## Quantum Field Theory 1

**Lecture times:** NTFMC B435, Monday $11^{00}$ to $13^{00}$ and Wednesday $15^{00}$ to $17^{00}$

| day | Subject |
|---|---|
| 09/04 | Overview, Introduction; discussion of times; discussion of prerequisites |
| 09/06 | Standard Model, particle content, indistinguishable particle, natural units, QM |
| 09/11 | Special Relativity and tensor algebra |
| 09/13 | Classical Field theory |
| 09/18 | *skipped due to the conference "Matter to the Deepest"* |
| 09/20 | *skipped due to the conference "Matter to the Deepest"* |
| 09/25 | Continuous symmetries, Noether's Theorem and conserved currents |
| 09/27 | Discussion/Exercises/Catch-up: LTs, QM, EoM, Rotations |
| 10/02 | Hamiltonian formalism and Canonical Quantization |
| 10/04 | Particles and anti-particles |
| 10/09 | *skipped due to 3rd CERN Baltic Conference* |
| 10/11 | *skipped due to 3rd CERN Baltic Conference* |
| 10/16 | Heisenberg and Schrödinger pictures |
| 10/18 | Discussion/Exercises/Catch-up: Casimir effect, NRQFT, Symmetries and complex field |
| 10/23 | Propagator |
| 10/25 | Interactions |
| 10/30 | Feynman diagrams |
| 10/25 | Discussion/Exercises/Catch-up: ... |
| 10/30 | Scatterings |
| 11/01 | *free – due to All Saints day* |
| 11/06 | Discussion/Exercises/Catch-up: ... |
| 11/08 | Scatterings |
| 11/13 | Cross section |
| 11/15 | Discussion/Exercises/Catch-up: optical theorem |
| 11/20 | Spinors |
| 11/22 | Dirac equation |
| 11/27 | Solution of the Dirac equation |
| 11/29 | Discussion/Exercises/Catch-up: gamma matrices, trace identities, $g = 2$ |
| 12/04 | Quantizing spin $\frac{1}{2}$ field |
| 12/06 | Quantizing spin 1 field |
| 12/11 | QED |
| 12/13 | Discussion/Exercises/Catch-up: QED processes, PDFs, collider processes |
| 12/18 | Non-Abelian gauge theories |
| 12/20 | Outlook for QFT II |

**Attendance** optional; bonus points for active participation possible.

**Homework** suggested; will count towards the grade; less credit for late homework;

**Grading:** 100 points = 100%,
available points:

35 homework

70 final exam: written and oral; 50% required to pass the course.

**email:** `tgajdosik@yahoo.com`

**webpage:** `http://web.vu.lt/ff/t.gajdosik/qft-1/`
Capital and small letters are important!

**Books** are available

## Reading assignments

The understanding of Special Relativity is needed for most parts of modern physics, although it might be hidden, like in electro-magnetism. But it is *essential* for particle physics. Therefore I *strongly* recommend the reading of the very short and very good introduction into Special Relativity by David Hogg [2]. In the lecture I want to stress additional features, which are not covered by David Hogg, but I will rely on the basic understanding, as it is taught by David Hogg.

A similar situation is with the presentation of Feynman diagrams. There the reading of Griffiths [3] and Zee [7] is required, the summary of diagrams in [10] or [12] is helpful. Of course, the questions resulting from the reading can be discussed in the weekly discussion hours.

## Homework

Without calculating some problems any lecture in theorectical physics remains a fairy tale. In that sense the homework is required to profit from this lecture. The solving of problems helps to understand, whether the student has understood the material or not. At the exam it is too late to recognise, that one has not learned the required material.

The students are invited to come before the homework is due to discuss the problems and ask. I will gladly help them to understand the problem and guide them to the solution. The best way to arrange for a meeting is to write an email to arrange a time, as I can not guarantee that I will have always immediately time for the questions or that I will be always in my room (NTFMC A321).

I plan to give less points for homework that is brought later than its due date. It will nevertheless help to do the homework, even if it is late, as the exam will have questions and problems to solve similar to the homework, too.

## Exam

The exam will be a written test, that I want to discuss afterwards with the student.

# References

[1] D. Tong,
*Lectures on Quantum Field Theory*
`http://www.damtp.cam.ac.uk/user/tong/qft.html`

[2] Lecture notes by David Hogg:
`http://cosmo.nyu.edu/hogg/sr/sr.pdf`

[3] David Griffiths,
*Introduction to Elementary Particles*
John Wiley & Sons, Inc.; ISBN 0-471-60386-4 (1987)

[4] M. Robinson,
*Symmetry and the standard model: Mathematics and particle physics*,
doi:10.1007/978-1-4419-8267-4

[5] P. B. Pal,
*Dirac, Majorana and Weyl fermions*
arXiv:1006.1718 [hep-ph].

[6] M. D. Schwartz, *Quantum Field Theory and the Standard Model*
Cambridge University Press; ISBN 978-1-107-03473-0 (2014)

[7] A. Zee,
*Quantum Field Theory in a Nutshell*
Princeton University Press; ISBN 0-691-01019-6 (2003)

[8] Michael E. Peskin and Daniel V. Schroeder,
*An Introduction to Quantum Field Theory*
Reading, USA: Addison-Wesley; ISBN 0-201-50397-2 (1995)

[9] I. J. R. Aitchison and A. J. G. Hey,
*Gauge theories in particle physics: A practical introduction.*
*Vol. 1: From relativistic quantum mechanics to QED*,
Bristol, UK: IOP (2003) 406 p
*Vol. 2: Non-Abelian gauge theories: QCD and the electroweak theory*,
Bristol, UK: IOP (2004) 454 p

[10] J. C. Romao and J. P. Silva,
*A resource for signs and Feynman diagrams of the Standard Model*
arXiv:1209.6213 [hep-ph].

[11] F. Olness and R. Scalise,
*Regularization, Renormalization, and Dimensional Analysis:*
*Dimensional Regularization meets Freshman E & M*,
Am. J. Phys. **79** (2011) 306 [arXiv:0812.3578 [hep-ph]].

[12] Stefan Pokorsky,
*Gauge Field Theories*
Cambridge University Press; ISBN 0-521-47816-2 (2000)

[13] The particle adventure:
`http://www.particleadventure.org/`

[14] F. Jegerlehner,
*Renormalizing the standard model*,
Conf. Proc. C **900603** (1990) 476.

[15] Steven Weinberg,
*The Quantum Theory of Fields, I and II*
Cambridge University Press; ISBN 0-521-58555-4 (1995)

[16] Steven Weinberg,
*The Quantum Theory of Fields, III*
Cambridge University Press; ISBN 0-521-66000-9 (2000)

[17] W. Siegel,
*Fields*,
hep-th/9912205; `http://insti.physics.sunysb.edu/~siegel/plan.html` (2002)

**Homework: Four-Vectors, Special Relativity** **— due 2023/09/13, 15:00**

exercises from David Griffiths, Chapter 3

3.9. Given two four-vectors, $a^\mu = (3, 4, 1, 2)$ and $b^\mu = (5, 0, 3, 4)$, find

- (a) $a_\mu$, $b_\mu$ 0.05 POINTS
- (b) $(\vec{a})^2$, $(\vec{b})^2$ 0.05 POINTS
- (c) $\vec{a} \cdot \vec{b}$ 0.05 POINTS
- (d) $a^2$, $b^2$ 0.05 POINTS
- (e) $a \cdot b$ 0.05 POINTS
- (f) Characterize $a^\mu$ and $b^\mu$ as timelike, spacelike, or lightlike. 0.05 POINTS

3.10. A second–rank tensor is called *symmetric* if it is unchanged when you switch the indices ($s^{\nu\mu} = s^{\mu\nu}$); it is called *antisymmetric* if it changes sign ($a^{\nu\mu} = -a^{\mu\nu}$).

- (a) How many independent elements are there in a symmetric tensor? (Since $s^{12} = s^{21}$, these would count as only *one* independent element.) 0.05 POINTS
- (b) How many independent elements are there in an antisymmetric tensor? 0.05 POINTS
- (c) Show that symmetry is preserved by Lorentz transformations — that is, if $s^{\mu\nu}$ is symmetric, so too is $s'^{\mu\nu}$. What about *anti*symmetry? 0.05 POINTS
- (d) If $s^{\mu\nu}$ is symmetric, show that $s_{\mu\nu}$ is also symmetric.
  If $a^{\mu\nu}$ is antisymmetric, show that $a_{\mu\nu}$ is antisymmetric. 0.05 POINTS
- (e) If $s^{\mu\nu}$ is symmetric and $a^{\mu\nu}$ is antisymmetric, show that $s^{\mu\nu}a_{\mu\nu} = 0$. 0.05 POINTS
- (f) Show that any second-rank tensor ($t^{\mu\nu}$) can be written as the sum of an antisymmetric part ($a^{\mu\nu}$) and a symmetric part ($s^{\mu\nu}$): ($t^{\mu\nu} = a^{\mu\nu} + s^{\mu\nu}$). Construct ($a^{\mu\nu}$) and ($s^{\mu\nu}$) explicitly, given ($t^{\mu\nu}$). 0.05 POINTS

3.19. Particle $A$, at rest, decays into particles $B$ and $C$ ($A \to B + C$).

- (a) Find the energy of the outgoing particles in terms of the various masses. 0.3 POINTS
- (b) Find the magnitude of the outgoing momenta. 0.3 POINTS
- (c) Note that the triangle function $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz$ factors: $\lambda(a^2, b^2, c^2) = (a + b + c)(a - b + c)(a + b - c)(a - b - c)$. Thus $|\vec{p}_B|$ goes to zero when $m_A = m_B + m_C$, and runs imaginary when $m_A < (m_B + m_C)$. Explain. 0.3 POINTS

3.21. A pion at rest decays into a muon and a neutrino ($\pi^- \to \mu^- + \bar{\nu}_\mu$). On the average, how far will the muon travel (in vacuum) before disintegrating? 0.35 POINTS

- (*) The length of a muon track is measured to be about 0.6 mm. How do you explain this? 0.15 POINTS

3.25. In a two–body scattering event, ($A + B \to C + D$), it is convenient to introduce the *Mandelstam variables*

$$s = (p_A + p_B)^2 \qquad t = (p_A - p_C)^2 \qquad u = (p_A - p_D)^2$$

- (a) Show that $s + t + u = m_A^2 + m_B^2 + m_C^2 + m_D^2$. 0.25 POINTS

The *theoretical* virtue of the Mandelstam variables is that they are Lorentz invariants, with the same value in any inertial system. *Experimentally*, though, the more accessible parameters are energies and scattering angles.

- (b) Find the CM energy of A, in terms of $s$, $t$, $u$, and the masses. 0.25 POINTS
- (c) Find the Lab ($B$ at rest) energy of A. 0.25 POINTS
- (d) Find the total CM energy ($E_{\text{TOT}} = E_A + E_B = E_C + E_D$). 0.25 POINTS

3.26. For elastic scattering of identical particles, $A + A \to A + A$, show that the Mandelstam variables (Problem 3.25) become

$$s = 4(|\vec{p}|^2 + m^2) \qquad t = -2|\vec{p}|^2(1 - \cos\theta) \qquad u = -2|\vec{p}|^2(1 + \cos\theta)$$

where $\vec{p}$ is the CM momentum of an incident particle, and $\theta$ is the scattering angle. 0.4 POINTS

**Recap: Units, Special Relativity** **— due 2023/09/27, 15:00**

Units Using natural units (for particle physics), find the conversion of the SI units of

(a) Length: express 1m in GeV. 0.05 POINTS

(b) Time: express 1s in GeV. 0.05 POINTS

(c) Energy: express 1J in GeV. 0.05 POINTS

(d) Mass: express 1kg in GeV. 0.05 POINTS

(e) Temperature: express 1K in GeV. 0.05 POINTS

LT From the invariance of $x^\mu g_{\mu\nu} x^\nu$ derive the properties of the Lorentz transformation (LT) $\Lambda^\mu_{\ \nu}$. Construct the LT for boosts. 0.25 POINTS

EM 1 Derive Maxwell's equations from the Lagrangian

$$\mathcal{L} = -\tfrac{1}{2}(\partial_\mu A_\nu)(\partial^\mu A^\nu) + \tfrac{1}{2}(\partial_\mu A^\mu)^2 \tag{1.18}$$

0.5 POINTS

EM 2 From the definition of the field strength $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ derive its Lorentztransformation. Start from the transformation of $A_\mu$. 0.1 POINTS

**Tong example sheet 1** **— due 2023/10/18, 15:00**

1. 0.5 POINTS
2. 0.5 POINTS
3. 0.5 POINTS
4. 0.5 POINTS
5. 0.5 POINTS
6. 1.0 POINTS
7. 1.0 POINTS
8. 1.0 POINTS
9. optional ! ... means additional points, but a lot of additional time (1.5) POINTS

**Tong example sheet 2** **— due 2023/11/08, 15:00**

1. 0.5 POINTS
2. 0.5 POINTS
3. 0.5 POINTS
4. 0.5 POINTS
5. 0.5 POINTS
6. 1.0 POINTS
7. optional ! ... means additional points, but a lot of additional time (1.5) POINTS
8. 0.5 POINTS
9. 0.5 POINTS
10. optional ! ... means additional points, but a lot of additional time (1.5) POINTS

**Tong example sheet 3** **— due 2023/11/22, 15:00**

1. 0.5 POINTS
2. 0.5 POINTS
3. 1.0 POINTS
4. 0.5 POINTS
5. 0.5 POINTS
6. 0.5 POINTS
7. 1.0 POINTS
8. is actually 9. on the page 4 of sheet 3. 1.0 POINTS

**Tong example sheet 4** **— due 2023/12/06, 15:00**

1. 0.5 POINTS
2. 0.5 POINTS
3. 0.5 POINTS
4. 0.5 POINTS
5. 0.5 POINTS
6. optional ! ... means additional points, but a lot of additional time (1.5) POINTS
7. 1.0 POINTS
8. 0.5 POINTS
9. optional ! ... means additional points, but a lot of additional time
   "Can you do better?" means: calculate the amplitude for $\gamma\gamma \to \phi\phi^*$ in scalar QED. (1.5) POINTS
10. optional ! ... means additional points, but a lot of additional time (1.5) POINTS