Quantum Field Theory 1

Lecture times: NTFMC B435, Monday 15^{00} to 17^{00} and   Wednesday 15^{00} to 17^{00}

day	Subject
09/02	Overview, Introduction; discussion of times; discussion of prerequisites
09/06	Standard Model, particle content, indistinguishable particle, natural units, QM
09/08	Special Relativity and tensor algebra
09/13	Classical Field theory
09/15	Continuous symmetries, Noether's Theorem and conserved currents
09/20	Discussion/Exercises/Catch-up: LTs, QM, EoM, Rotations
09/22	Hamiltonian formalism and Canonical Quantization
09/27	Particles and anti-particles
09/29	Heisenberg and Schrödinger pictures
10/04	Discussion/Exercises/Catch-up: Casimir effect, NRQFT, Symmetries and complex field
10/06	Propagator
10/11	Interactions
10/13	Feynman diagrams
10/18	Discussion/Exercises/Catch-up: ...
10/20	Scatterings
10/25	Cross section
10/27	Discussion/Exercises/Catch-up: optical theorem
11/01	*free: All Saints day*
11/03	Spinors
11/08	Dirac equation
11/10	Solution of the Dirac equation
11/15	Discussion/Exercises/Catch-up: gamma matrices, trace identities, $g = 2$
11/17	Quantizing spin $\frac{1}{2}$ field
11/22	Quantizing spin 1 field
11/24	QED
11/29	Discussion/Exercises/Catch-up: QED processes, PDFs, collider processes
12/01	Non-Abelian gauge theories
12/06	Outlook for QFT II
12/08	backup time, questions
12/13	backup time, questions
12/15	backup time, questions
12/20	backup time, questions

Attendance   optional; bonus points for active participation possible.

Homework   suggested; will count towards the grade; less credit for late homework;

Grading: 100 points = 100%,
available points:

    35   homework

    70   final exam: written and oral; 50% required to pass the course.

email: `tgajdosik@yahoo.com`

webpage: `http://web.vu.lt/ff/t.gajdosik/qft-1/`
Capital and small letters are important!

Books   are available

Reading assignments

The understanding of Special Relativity is needed for most parts of modern physics, although it might be hidden, like in electro-magnetism. But it is *essential* for particle physics. Therefore I *strongly* recommend the reading of the very short and very good introduction into Special Relativity by David Hogg [2]. In the lecture I want to stress additional features, which are not covered by David Hogg, but I will rely on the basic understanding, as it is taught by David Hogg.

    A similar situation is with the presentation of Feynman diagrams. There the reading of Griffiths [3] and Zee [7] is required, the summary of diagrams in [10] or [12] is helpful. Of course, the questions resulting from the reading can be discussed in the weekly discussion hours.

Homework

Without calculating some problems any lecture in theorectical physics remains a fairy tale. In that sense the homework is required to profit from this lecture. The solving of problems helps to understand, whether the student has understood the material or not. At the exam it is too late to recognise, that one has not learned the required material.

    The students are invited to come before the homework is due to discuss the problems and ask. I will gladly help them to understand the problem and guide them to the solution. The best way to arrange for a meeting is to write an email to arrange a time, as I can not guarantee that I will have always immediately time for the questions or that I will be always in my room (NTFMC A321).

    I plan to give less points for homework that is brought later than its due date. It will nevertheless help to do the homework, even if it is late, as the exam will have questions and problems to solve similar to the homework, too.

Exam

The exam will be a written test, that I want to discuss afterwards with the student.

References

[1] D. Tong,
    Lectures on Quantum Field Theory
    `http://www.damtp.cam.ac.uk/user/tong/qft.html`

[2] Lecture notes by David Hogg:
    `http://cosmo.nyu.edu/hogg/sr/sr.pdf`

[3] David Griffiths,
    Introduction to Elementary Particles
    John Wiley & Sons, Inc.; ISBN 0-471-60386-4 (1987)

[4] M. Robinson,
     Symmetry and the standard model: Mathematics and particle physics,
     doi:10.1007/978-1-4419-8267-4

[5] P. B. Pal,
     Dirac, Majorana and Weyl fermions
     arXiv:1006.1718 [hep-ph].

[6] M. D. Schwartz, *Quantum Field Theory and the Standard Model*
     Cambridge University Press; ISBN 978-1-107-03473-0 (2014)

[7] A. Zee,
     Quantum Field Theory in a Nutshell
     Princeton University Press; ISBN 0-691-01019-6 (2003)

[8] Michael E. Peskin and Daniel V. Schroeder,
     An Introduction to Quantum Field Theory
     Reading, USA: Addison-Wesley; ISBN 0-201-50397-2 (1995)

[9] I. J. R. Aitchison and A. J. G. Hey,
     Gauge theories in particle physics: A practical introduction.
     Vol. 1: From relativistic quantum mechanics to QED,
     Bristol, UK: IOP (2003) 406 p
     Vol. 2: Non-Abelian gauge theories: QCD and the electroweak theory,
     Bristol, UK: IOP (2004) 454 p

[10] J. C. Romao and J. P. Silva,
     A resource for signs and Feynman diagrams of the Standard Model
     arXiv:1209.6213 [hep-ph].

[11] F. Olness and R. Scalise,
     Regularization, Renormalization, and Dimensional Analysis:
     Dimensional Regularization meets Freshman E & M,
     Am. J. Phys. **79** (2011) 306 [arXiv:0812.3578 [hep-ph]].

[12] Stefan Pokorsky,
     Gauge Field Theories
     Cambridge University Press; ISBN 0-521-47816-2 (2000)

[13] The particle adventure:
      http://www.particleadventure.org/

[14] F. Jegerlehner,
     Renormalizing the standard model,
     Conf. Proc. C **900603** (1990) 476.

[15] Steven Weinberg,
     The Quantum Theory of Fields, I and II
     Cambridge University Press; ISBN 0-521-58555-4 (1995)

[16] Steven Weinberg,
     The Quantum Theory of Fields, III
     Cambridge University Press; ISBN 0-521-66000-9 (2000)

[17] W. Siegel,
     Fields,
     hep-th/9912205; http://insti.physics.sunysb.edu/~siegel/plan.html (2002)

Homework: Four-Vectors, Special Relativity — due 2020/09/15, 15:00

exercises from David Griffiths, Chapter 3

3.9. Given two four-vectors, $a^\mu = (3, 4, 1, 2)$ and $b^\mu = (5, 0, 3, 4)$, find

    (a) a_μ, b_μ 0.05 POINTS

    (b) $(\vec{a})^2$, $(\vec{b})^2$ 0.05 POINTS

    (c) $\vec{a} \cdot \vec{b}$ 0.05 POINTS

    (d) a^2, b^2 0.05 POINTS

    (e) $a \cdot b$ 0.05 POINTS

    (f) Characterize a^μ and b^μ as timelike, spacelike, or lightlike.   0.05 POINTS

3.10. A second–rank tensor is called *symmetric* if it is unchanged when you switch the indices ($s^{\nu\mu} = s^{\mu\nu}$); it is called *antisymmetric* if it changes sign ($a^{\nu\mu} = -a^{\mu\nu}$).

    (a) How many independent elements are there in a symmetric tensor? (Since $s^{12} = s^{21}$, these would count as only *one* independent element.)   0.05 POINTS

    (b) How many independent elements are there in an antisymmetric tensor?   0.05 POINTS

    (c) Show that symmetry is preserved by Lorentz transformations — that is, if $s^{\mu\nu}$ is symmetric, so too is $s'^{\mu\nu}$. What about *anti*symmetry?   0.05 POINTS

    (d) If $s^{\mu\nu}$ is symmetric, show that $s_{\mu\nu}$ is also symmetric.
        If $a^{\mu\nu}$ is antisymmetric, show that $a_{\mu\nu}$ is antisymmetric.   0.05 POINTS

    (e) If $s^{\mu\nu}$ is symmetric and $a^{\mu\nu}$ is antisymmetric, show that $s^{\mu\nu} a_{\mu\nu} = 0$.   0.05 POINTS

    (f) Show that any second-rank tensor ($t^{\mu\nu}$) can be written as the sum of an antisymmetric part ($a^{\mu\nu}$) and a symmetric part ($s^{\mu\nu}$): ($t^{\mu\nu} = a^{\mu\nu} + s^{\mu\nu}$). Construct ($a^{\mu\nu}$) and ($s^{\mu\nu}$) explicitly, given ($t^{\mu\nu}$).   0.05 POINTS

3.19. Particle A, at rest, decays into particles B and C ($A \rightarrow B + C$).

    (a) Find the energy of the outgoing particles in terms of the various masses.   0.3 POINTS

    (b) Find the magnitude of the outgoing momenta.   0.3 POINTS

    (c) Note that the triangle function $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz$ factors: $\lambda(a^2, b^2, c^2) = (a + b + c)(a - b + c)(a + b - c)(a - b - c)$. Thus $|\vec{p}_B|$ goes to zero when $m_A = m_B + m_C$, and runs imaginary when $m_A < (m_B + m_C)$. Explain.   0.3 POINTS

3.21. A pion at rest decays into a muon and a neutrino ($\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$). On the average, how far will the muon travel (in vacuum) before disintegrating?   0.3 POINTS

    (*) The length of a muon track is measured to be about 0.6 mm. How do you explain this?
                                                         0.15 POINTS

3.25. In a two–body scattering event, ($A + B \rightarrow C + D$), it is convenient to introduce the *Mandelstam variables*

$$s = (p_A + p_B)^2 \qquad t = (p_A - p_C)^2 \qquad u = (p_A - p_D)^2$$

    (a) Show that $s + t + u = m_A^2 + m_B^2 + m_C^2 + m_D^2$.   0.25 POINTS

The *theoretical* virtue of the Mandelstam variables is that they are Lorentz invariants, with the same value in any inertial system. *Experimentally*, though, the more accessible parameters are energies and scattering angles.

    (b) Find the CM energy of A, in terms of s, t, u, and the masses.   0.25 POINTS

    (c) Find the Lab (B at rest) energy of A.   0.25 POINTS

    (d) Find the total CM energy ($E_{\text{TOT}} = E_A + E_B = E_C + E_D$).   0.25 POINTS

3.26. For elastic scattering of identical particles, $A + A \rightarrow A + A$, show that the Mandelstam variables (Problem 3.25) become

$$s = 4(|\vec{p}|^2 + m^2) \qquad t = -2|\vec{p}|^2(1 - \cos\theta) \qquad u = -2|\vec{p}|^2(1 + \cos\theta)$$

where $\vec{p}$ is the CM momentum of an incident particle, and θ is the scattering angle.   0.4 POINTS

Recap: Units, Special Relativity   **— due 2020/09/22, 15:00**

Units Using natural units (for particle physics), find the conversion of the SI units of

    (a) Length: express 1m in GeV.   0.05 POINTS

    (b) Time: express 1s in GeV.   0.05 POINTS

    (c) Energy: express 1J in GeV.   0.05 POINTS

    (d) Mass: express 1kg in GeV.   0.05 POINTS

    (e) Temperature: express 1K in GeV.   0.05 POINTS

LT From the invariance of $x^\mu g_{\mu\nu} x^\nu$ derive the properties of the Lorentz transformation (LT) $\Lambda^\mu{}_\nu$. Construct the LT for boosts.   0.25 POINTS

EM 1 Derive Maxwell's equations from the Lagrangian

$$\mathcal{L} = -\tfrac{1}{2}(\partial_\mu A_\nu)(\partial^\mu A^\nu) + \tfrac{1}{2}(\partial_\mu A^\mu)^2 \tag{1.18}$$

                                                             0.5 POINTS

EM 2 From the definition of the field strength $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ derive its Lorentztransformation. Start from the transformation of A_μ.   0.1 POINTS

Tong example sheet 1   **— due 2020/09/29, 15:00**

    1.   0.5 POINTS

    2.   0.5 POINTS

    3.   0.5 POINTS

    4.   0.5 POINTS

    5.   0.5 POINTS

    6.   1.0 POINTS

    7.   1.0 POINTS

    8.   1.0 POINTS

    9. optional ! ... means additional points, but a lot of additional time   (1.5) POINTS

Tong example sheet 2   **— due 2020/10/13, 15:00**

    1.   0.5 POINTS

    2.   0.5 POINTS

    3.   0.5 POINTS

    4.   0.5 POINTS

    5.   0.5 POINTS

    6.   1.0 POINTS

    7. optional ! ... means additional points, but a lot of additional time   (1.5) POINTS

    8.   0.5 POINTS

    9.   0.5 POINTS

    10. optional ! ... means additional points, but a lot of additional time   (1.5) POINTS

Tong example sheet 3   **— due 2020/10/27, 15:00**

    1.   0.5 POINTS

    2.   0.5 POINTS

    3.   1.0 POINTS

    4.   0.5 POINTS

    5.   0.5 POINTS

    6.   0.5 POINTS

    7.   0.5 POINTS

    8. is actually 9. on the page 4 of sheet 3.   1.0 POINTS

Tong example sheet 4   **— due 2020/11/24, 15:00**

    1.   0.5 POINTS

    2.   0.5 POINTS

    3.   0.5 POINTS

    4.   0.5 POINTS

    5.   0.5 POINTS

    6. optional ! ... means additional points, but a lot of additional time   (1.5) POINTS

    7.   1.0 POINTS

    8.   0.5 POINTS

    9. optional ! ... means additional points, but a lot of additional time
       "Can you do better?" means: calculate the amplitude for $\gamma\gamma \to \phi\phi^*$ in scalar QED.   (1.5) POINTS

   10. optional ! ... means additional points, but a lot of additional time   (1.5) POINTS