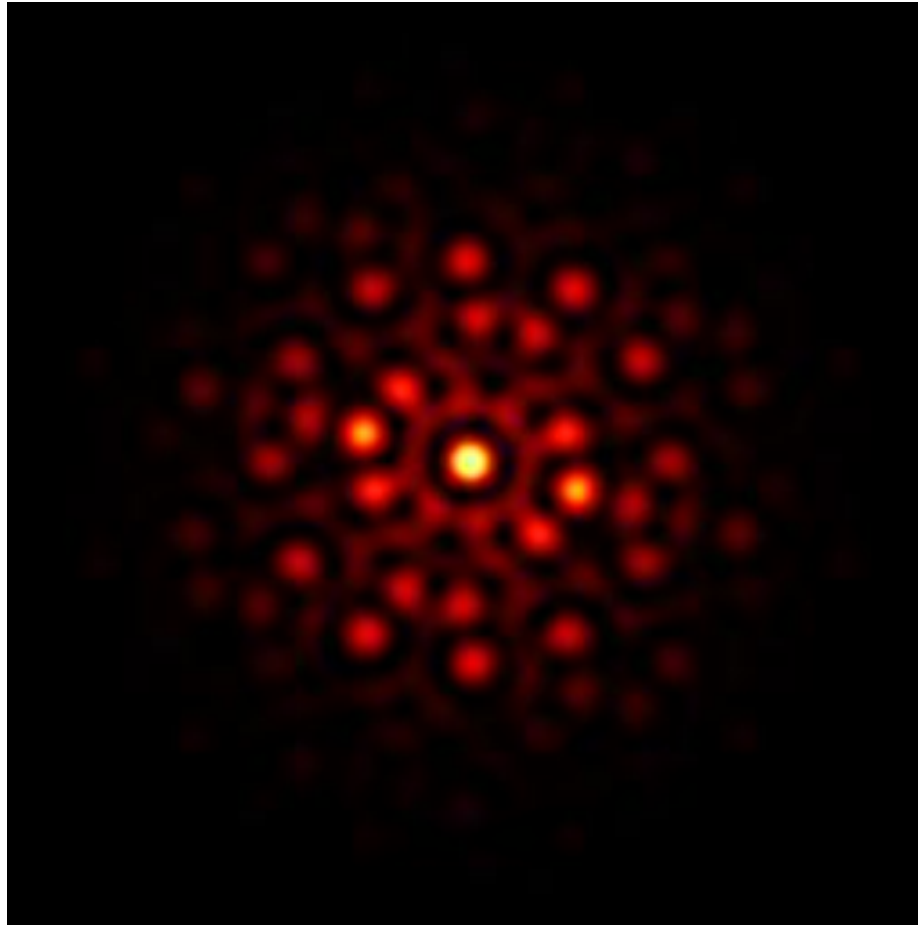
Keturi pluoštai:  $\vartheta = \pm 0.03^\circ$ ,  $\Delta\varphi = 90^\circ$

# Beselio pluoštai



Šeši pluoštai:  $\vartheta = \pm 0.03^\circ$ ,  $\Delta\varphi = 60^\circ$

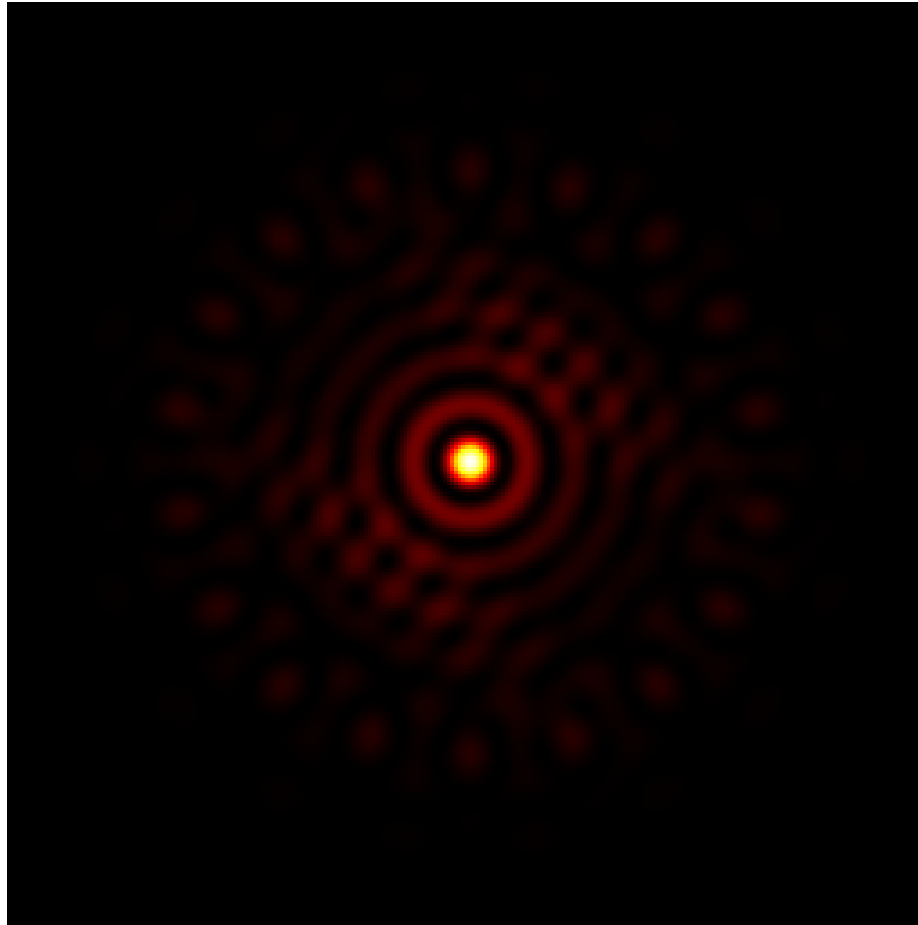
# Beselio pluoštai



Dvylika pluoštų:  $\vartheta = \pm 0.03^\circ$ ,  $\Delta\varphi = 30^\circ$

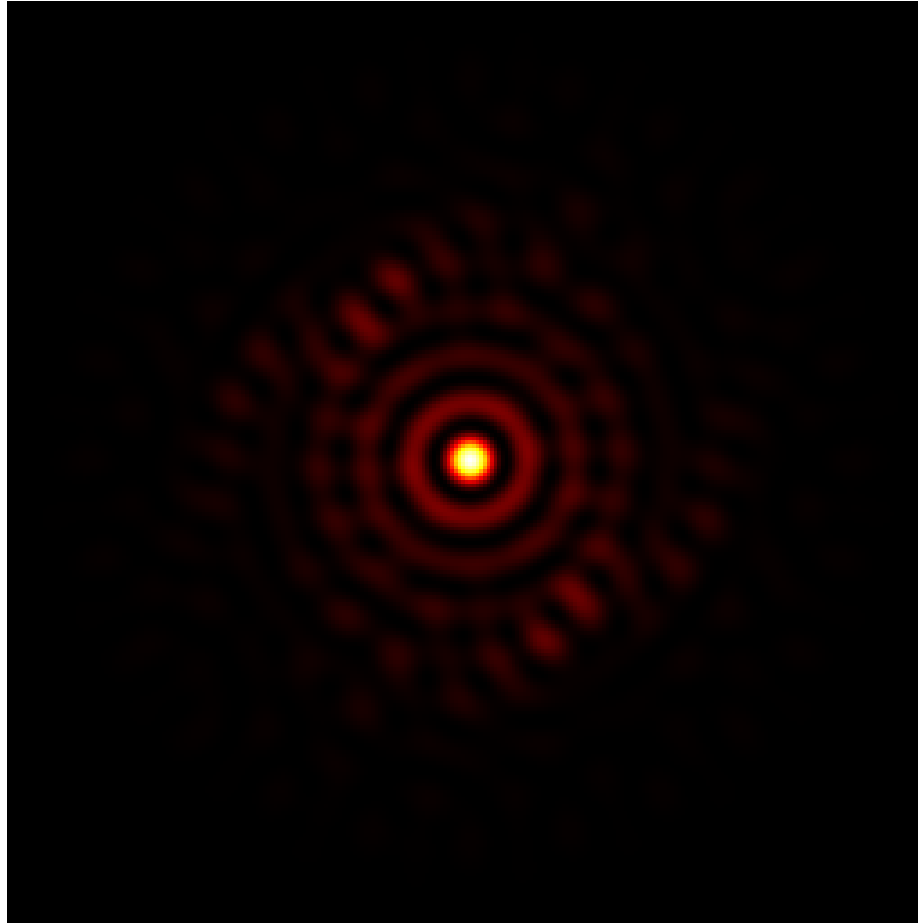


# Beselio pluoštai



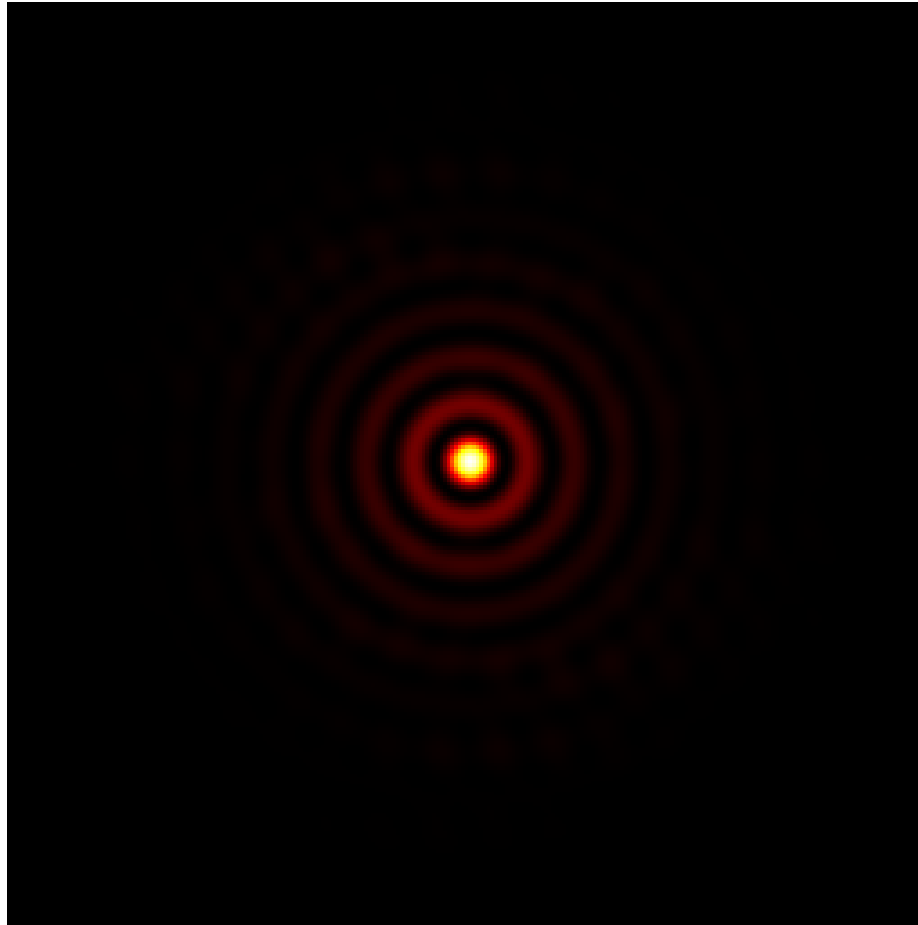
Aštuoniolika pluoštų:  $\vartheta = \pm 0.03^\circ$  ,  $\Delta\varphi = 20^\circ$

# Beselio pluoštai



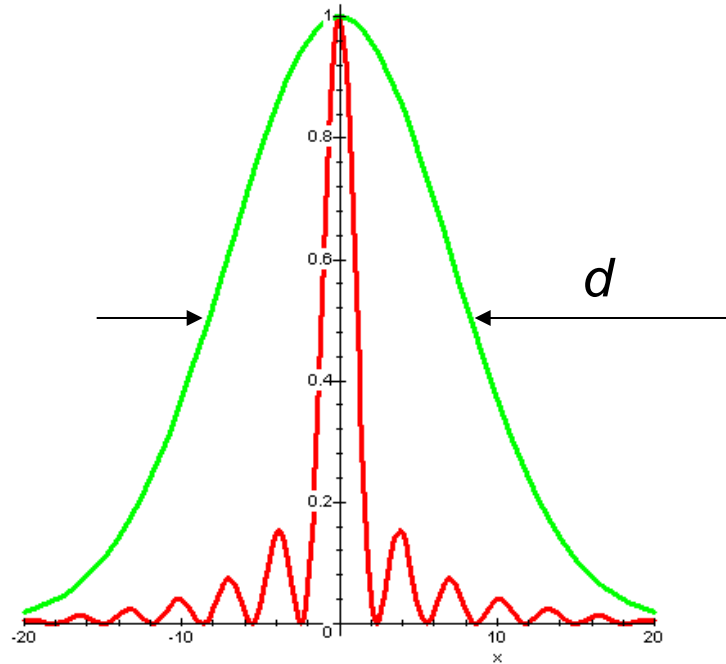
Dvidešimt keturi pluoštai:  $\vartheta = \pm 0.03^\circ$  ,  $\Delta\varphi = 15^\circ$

# Beselio pluoštai



Trisdešimt šeši pluoštai:  $\vartheta = \pm 0.03^\circ$ ,  $\Delta\varphi = 10^\circ$

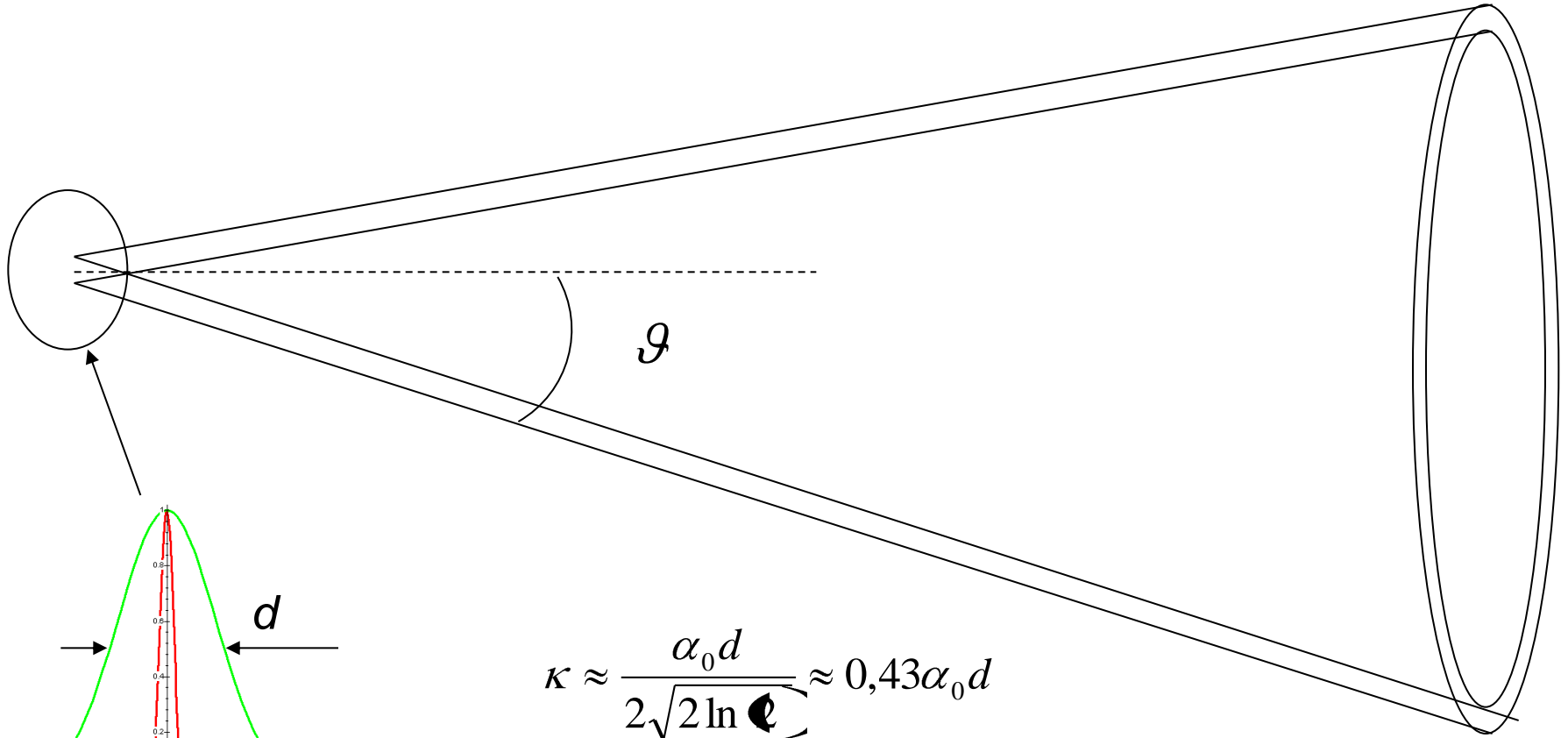
# Beselio pluoštai



$$A_n(\theta, \varphi) = a_n J_n(\alpha_0 \rho) \exp(in\varphi - \rho^2/d^2)$$

$$\alpha_0 = k \sin \theta = \omega/c \sin \theta$$

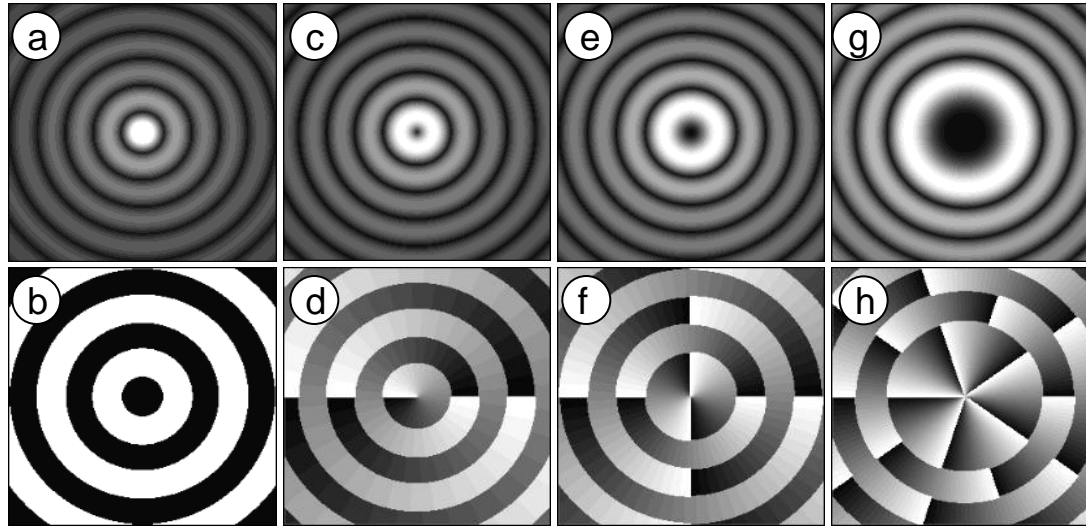
# Beselio pluoštai



$$\kappa \approx \frac{\alpha_0 d}{2\sqrt{2 \ln e}} \approx 0,43\alpha_0 d$$

$$\alpha_0 = k \sin \vartheta = \frac{\omega}{c} \sin \vartheta$$

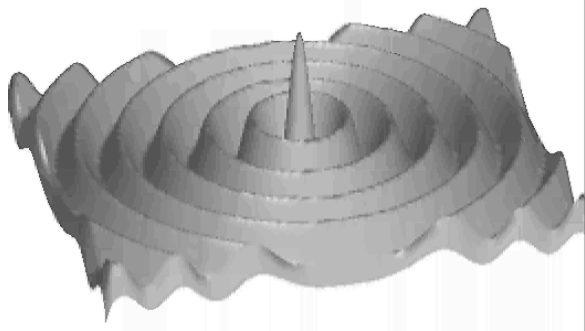
# Beselio pluoštai



Beselio pluoštų intensyvumo skirstiniai (a, c, e, g) ir faziniai portretai (b, d, f, h). Atitinkamai paveikslėliuose pavaizduoti nulinės (a, b), pirmos (c, d), antros (e, f) ir penktos (g, h) eilės Beselio pluoštai. Fazinių portretų paveikslėliuose juoda spalva atitinka fazę lygią nuliui, o balta atitinka fazę lygia . Tarpinės fazės vertės atitinka pilka spalva.

# Beselio pluoštai

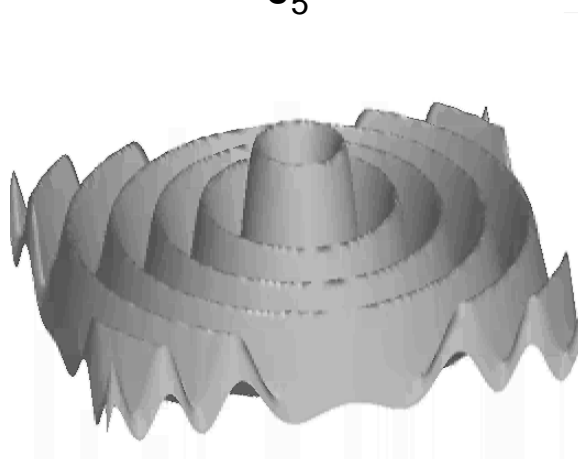
$J_0$



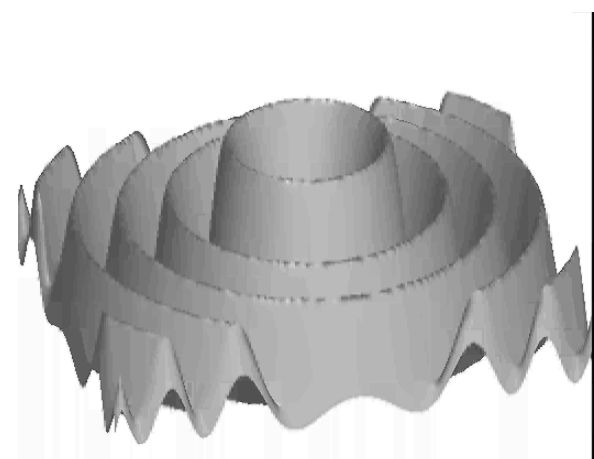
$J_1$



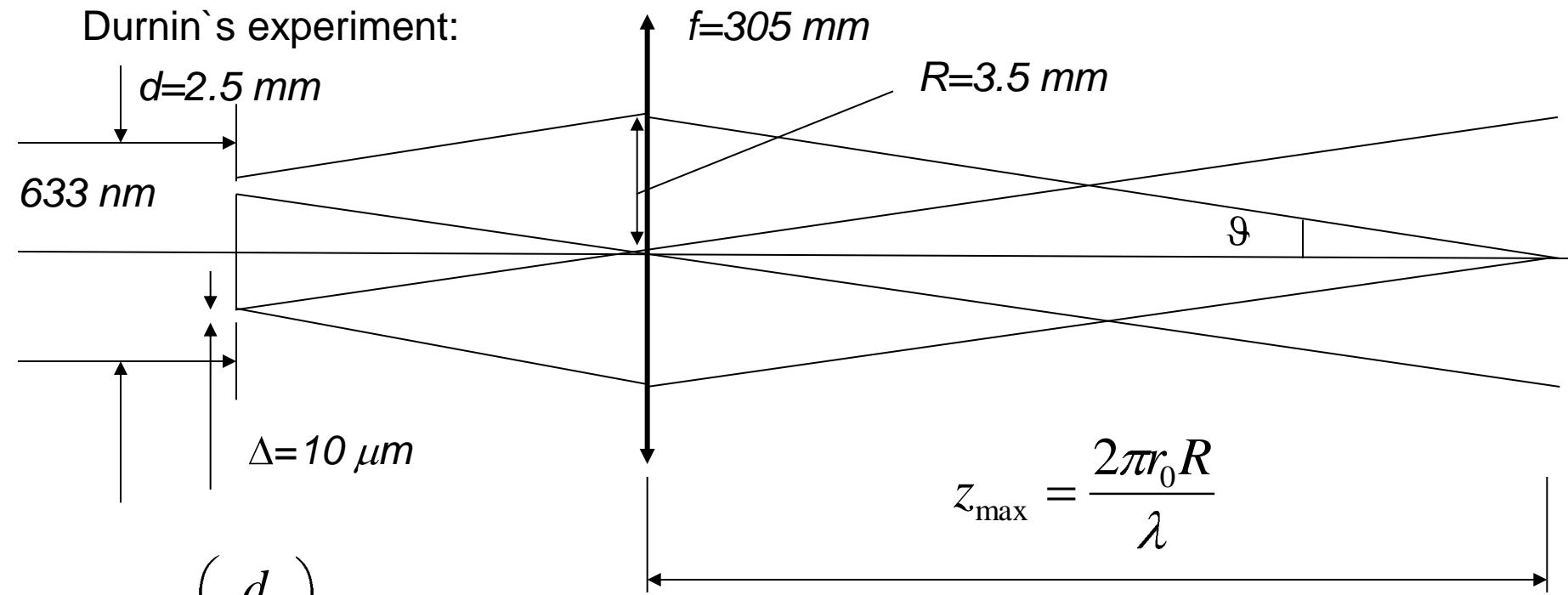
$J_5$



$J_{10}$



# Beselio pluoštai



$$\vartheta = \arctg\left(\frac{d}{2f}\right)$$

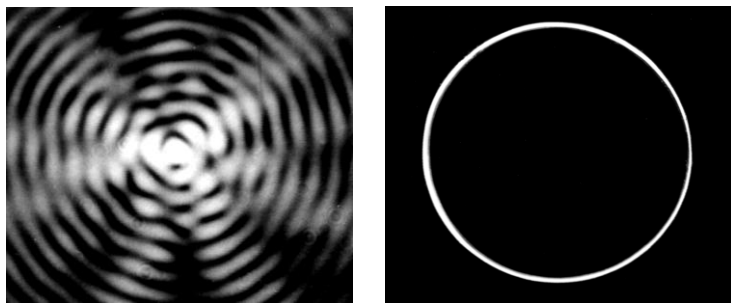
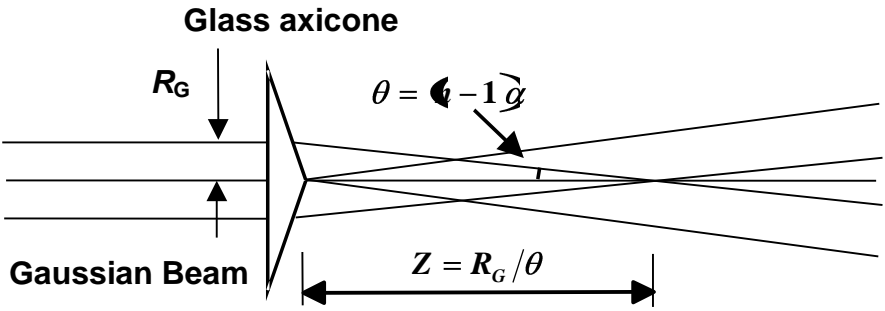
$$r_0 \approx \frac{1}{k_{\perp}} \quad k_{\perp} = \left(\frac{2\pi}{\lambda}\right) \sin \vartheta$$

Gauso pluoštui:  $z_{\text{max}} = 3 \text{ mm}$

J. Durnin, "Exact solutions for nondiffracting beams. I. The scalar theory", J. Opt. Soc. Am. A, vol. 4, p.651-654 (1987).



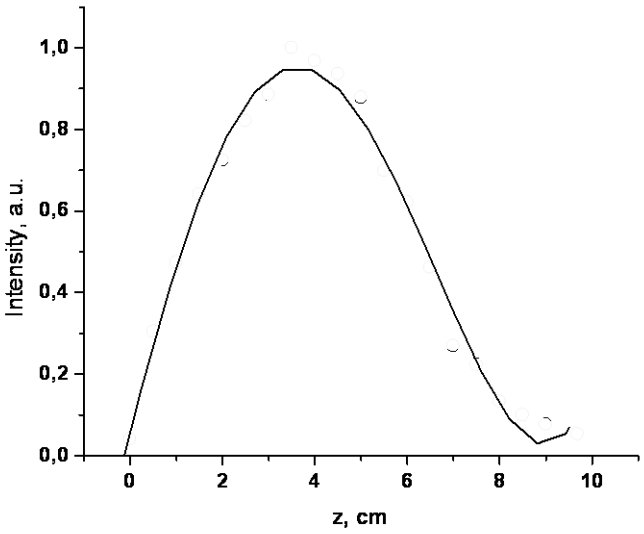
# Beselio pluoštai



a

b

Beselio pluošto suformuoto aksikonu intensyvumo skirstinys (a) ir jo Furje spektras (b)



$$z_{\max} = R / \mathcal{G}$$

# Beselio pluoštai

$$A_n(r, \varphi) = a \left( \frac{r}{d} \right)^n \exp \left[ i \left( \frac{r}{d} \right)^2 + in\varphi \right], \quad n = 0, 1, 2, \dots$$

Lagero ir Gauso pluošto kompleksinė amplitudė cilindrinėse koordinatėse

$$\exp(-i\beta_0 r) \quad \text{Kūginės prizmės perdavimo funkcija} \quad \beta_0 \cong \frac{2\pi}{\lambda} (n_a - 1) d$$

$$B_n(r, \varphi) = A_n(r, \varphi) \exp(-i\beta_0 r) = a \left( \frac{r}{d} \right)^n \exp \left[ i \left( \frac{r}{d} \right)^2 + i(n\varphi - \beta_0 r) \right]$$

Lagero ir Gauso pluošto kompleksinė amplitudė už kūginės prizmės

$$B_n(\xi, \varphi, z_0) = a \left( \frac{m}{2} \right)^{n/2} e^{i(\varphi - n\pi/2 - \pi/4)} \left[ F_1 + F_2 \right] J_n(n\xi) - i \left[ F_1 - F_2 \right] J_{n-1}(n\xi)$$

$$\xi = r/d \quad z_0 = \frac{mz}{2L_d} \quad L_d = \frac{kd^2}{2} \quad m = \beta_0 d$$

$$F_1 = \left( \frac{z_0 + \xi}{z_0} \right)^{n+1/2} e^{-\left( \frac{z_0 + \xi}{z_0} \right)^2}$$

$$x > 0 \quad H_0(x) = 1$$

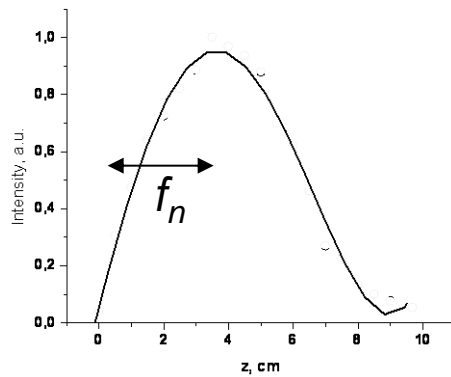
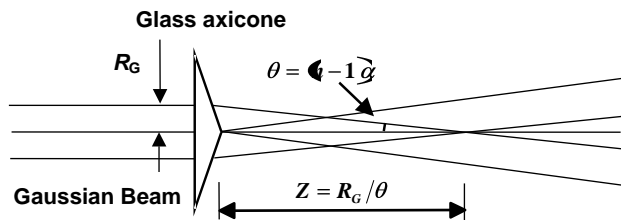
$$x \leq 0 \quad H_0(x) = 0$$

$$F_2 = \left( \frac{z_0 - \xi}{z_0} \right)^{n+1/2} e^{-\left( \frac{z_0 - \xi}{z_0} \right)^2} H_0\left( \frac{z_0 - \xi}{z_0} \right)$$

# Beselio pluoštai

$$I_n(r, z_0) = \frac{\pi m}{2} a^2 \left[ F_1 + F_2 \right] J_n^2(n\xi) + \left[ F_1 - F_2 \right] J_{n-1}^2(n\xi)$$

$$f_n = \frac{L_d}{m} \sqrt{2n+1} = \frac{d\sqrt{2n+1}}{2\alpha(n_a - 1)}$$



# Beselio pluoštai

$$I = \left| R_0 e^{ik_x x} + a \left( \frac{r}{d} \right)^n \exp \left[ i \left( k_x x - n\varphi - \beta_0 r \right) \right] \right|^2 =$$

$$= R_0^2 + a^2 \left( \frac{r}{d} \right)^{2n} e^{-2r^2/d^2} + 2R_0 a \left( \frac{r}{d} \right)^n e^{-r^2/d^2} \cos(k_x x - n\varphi + \beta_0 r)$$

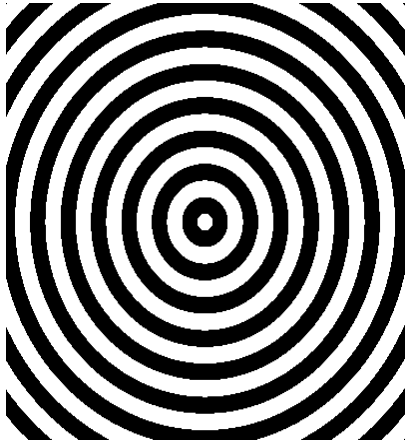
Intensyvumo skirstinys, kai interferuoja Beselio pluoštas ir atraminė banga (holograma)

$$T = \frac{1}{2} \left( 1 - \cos(k_x x - n\varphi + \beta_0 r) \right) \quad \text{Pralaidumo funkcija}$$

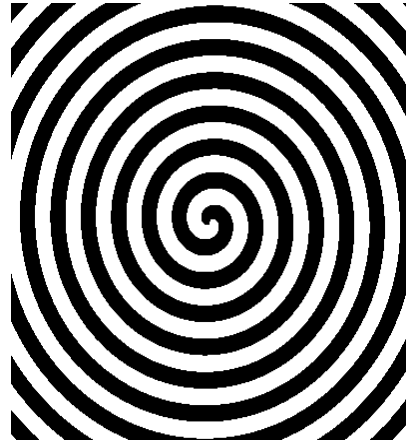


$$E_T = \frac{a}{2} e^{-r^2/d^2} - \frac{a}{4} e^{-r^2/d^2} e^{i(k_x x - n\varphi + \beta_0 r)} - \frac{a}{2} e^{-r^2/d^2} e^{-i(k_x x - n\varphi + \beta_0 r)}$$

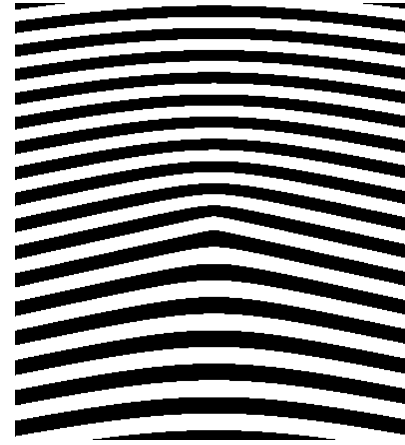
# Beselio pluoštai



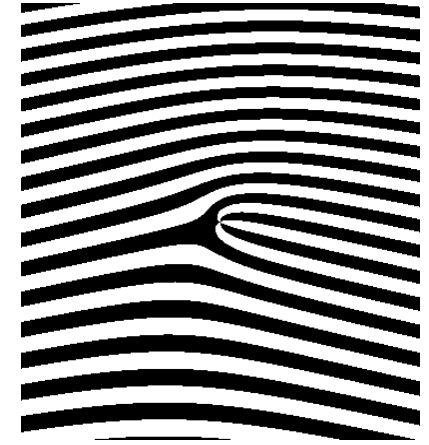
(a)



(b)



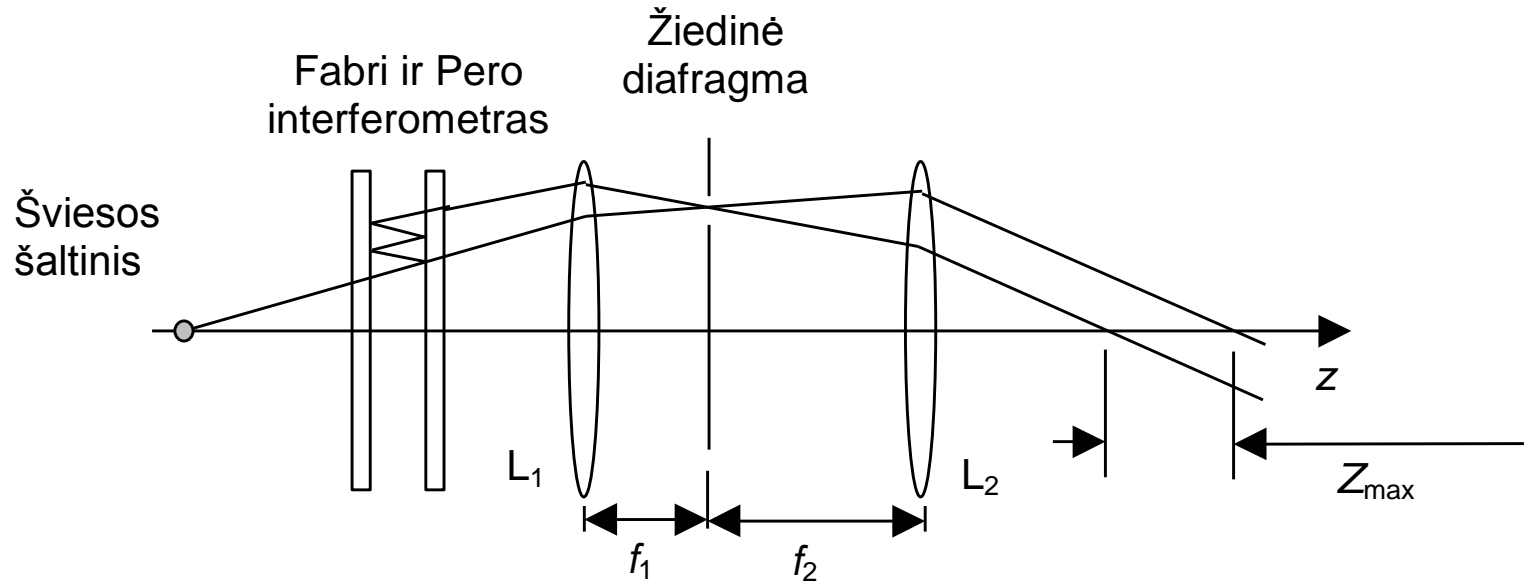
(c)



(d)

Amplitudinių binarinių (juoda ir balta) hologramų, skirtų generuoti įvairios eilės Beselio pluoštams, pavyzdžiai. (a) ir (b) bendraašės hologramos atitinkamai  $J_0$  ir  $J_1$  Beselio pluoštų formavimui. (c) ir (d) nebendraašės hologramos skirtos  $J_0$  ir  $J_3$  Beselio pluoštų formavimui.

# Beselio pluoštai



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