1. Special Relativity (SR) — Introduction

Lectures

- Introduction; four-vectors, invariants, characterisation
- energy, momentum, mass
- collisions
- Lorentz transformations

Links

- Lecture notes by David Hogg: `http://cosmo.nyu.edu/hogg/sr/sr.pdf`
  - or: `http://web.vu.lt/ff/t.gajdosik/wop/sr.pdf`
- Tatsu Takeuchi: `http://www.phys.vt.edu/~takeuchi/relativity/notes/`
- "Special Relativity for Particle Physics":
  `http://web.vu.lt/ff/t.gajdosik/wop/sr4wop.pdf`

1. Special Relativity (SR) — Introduction

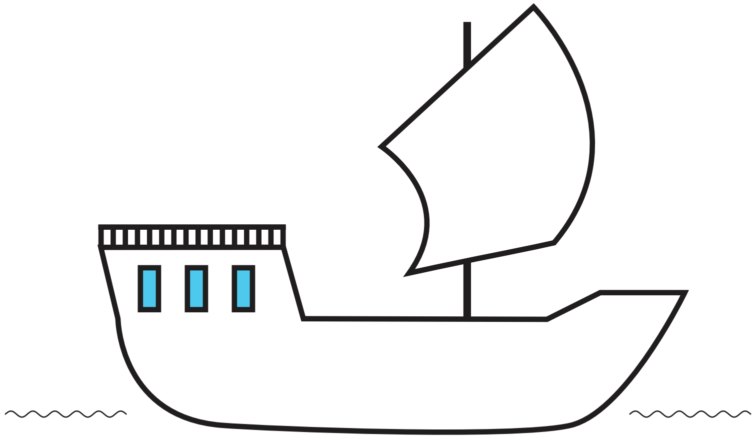
History

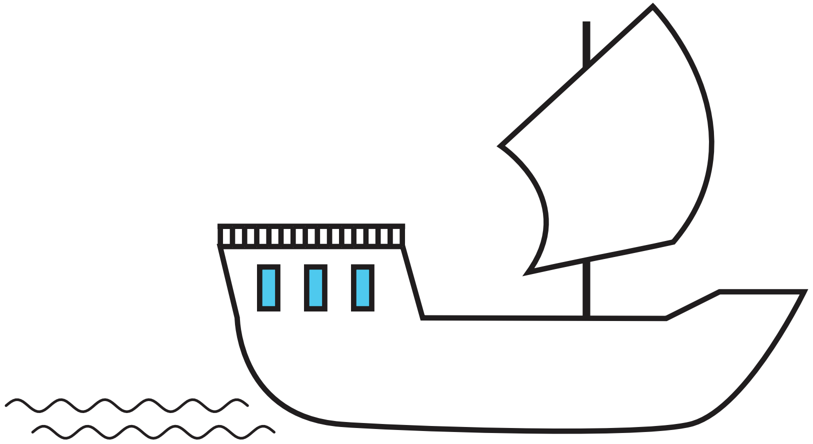
- 1632: Galileo Galilei describes the principle of relativity:
  - ''Dialogue concerning the Two Chief World Systems''
- 1861: Maxwell's equations
- 1887: Michelson-Morley experiment
- 1889 / 1892: Lorentz – Fitzgerald transformation
- 1905: Albert Einstein publishes the Theory of Special Relativity:
  - ''On the Electrodynamics of Moving Bodies''
- 1908: Hermann Minkovsky introduces 4D space-time

Galilean Invariance:

Every physical theory should mathematically look the same to every inertial observer

- for Galileo it was the mechanics and kinematics:
  - water dropping down
  - throwing a ball or a stone
  - insects flying
  - jumping around





1. Special Relativity (SR) — four-vectors

Galilean Invariance / Galilean transformations: $t \to t'$, $\vec{x} \to \vec{x}'$

Two inertial observers, O and O',

- measure the same absolute time (i.e.: 1 second = 1 second$'$)
  - Time translations : $t' = t + \tau$, $\vec{x}' = \vec{x}$

    in index notation: $t' = t + \tau$, $x'_j = x_j$
- have at $t = 0$ a relative distance $\Delta\vec{r}$
  - Spatial translations : $t' = t$, $\vec{x}' = \vec{x} + \Delta\vec{r}$

    in index notation: $t' = t$, $x'_j = x_j + \Delta r_j$
- have coordinate systems that are rotated by a relative rotation $\mathbf{R}$
  - Rotations : $t' = t$, $\vec{x}' = \mathbf{R} \cdot \vec{x}$, where $\mathbf{R}$ is an orthogonal matrix

    in index notation: $t' = t$, $x'_j = \mathbf{R}_{jk} x_k = \sum_{k=1}^{3} \mathbf{R}_{jk} x_k$
- have a constant relative velocity $\vec{v}$ (which can be zero, too)
  - Boosts : $t' = t$, $\vec{x}' = \vec{x} + \vec{v}t$

    in index notation: $t' = t$, $x'_j = x_j + v_j t$

1. Special Relativity (SR) — four-vectors

Vectors, Tensors, and notation

in the plane — i.e. in the 2D (Euclidean) space

- we can pick a coordinate system and describe points with coordinates
  - Cartesian coordinates (x, y)
  - Polar coordinates (r, θ)

- a vector can be understood as a difference of points

- position vector: difference between the position and the origin

- we can write the vector $\vec{v}$
  - as a row (v_x, v_y)
  - or as a column $\binom{v_x}{v_y}$
  - or in index notation v_i or v^i, where we identify $v_x = v_1$ and $v_y = v_2$

1. Special Relativity (SR) — four-vectors

Vectors, Tensors, and notation

multiplying vectors

- with a number, not a problem: $c * \vec{a} = (c * a_x, c * a_y)$
- with another vector: what do we want to get?
  - a number $\Rightarrow$ scalar product: $\vec{a} \cdot \vec{b} := a_x * b_x + a_y * b_y$
  - another vector: there is no unique prescription ...
  - a tensor $\Rightarrow$ tensor product: $\vec{a} \otimes \vec{b}$
    - in index notation: $a_j \otimes b_k = a_j b_k = (a \otimes b)_{jk}$

what is a tensor?

- an object that looks like the tensor product of vectors ...
- easiest imaginable in indexnotation:
  - a tensor is an object with indices $t_{jk\ell}$ or $t^{jk\ell}$ or $t^j{}_{k\ell}$
- special tensors
  - a vector is a tensor of rank one: it has one index
  - a matrix is a tensor of rank two: it has two indices

Vectors, Tensors, and notation

multiplying tensors

- one index of each can be treated like a scalar product
  $\Rightarrow$ matrix multiplication
  - with $a = a_{jk}$ and $b = b_{mn}$: $a \cdot b = \sum_k a_{jk} * b_{kn}$
    - here a and b can be understood as matrices
- in order to simplify the writing, we can omit the $\sum$ symbol
  $\Rightarrow$ Einsteins summation convention
  - one sums over repeated indices: $a_{jk} * b_{kn} := \sum_k a_{jk} * b_{kn}$

index position can be used to distinguish objects

- example:
  - columnvector $\vec{v} = v^i = \binom{v_x}{v_y}$
  - rowvector $(\vec{v})^\top = v_i = (v_x, v_y)$
  - ★ then a matrix has to have upper and lower index!

1. Special Relativity (SR) — four-vectors

Vectors, Tensors, and notation

in more dimensional space we just have more coordinates

- In 3D space (our 3D world):
  - $\vec{v} = (v_x, v_y, v_z) = v_i$ (in cartesian coordinates)
- In 4D Minkovsky space people do **not** write an arrow:
  - momentum $p = (E = p^t, p^x, p^y, p^z) = (p^0, p^1, p^2, p^3) = p^\mu$
    - and the index is usually a greek letter: μ, ν, ρ, etc.
  - position $r = (ct, x, y, z) = (x^0, x^1, x^2, x^3) = r^\mu$
    - time $ct = x^0$ is measured like spacial distances in meters
    - The constant speed of light c is used as the conversion factor between seconds and meters

For the rest of the lecture we set $c = 1$. (i.e.: $3 \cdot 10^8\text{m} = 1\text{s}$)

- so we measure time in seconds and distances in light-seconds (=300.000km)
- or distances in meters and time in "3 nanoseconds" (the time light needs to travel 1m)

1. Special Relativity (SR) — Invariants

What are invariant objects?

- Objects that are the same for every inertial observer
- Examples in 3D: rotations or translations
  - the distances ℓ between points: $\ell^2 = (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2$
  - the angle α between directions: $\cos\alpha = (\vec{a} \cdot \vec{b})/(|\vec{a}| * |\vec{b}|)$
- In 4D Minkovsky space: $(\Delta s)^2 = (\Delta t)^2 - (\Delta x)^2 - (\Delta y)^2 - (\Delta z)^2$
  - The time t is measured like spacial distances in meters
  - The constant speed of light c is used as the conversion factor between seconds and meters
- Any scalar product of four-vectors in Minkovsky space:

$$(p.q) = p^\mu q^\nu g_{\mu\nu} = p^0 q^0 - p^1 q^1 - p^2 q^2 - p^3 q^3$$

What is the use of scalar products?

- Scalars are the same in every inertial frame
  - If one knows its value in one frame, one knows it in every frame
  - Use the most comfortable frame to calculate the value of a scalar!

- Events A and B happen at a certain time in a certain place:
  - In every frame they can be described by four-vectors $a^\mu = (a^0, a^1, a^2, a^3)$ and $b^\mu = (b^0, b^1, b^2, b^3)$
  - Their relative position $d^\mu = a^\mu - b^\mu$ is frame dependent
  - But their "4-distance" $d^2 = (d \cdot d)$ is invariant
  - d^2 classifies the causal connection of A and B

Classification of d^2

- If $d^2 > 0$ they are time-like separated:
  - one event happens before the other in every frame
  - there is a frame, where A and B happen at the same position
  - in this frame $d^\mu = (\Delta t, 0, 0, 0)$ with $\Delta t = \sqrt{d^2}$
- If $d^2 = 0$ they are light-like related. If $A \neq B$:
  - there is no frame, where A and B happen at the same time
  - there is no frame, where A and B happen at the same position
  - there is a frame, where $d^\mu = (\eta, \eta, 0, 0)$ with η arbitrary
- If $d^2 < 0$ they are space-like separated:
  - there is a frame, where A and B happen at the same time
  - in this frame $d^\mu = (0, \Delta s, 0, 0)$, with $\Delta s = \sqrt{-d^2}$, if the x-axis is oriented in the direction $\overline{AB}$

Special scalar products

- Particles are described by their energy-momentum four-vector:

$$p^\mu = (p^0, p^1, p^2, p^3) = (E, p_x, p_y, p_z) = (E, \vec{p})$$

  - The mass of the particle is defined in its rest-frame: $\vec{p} = 0$
  - There, the energy-momentum four-vector is $p^\mu = (m, 0)$
  - Since $p^2 = (p \cdot p)$ is a scalar, it is the same in every frame
  - In the rest-frame $p^2 = m^2 - \vec{0}^2 = m^2$
  - Therefore in every frame

$$m^2 = E^2 - \vec{p}^2 \quad !$$

- This can be applied to collisions, too: $(p_1 + p_2)^2$ is constant
  - In the rest-frame of $(p_1 + p_2)$ we have $\vec{p}_1 + \vec{p}_2 = 0 \;\Rightarrow (p_1 + p_2)^2 = (E_1 + E_2)^2$
    * E_1 and E_2 are the energy values of p_1 and p_2 in the rest-frame of $(p_1 + p_2)$!