

Homework: elementary particle dynamics**possible after epd1**

- 2.1. Calculate the ratio of the gravitational attraction to the electrical repulsion between two stationary electrons. (Do I need to tell you how far apart they are?) 0.5 POINTS
- 2.2. Sketch the lowest-order Feynman diagram representing *Delbruck scattering*: $\gamma + \gamma \rightarrow \gamma + \gamma$. (This process, the scattering of light by light, has no analog in classical electrodynamics.) 0.5 POINTS
- 2.3. Draw all the fourth-order (four vertex) diagrams for Compton scattering $\gamma + e^- \rightarrow \gamma + e^-$. (There are 17 of them; disconnected diagrams don't count.) 1.5 POINTS

possible after epd3

- 2.6. Draw all the lowest-order diagrams contributing to the process $e^+ + e^- \rightarrow W^+ + W^-$. (One of them involves the direct coupling of Z to W 's and another the coupling of γ to W 's, so when LEP (the electron-positron collider at CERN) achieved sufficient energy to make two W 's, in 1996, these exotic processes could be studied experimentally. See B. Schwarzschild, *Physics Today* (September 1996), p. 21. 0.5 POINTS
- 2.8. Some decays involve two (or even all three) different forces. Draw possible Feynman diagrams for the following processes:

(a) $\mu \rightarrow e + e + e^+ + \nu_\mu + \bar{\nu}_e$ 0.25 POINTS

What interactions are involved? (Both these decays have been observed, by the way.)

possible after epd4

- 2.5. (a) Which decay do you think would be more likely, 0.5 POINTS

$$\Xi^- \rightarrow \Lambda + \pi^- \quad \text{or} \quad \Xi^- \rightarrow n + \pi^-$$

Explain your answer, and confirm it by looking up the experimental data.

- (b) Which decay of the $D^0(c\bar{u})$ meson is most likely, 0.5 POINTS

$$D^0 \rightarrow K^- + \pi^+ , \quad D^0 \rightarrow \pi^- + \pi^+ , \quad \text{or} \quad D^0 \rightarrow K^+ + \pi^-$$

Which is *least* likely? Draw the Feynman diagrams, explain your answer and check the experimental data.

(One of the successful predictions of the Cabibbo/GIM/KM model was that charmed mesons should decay preferentially into strange mesons, even though *energetically* the 2π mode is favored.)

- 2.8. Some decays involve two (or even all three) different forces. Draw possible Feynman diagrams for the following processes:

(b) $\Sigma^+ \rightarrow p + \gamma$ 0.5 POINTS

What interactions are involved? (Both these decays have been observed, by the way.)

- 2.12. The W^- was discovered in 1983 at CERN, using proton/antiproton scattering:

$$p + \bar{p} \rightarrow W^- + X$$

where X represents one or more particles. What is the most likely X , for this process? Draw a Feynman diagram for your reaction. and explain why your X is more probable than the various alternatives. 0.5 POINTS

possible after sr2

- 2.4. Determine the mass of the virtual photon in each of the lowest-order diagrams for Bhabha scattering $e^+ + e^- \rightarrow e^+ + e^-$ (assume the electron and positron are at rest). What is the photons velocity? (Note that these answers are impossible for *real* photons.) 0.5 POINTS

Homework: relativistic kinematics

possible after sr1

- 3.9. Given two four-vectors, $a^\mu = (3, 4, 1, 2)$ and $b^\mu = (5, 0, 3, 4)$, find
- (a) a_μ, b_μ 0.1 POINTS
 - (b) $(\vec{a})^2, (\vec{b})^2$ 0.1 POINTS
 - (c) $\vec{a} \cdot \vec{b}$ 0.1 POINTS
 - (d) a^2, b^2 0.1 POINTS
 - (e) $a \cdot b$ 0.1 POINTS
 - (f) Characterize a^μ and b^μ as timelike, spacelike, or lightlike. 0.1 POINTS
- 3.10. A second-rank tensor is called *symmetric* if it is unchanged when you switch the indices ($s^{\nu\mu} = s^{\mu\nu}$); it is called *antisymmetric* if it changes sign ($a^{\nu\mu} = -a^{\mu\nu}$).
- (a) How many independent elements are there in a symmetric tensor? (Since $s^{12} = s^{21}$, these would count as only *one* independent element.) 0.1 POINTS
 - (b) How many independent elements are there in an antisymmetric tensor? 0.1 POINTS
 - (c) Show that symmetry is preserved by Lorentz transformations — that is, if $s^{\mu\nu}$ is symmetric, so too is $s'^{\mu\nu}$. What about *antisymmetry*? 0.1 POINTS
 - (d) If $s^{\mu\nu}$ is symmetric, show that $s_{\mu\nu}$ is also symmetric.
If $a^{\mu\nu}$ is antisymmetric, show that $a_{\mu\nu}$ is antisymmetric. 0.1 POINTS
 - (e) If $s^{\mu\nu}$ is symmetric and $a^{\mu\nu}$ is antisymmetric, show that $s^{\mu\nu}a_{\mu\nu} = 0$. 0.1 POINTS
 - (f) Show that any second-rank tensor ($t^{\mu\nu}$) can be written as the sum of an anti-symmetric part ($a^{\mu\nu}$) and a symmetric part ($s^{\mu\nu}$): ($t^{\mu\nu} = a^{\mu\nu} + s^{\mu\nu}$). Construct ($a^{\mu\nu}$) and ($s^{\mu\nu}$) explicitly, given ($t^{\mu\nu}$). 0.1 POINTS

Remark: It helps to read the corresponding chapters in Griffiths ...

if this is not enough for 3.10, it might help to read "Special Relativity for Particle Physics": <http://web.vu.lt/ff/t.gajdosik/files/2014/01/sr4wop.pdf>

- 3.15. A pion traveling at speed v decays into a muon and a neutrino, $\pi^- \rightarrow \mu^- + \bar{\nu}$. If the neutrino emerges at 90° to the original pion direction, at what angle does the muon come off? 0.5 POINTS

possible after sr2

3.3. Transformation between the frames S' and S , which are moving with the speed v relative to each other:

(a) How do *volumes* transform?

Specifically, if a container has volume V' in its own rest frame, S' , what is its volume V as measured by an observer in S , with respect to which it is moving at speed v . 0.2 POINTS

(b) How do *densities* transform?

If a container holds ρ' molecules per unit volume in its own rest frame, S' , how many molecules per unit volume does it carry in S ? 0.2 POINTS

3.4. Cosmic ray muons are produced high in the atmosphere (at 8000 m, say) and travel toward the earth at very nearly the speed of light, (0.998 c , say). The speed of light is roughly $3. \times 10^8$ m/s.

(a) Given the lifetime of the muon (2.2×10^{-6} s), how far would the average muon go before disintegrating, according to preresativistic physics? Would the muon make it to ground level? 0.2 POINTS

(b) Now answer the same question using *relativistic* physics. (Because of time dilation, the muons last longer, so they travel farther.) 0.2 POINTS

(c) Pions are also produced in the upper atmosphere. In fact, the sequence is: a proton (from outer space) hits a proton (in the atmosphere) $\rightarrow p + p +$ pions. The pions then decay into muons: $\pi^\pm \rightarrow \mu^\pm + \nu_\mu(\bar{\nu}_\mu)$. But the lifetime of the pion is much shorter, (2.6×10^{-8} s). Assuming the pions have the same speed (0.998 c), will the average pion reach ground level? 0.2 POINTS

(*) Now analyze the same process from the perspective of the *muon*. (In *its* reference frame it only lasts 2.2×10^{-6} s; how then does it make it to the ground?) 0.2 POINTS

3.5. Half the muons in a monoenergetic beam decay in the first 600 m. How fast are they going? 0.5 POINTS

3.6. As the outlaws escape in their getaway car, which goes $\frac{3}{4} c$, the cop fires a bullet from the pursuit car, which goes $\frac{1}{2} c$. The muzzle velocity (speed relative to the gun) of the bullet is $\frac{1}{3} c$. Does the bullet reach its target

(a) According to preresativistic physics? 0.2 POINTS

(b) According to relativity? 0.2 POINTS

3.11. A particle is traveling at $\frac{3}{5} c$ in $\hat{x}$ -direction. Determine its proper velocity $\eta^\mu = \frac{1}{m}p^\mu$ (all four components). 0.5 POINTS

3.13. Is p^μ timelike, spacelike, or lightlike for a (real) particle with mass m ?

How about a massless particle?

How about a virtual particle?

0.3 POINTS

3.23. A particle travelling at speed u approaches an identical particle at rest.

- (a) What is the speed v of each particle in the CM frame?
(Classically, of course, it would just be $u/2$.) 0.2 POINTS
- (b) Find $\gamma \equiv 1/\sqrt{1-v^2/c^2}$ in terms of $\gamma' \equiv 1/\sqrt{1-u^2/c^2}$. 0.2 POINTS
- (c) Use your result in part (b) to express the kinetic energy of each particle in the CM frame, and thus re-derive eq. (3.54). 0.2 POINTS

possible after sr3

3.16. Particle A (energy E) hits particle B (at rest), producing particles $C_1, C_2, \dots, C_n$. Calculate the threshold (i.e. the minimum E) for this reaction, in terms of the various particle masses. 1 POINTS

3.17. Use the result of Problem 3.16 to find the threshold energies for the following reactions, assuming that the target proton is stationary:

- (a) $p + p \rightarrow p + p + \pi^0$ 0.1 POINTS
- (b) $p + p \rightarrow p + p + \pi^+ + \pi^-$ 0.1 POINTS
- (c) $\pi^- + p \rightarrow p + \bar{p} + n$ 0.1 POINTS
- (d) $\pi^- + p \rightarrow K^0 + \Sigma^0$ 0.1 POINTS
- (e) $p + p \rightarrow p + \Sigma^+ + K^0$ 0.1 POINTS

3.18. The first man-made Ω^- (Fig. 1.9) was created by firing a high-energy proton at a stationary hydrogen atom to produce a K^+/K^- pair: $p + p \rightarrow p + p + K^+ + K^-$; the K^- in turn hit another stationary proton, $K^- + p \rightarrow \Omega^- + K^0 + K^+$. What minimum kinetic energy is required (for the incident proton), to make an Ω^- in this way? (Gell-Mann must have done this calculation to see whether the experiment would be feasible.) 1.5 POINTS

3.19. Particle A , at rest, decays into particles B and C ($A \rightarrow B + C$).

- (a) Find the energy of the outgoing particles in terms of the various masses. 0.5 POINTS
- (b) Find the magnitude of the outgoing momenta. 0.5 POINTS
- (c) Note that the triangle function $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz$ factors: $\lambda(a^2, b^2, c^2) = (a+b+c)(a-b+c)(a+b-c)(a-b-c)$. Thus $|\vec{p}_B|$ goes to zero when $m_A = m_B + m_C$, and runs imaginary when $m_A < (m_B + m_C)$. Explain. 0.5 POINTS

3.20. Use the result of Problem 3.19 to find the CM energy of each decay product in the following reactions:

- (a) $\pi^- \rightarrow \mu^- + \bar{\nu}$ 0.1 POINTS
- (c) $\pi^0 \rightarrow \gamma + \gamma$ 0.1 POINTS
- (c) $K^+ \rightarrow \pi^+ + \pi^0$ 0.1 POINTS
- (d) $\Lambda \rightarrow p + \pi^-$ 0.1 POINTS
- (e) $\Omega^- \rightarrow \Lambda + K^-$ 0.1 POINTS

- 3.21. A pion at rest decays into a muon and a neutrino ($\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$). On the average, how far will the muon travel (in vacuum) before disintegrating? 0.5 POINTS

(*) The length of a muon track is measured to be about 0.6 mm. How do you explain this? 0.3 POINTS

- 3.22. Particle A , at rest, decays into three or more particles: $A \rightarrow B + C + D + \dots$

(a) Determine the maximum and the minimum energies that particle B can have in such a decay, in terms of the various masses ($m_A, m_B, m_C, m_D, \dots$). 0.5 POINTS

(b) Find the maximum and minimum electron energies in muon decay, $\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$. 0.2 POINTS

- 3.25. In a two-body scattering event, ($A + B \rightarrow C + D$), it is convenient to introduce the *Mandelstam variables*

$$s = (p_A + p_B)^2 \quad t = (p_A - p_C)^2 \quad u = (p_A - p_D)^2$$

(a) Show that $s + t + u = m_A^2 + m_B^2 + m_C^2 + m_D^2$. 0.2 POINTS

The *theoretical* virtue of the Mandelstam variables is that they are Lorentz invariants, with the same value in any inertial system. *Experimentally*, though, the more accessible parameters are energies and scattering angles.

(b) Find the CM energy of A, in terms of s, t, u , and the masses. 0.2 POINTS

(c) Find the Lab (B at rest) energy of A. 0.2 POINTS

(d) Find the total CM energy ($E_{\text{TOT}} = E_A + E_B = E_C + E_D$). 0.2 POINTS

- 3.27. Work out the kinematics of Compton scattering: a photon of wavelength λ collides elastically with a charged particle of mass m . If the photon scatters at angle θ , find its outgoing wavelength, λ' ; use $E_\gamma = h/\lambda$. 0.5 POINTS

possible after sr4

- 3.7. Find the matrix $\mathbf{M}$ that inverts the Lorentz transformation Λ , eq. (3.12): $x^\mu = M^\mu_\nu x'^\nu$. Show that $\mathbf{M}$ is the matrix inverse of Λ : $\Lambda \mathbf{M} = 1$. 0.5 POINTS

- 3.8. Show that $\mathbf{I} = x_\mu x^\mu$, eq. (3.13), is invariant under the Lorentz transformation Λ , eq. (3.8). 0.5 POINTS

- 3.12. Consider a collision in which particle A (with 4-momentum p_A^μ) hits particle B (with 4-momentum p_B^μ), producing particles C (p_C^μ) and D (p_D^μ). Assume energy-momentum conservation in system S (i.e. $p_A^\mu + p_B^\mu = p_C^\mu + p_D^\mu$). Using the Lorentz transformation $\Lambda(\eta)$, eq. (3.12), show that energy and momentum are also conserved in system S' . 0.5 POINTS

exercises from David Hogg, Chapter 6, p. 34
possible after sr3

- 6.7. A particle of mass M , at rest, decays into two smaller particles of masses m_1 and m_2 . What are their energies and momenta? 0.2 POINTS
- 6.8. Solve problem 6.7 again for the case $m_2 = 0$. Solve the equations for p and E_1 and then take the limit $m_1 \rightarrow 0$. 0.2 POINTS
- 6.9. If a massive particle decays into photons, explain using 4-momenta why it cannot decay into a single photon, but must decay into two or more. Does your explanation still hold if the particle is moving at high speed when it decays? 1.0 POINTS
- 6.10. A particle of rest mass M , travelling at speed v in the x -direction, decays into two photons, moving in the positive and negative x -direction relative to the original particle. What are their energies? 0.5 POINTS
What are the photon energies and directions if the photons are emitted in the positive and negative y -direction relative to the original particle (i.e., perpendicular to the direction of motion, in the particle's rest frame). 0.5 POINTS

Homework: Symmetries

possible after Symmetries (or self study) , will help with Quantum Mechanics

4.1. Prove that I , R_+ , R_- , R_a , R_b , and R_c are *all* the symmetries of the equilateral triangle. 1.0 POINTS

4.2. Construct a "multiplication table" for the triangle group (of exercise 4.1). Is this an Abelian group? How can you tell, just by looking at the multiplication table? 1.0 POINTS

4.6. Consider a vector $\vec{a}$ in two dimensions. Suppose its components with respect to Cartesian axes x, y , are (a_x, a_y) . What are its components (a'_x, a'_y) in a system x', y' which is rotated, counterclockwise, by an angle θ , with respect to x, y ?

(a) Express your answer in the form of a 2×2 matrix $R(\theta)$: 0.2 POINTS

$$\begin{pmatrix} a'_x \\ a'_y \end{pmatrix} = R(\theta) \begin{pmatrix} a_x \\ a_y \end{pmatrix}$$

(b) Show that R is an orthogonal matrix. 0.1 POINTS

(c) What is its determinant? 0.1 POINTS

(d) The set of *all* such rotations constitutes a group; what is the name of this group? 0.1 POINTS

(e) By multiplying the matrices show that $R(\theta_1)R(\theta_2) = R(\theta_1 + \theta_2)$; 0.1 POINTS

(f) is this an Abelian group? 0.1 POINTS

4.7. Consider the matrix $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$.

(a) Is it in the group $O(2)$? 0.1 POINTS

(b) How about $SO(2)$? 0.1 POINTS

(c) What is its effect on the vector $\vec{a}$ of Problem 4.6? 0.1 POINTS

(d) Does it describe a possible rotation of the plane? 0.2 POINTS

4.8. Suppose we interpret the electron literally as a classical solid sphere of radius r , mass m , spinning with angular momentum $\frac{1}{2}\hbar$. What is the speed, v , of a point on its 'equator'? Experimentally, it is known that r is less than 10^{-16} cm. What is the corresponding equatorial speed? What do you conclude from this? 0.5 POINTS

- 4.19. (a) Show that $\sigma_x^2 = \sigma_y^2 = \sigma_z^2 = 1$. ('1' here really means the 2×2 unit matrix; if no matrix is specified, the unit matrix is understood.) 0.3 POINTS
- (b) Show that $\sigma_x \sigma_y = -\sigma_y \sigma_x = i\sigma_z$; $\sigma_y \sigma_z = -\sigma_z \sigma_y = i\sigma_x$; $\sigma_z \sigma_x = -\sigma_x \sigma_z = i\sigma_y$. 0.3 POINTS

These results are neatly summarized in the formula

$$\sigma_j \sigma_k = \delta_{jk} + i\epsilon_{jkl} \sigma_l$$

(summation over l implied), where δ_{jk} is the Kronecker delta:

$$\delta_{jk} = \begin{cases} 1, & \text{if } j = k \\ 0, & \text{otherwise} \end{cases}$$

and ϵ_{jkl} is the Levi-Civita symbol:

$$\epsilon_{jkl} = \begin{cases} 1, & \text{if } jkl = 123, 231, \text{ or } 312 \\ -1, & \text{if } jkl = 132, 213, \text{ or } 321 \\ 0, & \text{otherwise} \end{cases}$$

- 4.22. (a) Show that $U(\vec{\theta}) = e^{-i(\vec{\theta} \cdot \vec{\sigma})/2}$, in eq. (4.28), is *unitary*. 0.5 POINTS
- (b) Show that $\det U = 1$. [*Hint*: You can either do this directly (however, see footnote after eq. (4.29)), or else use the result of Problem 4.21.] 0.3 POINTS
- 4.23. The extension of everything in Section 4.4 to higher spin is relatively straightforward. For spin 1 we have three state ($m_s = +1, 0, -1$), which we can represent as column vectors

$$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix},$$

respectively. The only problem is to construct the 3×3 matrices $\hat{S}_x$, $\hat{S}_y$, and $\hat{S}_z$. The latter is easy:

- (a) Construct $\hat{S}_z$ for spin 1. 0.3 POINTS

To obtain $\hat{S}_x$ and $\hat{S}_y$ it is easiest to start with the "raising" and "lowering" operators, $\hat{S}_{\pm} = \hat{S}_x \pm i\hat{S}_y$, which have the property

$$\hat{S}_{\pm}|sm\rangle = \hbar\sqrt{s(s+1) - m(m \pm 1)}|s(m \pm 1)\rangle$$

- (b) Construct the matrices $\hat{S}_+$ and $\hat{S}_-$ for spin 1. 0.3 POINTS
- (c) Using (b) determine the spin-1 matrices $\hat{S}_x$ and $\hat{S}_y$. 0.3 POINTS
- (d) Do the same construction for spin $\frac{3}{2}$. 0.9 POINTS