Introduction to Quantum Field Theory

Lectures: 2 hours each week: NTFMC B435, Thursday 13^{00} to 15^{00}
and 2 hours for discussions: NTFMC A435, Monday 15^{00} to 17^{00}

day	Subject
09/10	Overview, Introduction; assignements of seminars; discussion of projects
09/17	Special Relativity Repetition: Invariants, Lorentztransformations, Poincaré group [1, 2, 3]
09/24	Special Relativity: Spinors [2, 3, 4]
10/01	Quantum Field Theory: Approaches: canonical, path-integral [3, 5, 6, 7, 8, 14]
10/08	Quantum Field Theory: Feynman diagrams [2, 5, 6, 7, 8, 9]
10/15	Quantum Field Theory: Renormalisation [5, 6, 7, 8, 10]
10/22	Quantum Field Theory: Gauge theory [3, 5, 6, 7, 8, 11]
10/29	Quantum Field Theory: QED [5, 6, 7, 8, 11]
11/05	Quantum Field Theory: QCD [5, 6, 7, 8, 11]
11/12	The Standard Model: Particle content [2, 3, 5, 12, 13]
11/19	The Standard Model: Higgs Mechanism [2, 3, 5, 12, 13]
11/26	The Standard Model: Particle detection [2, 12]
12/03	no lecture, 12/07 no discussions: (Visit to Vienna??) [fallback date]
12/10	Outlook: Supersymmetry (SUSY) — MSSM [6, 11, 15]
12/17	Outlook: Strings, Stringtheory, Superstrings [16, 17]
12/21	Questions, review of homework

Attendance optional; will count towards the grade

Homework suggested; will count towards the grade; less credit for late homework;

Grading: 100 points = 100%,
available points:

15 attendance /

15 seminar presentation / project presentation

20 homework

60 final exam: written and oral; 50% required to pass the course.

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webpage: `http://web.vu.lt/ff/t.gajdosik/mtf/`
Capital and small letters are important!

Books are available

Reading assignments

The understanding of Special Relativity is needed for most parts of modern physics, although it might be hidden, like in electro-magnetism. But it is *essential* for particle physics. Therefore I *strongly* recommend the reading of the very short and very good introduction into Special Relativity by David Hogg [1]. In the lecture I want to stress additional features, which are not covered by David Hogg, but I will rely on the basic understanding, as it is taught by David Hogg.

A similar situation is with the presentation of Feynman diagrams. There the reading of Griffiths [2] and Zee [6] is required. Of course, the questions resulting from the reading can be discussed in the weekly discussion hours.

Seminar presentation

The idea of the presentation is to involve the students into the discussion about particle physics and related areas.

The student chooses a subject for the presentation and clarifies with me, if the subject is suitable or not. If it is suitable the student will get a time during the discussion hours to present the subject to the fellow students. The presentation can be given in English or in Lithuanian. The presentation should be rather short, i.e. about 5 minutes, and it has to be presented using the computer.

- The presentation has to be prepared in a computer readable format:
  - **.pdf** or **.ps** are recommended, as the presentation will look the same, independent of the computer.
  - a powerpoint presentation might work, too, but then it has to be done for Windows XP;
- The presentation should be given orally. It is recommended, that the student does not just read a text, but explains the subjects freely in his own words.
- The student should be able to answer questions from his fellow students. That does not mean, that he has to have all the answers.

The presentation helps also practicing the necessary presentation of the masters thesis at the end of the students masters studies.

Homework

Without calculating some problems any lecture in theorectical physics remains a fairy tale. In that sense the homework is required to profit from this lecture. The solving of problems helps to understand, whether the student has understood the material or not. At the exam it is too late to recognise, that one has not learned the required material.

The students are invited to come before the homework is due to discuss the problems and ask. I will gladly help them to understand the problem and guide them to the solution. The best way to arrange for a meeting is to write an email to arrange a time, as I can not guarantee that I will have always immediately time for the questions or that I will be always in my room (NTFMC A321).

I plan to give less points for homework that is brought later than its due date. It will nevertheless help to do the homework, even if it is late, as the exam will have questions and problems to solve similar to the homework, too.

Exam

The exam will be a written test, that I want to discuss afterwards with the student.

References

[1] Lecture notes by David Hogg:
`http://cosmo.nyu.edu/hogg/sr/sr.pdf`

[2] David Griffiths,
Introduction to Elementary Particles
John Wiley & Sons, Inc.; ISBN 0-471-60386-4 (1987)

[3] M. Robinson,
Symmetry and the standard model: Mathematics and particle physics,
doi:10.1007/978-1-4419-8267-4

[4] P. B. Pal,
Dirac, Majorana and Weyl fermions
arXiv:1006.1718 [hep-ph].

[5] M. D. Schwartz, *Quantum Field Theory and the Standard Model*
Cambridge University Press; ISBN 978-1-107-03473-0 (2014)

[6] A. Zee,
Quantum Field Theory in a Nutshell
Princeton University Press; ISBN 0-691-01019-6 (2003)

[7] Michael E. Peskin and Daniel V. Schroeder,
An Introduction to Quantum Field Theory
Reading, USA: Addison-Wesley; ISBN 0-201-50397-2 (1995)

[8] I. J. R. Aitchison and A. J. G. Hey,
Gauge theories in particle physics: A practical introduction.
Vol. 1: From relativistic quantum mechanics to QED,
Bristol, UK: IOP (2003) 406 p
Vol. 2: Non-Abelian gauge theories: QCD and the electroweak theory,
Bristol, UK: IOP (2004) 454 p

[9] J. C. Romao and J. P. Silva,
A resource for signs and Feynman diagrams of the Standard Model
arXiv:1209.6213 [hep-ph].

[10] F. Olness and R. Scalise,
Regularization, Renormalization, and Dimensional Analysis:
Dimensional Regularization meets Freshman E & M,
Am. J. Phys. **79** (2011) 306 [arXiv:0812.3578 [hep-ph]].

[11] Stefan Pokorsky,
Gauge Field Theories
Cambridge University Press; ISBN 0-521-47816-2 (2000)

[12] The particle adventure:
`http://www.particleadventure.org/`

[13] F. Jegerlehner,
Renormalizing the standard model,
Conf. Proc. C **900603** (1990) 476.

[14] Steven Weinberg,
The Quantum Theory of Fields, I and II
Cambridge University Press; ISBN 0-521-58555-4 (1995)

[15] Steven Weinberg,
The Quantum Theory of Fields, III
Cambridge University Press; ISBN 0-521-66000-9 (2000)

[16] T. Mohaupt,
Introduction to string theory,
Lect. Notes Phys. **631** (2003) 173; `doi:10.1007/978-3-540-45230-0_5` [hep-th/0207249].

[17] W. Siegel,
Fields,
hep-th/9912205; `http://insti.physics.sunysb.edu/~siegel/plan.html` (2002)

Homework: Poincaré group **— due 2020/09/24, 13:00**

P.1. Using the generator for Lorentz transformations $(M_{\alpha\beta})^{\mu}{}_{\nu} = -i(\delta^{\mu}_{\alpha} g_{\beta\nu} - \delta^{\mu}_{\beta} g_{\alpha\nu})$

(a) On what quantities do these generators act? 0.2 POINTS

(b) How would the generator look when it should act onto a scalar? 0.2 POINTS

(c) Using x^{μ} as a column vector, write the the six matrices $(M_{\alpha\beta})^{\mu}{}_{\nu}$ for $\alpha < \beta$. 0.6 POINTS

(d) Check the Lie algebra of the Lorentz group

$$[M_{\alpha\beta}, M_{\gamma\delta}]^{\mu}{}_{\nu} = i(g_{\alpha\gamma} M_{\beta\delta} - g_{\beta\gamma} M_{\alpha\delta} - g_{\alpha\delta} M_{\beta\gamma} + g_{\beta\delta} M_{\alpha\gamma})^{\mu}{}_{\nu}$$

using the matrices from (c). 1 POINTS

P.2. Using the two Weyl-spinor generators

$$\begin{aligned} (\sigma^{\alpha\beta})_a{}^b &= -\tfrac{i}{4}(\sigma^{\alpha}\bar{\sigma}^{\beta} - \sigma^{\beta}\bar{\sigma}^{\alpha})_a{}^b \\ \text{and} \qquad (\bar{\sigma}^{\alpha\beta})^{\dot{a}}{}_{\dot{b}} &= -\tfrac{i}{4}(\bar{\sigma}^{\alpha}\sigma^{\beta} - \bar{\sigma}^{\beta}\sigma^{\alpha})^{\dot{a}}{}_{\dot{b}} \end{aligned}$$

(a) On what quantities do these two different generators act? 0.1 POINTS

(b) In what do these quantities from (a) differ? 0.1 POINTS

(c) Writing the two spinors from (a) and (b) as column vectors, write two sets of six matrices for $(\sigma^{\alpha\beta})_a{}^b$ and $(\bar{\sigma}^{\alpha\beta})^{\dot{a}}{}_{\dot{b}}$, each, taking $\alpha < \beta$. 0.6 POINTS

(d) Check the Lie algebra of the Lorentz group

$$[M^{\alpha\beta}, M^{\gamma\delta}] = i(g^{\alpha\gamma} M^{\beta\delta} - g^{\beta\gamma} M^{\alpha\delta} - g^{\alpha\delta} M^{\beta\gamma} + g^{\beta\delta} M^{\alpha\gamma})$$

using the matrices from (c): $M^{\alpha\beta} = \sigma^{\alpha\beta}$ or $M^{\alpha\beta} = \bar{\sigma}^{\alpha\beta}$. 1 POINTS

(e) Comment on the differences between $\sigma^{\alpha\beta}$ and $\bar{\sigma}^{\alpha\beta}$ and on the relation between boosts and rotations. 0.2 POINTS

Reading assignment: David Griffiths, Chapter 7.1 to 7.3, *The Dirac equation, etc.*
for more interested students: P. B. Pal, *Dirac, Majorana and Weyl fermions*, arXiv:1006.1718 [hep-ph].

Homework: Spinors — **due 2020/10/08, 13:00**
David Griffiths, Chapter 7, pp. 268-273, 7.2, 7.3, 7.4, 7.6, 7.13, 7.14, 7.15, 7.16, 7.17, 7.24, 7.29

7.2. Show that $\gamma^0 = \begin{pmatrix} \mathbf{1}_2 & 0 \\ 0 & -\mathbf{1}_2 \end{pmatrix}$, $\gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}$, satisfy $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$. 0.2 POINTS

7.3. Derive the normalization N (and show that it is the same) for the spinors

$$u^{(1)} = N \begin{pmatrix} 1 \\ 0 \\ \frac{p_z}{E+m} \\ \frac{p_x+ip_y}{E+m} \end{pmatrix} \qquad u^{(2)} = N \begin{pmatrix} 0 \\ 1 \\ \frac{p_x-ip_y}{E+m} \\ \frac{-p_z}{E+m} \end{pmatrix}$$

$$v^{(1)} = u^{(4)}(-E,-p) = N \begin{pmatrix} \frac{p_x-ip_y}{E+m} \\ \frac{-p_z}{E+m} \\ 0 \\ 1 \end{pmatrix} \qquad v^{(2)} = -u^{(3)}(-E,-p) = -N \begin{pmatrix} \frac{p_z}{E+m} \\ \frac{p_x+ip_y}{E+m} \\ 1 \\ 0 \end{pmatrix}$$

using the normalization condition $u^{(i)\dagger}u^{(i)} = v^{(i)\dagger}v^{(i)} = 2E$. 0.2 POINTS

7.4. Show that $u^{(1)}$ and $u^{(2)}$ are *orthogonal*, in the sense that $u^{(1)\dagger}u^{(2)} = 0$. Likewise, show that $v^{(1)}$ and $v^{(2)}$ are orthogonal. Are $u^{(1)}$ and $v^{(1)}$ orthogonal? 0.1 POINTS

7.6. If the z-axis (= 3-axis) points along the direction of motion, calculate the spinors $u^{(1)}$, $u^{(2)}$, $v^{(1)}$ and $v^{(2)}$. Confirm that these are eigenspinors of $S_z = \frac{\hbar}{2}\gamma^0\gamma^3\gamma^5$, and find their eigenvalues. 0.3 POINTS

7.13. (a) Starting from $\mathbf{S}(\Lambda) = a_+ + a_-\gamma^0\gamma^1$ with $a_\pm = \pm\sqrt{\frac{1}{2}(\gamma \pm 1)}$ and Λ being a Lorentz transformation only along the 1-axis, calculate $\mathbf{S}^\dagger\mathbf{S}$, and confirm that it is not the unity matrix. 0.1 POINTS

(b) Show that $\mathbf{S}^\dagger\gamma^0\mathbf{S} = \gamma^0$. 0.1 POINTS

(*) Show that $\bar{\mathbf{S}}(\Lambda) := \gamma^0\mathbf{S}^\dagger(\Lambda)\gamma^0 = \mathbf{S}^{-1}(\Lambda)$ 0.1 POINTS

7.14. Show that $\bar\psi\gamma^5\psi$ is invariant under $\psi \to \psi' = S\psi$ with S from 7.11.a. 0.2 POINTS

7.15. Show that the adjoint spinors $\bar{u}^{(1,2)}$ and $\bar{v}^{(1,2)}$ satisfy the equations 0.4 POINTS

$$\bar{u}(\gamma^\mu p_\mu - m) = 0 \qquad \text{and} \qquad \bar{v}(\gamma^\mu p_\mu + m) = 0 \ .$$

7.16. Show that the normalisation condition from 7.3., expressed with the adjoint spinors, becomes $\bar{u}u = -\bar{v}v = 2mc$. 0.2 POINTS

7.17. Show that $V^\mu = \bar\psi\gamma^\mu\psi$ is a four-vector, by confirming that its components transform according to the Lorentztransformation $V'^\mu = \Lambda^\mu_\nu V^\nu$ by transforming the spinors. 0.4 POINTS

7.24. Using $u^{(1)}$, $u^{(2)}$, and $v^{(1)}$, $v^{(2)}$, prove the completeness relations 0.2 POINTS

$$\sum_{s=1,2} u^{(s)}\bar{u}^{(s)} = (\gamma^\mu p_\mu + m) \quad \text{and} \quad \sum_{s=1,2} v^{(s)}\bar{v}^{(s)} = (\gamma^\mu p_\mu - m) \ .$$

7.29. (a) Show that $\gamma^0\gamma^{\nu\dagger}\gamma^0 = \gamma^\nu$, for $\nu = 0$, 1, 2, or 3. 0.1 POINTS

(b) If Γ is any product of γ-matrices ($\Gamma = \gamma^a\gamma^b\cdots\gamma^c$) show that $\bar\Gamma = \gamma^0\Gamma^\dagger\gamma^0$ is the same product in reverse order, $\bar\Gamma = \gamma^c\cdots\gamma^b\gamma^a$. 0.2 POINTS

Homework: Particle kinematics best done as early as possible — **due 2020/10/22, 13:00**
David Griffiths, Chapter 3, pp. 100-102, n. 3.14, n. 3.16, n. 3.18, n. 3.19, n. 3.22 and n. 3.22:

3.14. Particle A (energy E) hits particle B (at rest), producing particles C_1, C_2, $\dots C_n$. Calculate the threshold (i.e. the minimum E) for this reaction, in terms of the various particle masses. 0.4 POINTS

3.16. Particle A, at rest, decays into particles B and C ($A \to B + C$).

(a) Find the energy of the outgoing particles in terms of the various masses. 0.3 POINTS

(b) Find the magnitude of the outgoing momenta. 0.3 POINTS

(c) $|\vec{p}_B|$ goes to zero when $m_A = m_B + m_C$, and "runs imaginary" when $m_A < m_B + m_C$. Explain. 0.1 POINTS

3.18. (a) A pion at rest decays into a muon and a neutrino ($\pi^- \to \mu^- + \bar{\nu}_\mu$). On the average, how far will the muon travel (in vacuum) before disintegrating? **Calculate** the speed of the muon! 0.4 POINTS

(b) The length in the muon track in Figure 1.7 is about 0.6 mm (the photograph has been enlarged). How do you explain this? 0.2 POINTS

3.19. Particle A, at rest, decays into three or more particles: $A \to B + C + D + \dots$.

(a) Determine the maximum and the minimum energies that particle B can have in such a decay, in terms of the various masses ($m_A, m_B, m_C, m_D, \dots$). 0.5 POINTS

(b) Find the maximum and minimum electron energies in muon decay, $\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$. 0.2 POINTS

3.22. In a two–body scattering event, ($A + B \to C + D$), it is convenient to introduce the *Mandelstam variables*

$$s = (p_A + p_B)^2 \qquad t = (p_A - p_C)^2 \qquad u = (p_A - p_D)^2$$

(a) Show that $s + t + u = m_A^2 + m_B^2 + m_C^2 + m_D^2$. 0.2 POINTS

The *theoretical* virtue of the Mandelstam variables is that they are Lorentz invariants, with the same value in any inertial system. *Experimentally*, though, the more accessible parameters are energies and scattering angles.

(b) Find the CM energy of A, in terms of s, t, u, and the masses. 0.2 POINTS

(c) Find the Lab (B at rest) energy of A. 0.2 POINTS

(d) Find the total CM energy ($E_{\mathrm{TOT}} = E_A + E_B = E_C + E_D$). 0.2 POINTS

3.23. For elastic scattering of identical particles, $A + A \to A + A$, show that the Mandelstam variables (Problem 3.22) become

$$s = 4(|\vec{p}|^2 + m^2) \qquad t = -2|\vec{p}|^2(1 - \cos\theta) \qquad u = -2|\vec{p}|^2(1 + \cos\theta)$$

where $\vec{p}$ is the CM momentum of an incident particle, and θ is the scattering angle. 0.4 POINTS

Reading assignment: A. Zee, Chapter I.7. *Feynman Diagrams* and
David Griffiths, Chapter 6.3, *The Feynman Rules for a Toy Theory*

Homework: Feynman Diagrams best done soon after 2020/09/24 — **due 2020/11/05, 13:00**
David Griffiths, Chapter 6, p. 222, n. 6.11 and n. 6.12 (a):

6.11. (a) Is $A \to B + B$ a possible process in the ABC theory? 0.5 POINTS

(b) Suppose a diagram has n_A external A lines, n_B external B lines, and n_C external C lines. Develop a simple criterion for determining whether it is an allowed reaction. 1.5 POINTS

(c) Assuming A is heavy enough, what are the most likely decay modes, after $A \to B + C$? Draw a Feynman diagram for each decay. 1 POINTS

6.12. (a) Draw all lowest order diagrams for $A + A \to A + A$. (There are six of them.)
1.2 POINTS

Reading suggestion: Romao and Silva, *A resource for signs and Feynman diagrams of the Standard Model* [9] or Stefan Pokorsky, *Gauge Field Theories* [11], Appendix C, pp. 563 to 582. You should get the idea, how the parts of Feynman diagrams in the Standard Model look like. You can get these Feynman rules from other sources, too — where ever you like.

Homework: Processes in the Standard Model
possible to do it after 2020/09/23 — **due 2020/11/19, 13:00**
Use the Feynman Rules for the Standard Model.

SM.1 Draw all lowest order diagrams for $e^- + e^+ \to t + \bar{t}$. 1 POINTS

SM.2 Draw all lowest order diagrams for $H \to \gamma + \gamma$. You will notice that the Higgs particle does not couple directly to the photon. Nevertheless, this channel is one of the most promising channels for the Higgs discovery at the LHC. 2 POINTS

SM.3 Draw diagrams for the LHC with $p + p \to t(\bar{t}) + X$. Treat the proton as a bunch of different partons, i.e. parts of the proton, like up-quark, down-quark, gluons, sea-quarks, and other virtual particles. And the outcome should be either a top- or an antitop-quark and something, where this something is not specified. In the specific Feynman diagram, this X will be something specific, but think of all possibilities.
2 POINTS