

## Concepts of modern theoretical physics

**Lectures:** 2 hours each week: room 511, Tuesday 13<sup>00</sup> to 15<sup>00</sup>  
**and 2 hours for discussions:** room 511, Monday 11<sup>00</sup> to 13<sup>00</sup>

day Subject

- 09/10 Overview, Introduction; assignments of seminars
- 09/10 Special Relativity Repetition: Invariants, Lorentztransformations, Poincaré group [1, 2, 3]
- 09/11 Special Relativity: Spinors [2, 3, 4]
- 09/18 Quantum Field Theory: Approaches: canonical, path-integral [3, 5, 6, 7, 13]
- 09/25 Quantum Field Theory: Feynman diagrams [2, 5, 6, 7, 8]
- 10/02 Quantum Field Theory: Renormalisation [5, 6, 7, 9]
- 10/09 Quantum Field Theory: Gauge theory [3, 5, 6, 7, 10]
- 10/16 Quantum Field Theory: QED [5, 6, 7, 10]
- 10/23 Quantum Field Theory: QCD [5, 6, 7, 10]
- 10/30 — Break / Rest / Questions —
- 11/06 The Standard Model: Particle content [2, 3, 11, 12]
- 11/13 The Standard Model: Higgs Mechanism [2, 3, 11, 12]
- 11/20 The Standard Model: Particle detection [2, 11]
- 11/27 Outlook: Supersymmetry (SUSY) — MSSM [5, 10, 14]
- 12/04 Outlook: Strings, Stringtheory, Superstrings [15, 16]
- 12/11 Questions, review of homework

**Attendance** optional; will count towards the grade

**Homework** suggested; will count towards the grade; less credit for late homework;

**Grading:** 100 points = 100%,  
available points:

15 attendance

15 seminar presentation

20 homework

60 final exam: written and oral; 50% required to pass the course.

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Capital and small letters are important!

**Books** are available

## Reading assignments

The understanding of Special Relativity is needed for most parts of modern physics, although it might be hidden, like in electro-magnetism. But it is *essential* for particle physics. Therefore I *strongly* recommend the reading of the very short and very good introduction into Special Relativity by David Hogg [1]. In the lecture I want to stress additional features, which are not covered by David Hogg, but I will rely on the basic understanding, as it is taught by David Hogg.

A similar situation is with the presentation of Feynman diagrams. There the reading of Griffiths [2] and Zee [5] is required. Of course, the questions resulting from the reading can be discussed in the weekly discussion hours.

## Seminar presentation

The idea of the presentation is to involve the students into the discussion about particle physics and related areas.

The student chooses a subject for the presentation and clarifies with me, if the subject is suitable or not. If it is suitable the student will get a time during the discussion hours to present the subject to the fellow students. The presentation can be given in English or in Lithuanian. The presentation should be rather short, i.e. about 5 minutes, and it has to be presented using the computer.

- The presentation has to be prepared in a computer readable format:
  - **.pdf** or **.ps** are recommended, as the presentation will look the same, independent of the computer.
  - a powerpoint presentation might work, too, but then it has to be done for Windows XP;
- The presentation should be given orally. It is recommended, that the student does not just read a text, but explains the subjects freely in his own words.
- The student should be able to answer questions from his fellow students. That does not mean, that he has to have all the answers.

The presentation helps also practicing the necessary presentation of the masters thesis at the end of the students masters studies.

## Homework

Without calculating some problems any lecture in theoretical physics remains a fairy tale. In that sense the homework is required to profit from this lecture. The solving of problems helps to understand, whether the student has understood the material or not. At the exam it is too late to recognise, that one has not learned the required material.

The students are invited to come before the homework is due to discuss the problems and ask. I will gladly help them to understand the problem and guide them to the solution. The best way to arrange for a meeting is to write an email to arrange a time, as I can not guarantee that I will have always immediately time for the questions or that I will be always in my room (305).

I plan to give less points for homework that is brought later than its due date. It will nevertheless help to do the homework, even if it is late, as the exam will have questions and problems to solve similar to the homework, too.

## Exam

The exam will be a written test, that I want to discuss afterwards with the student.

## References

- [1] Lecture notes by David Hogg:  
<http://cosmo.nyu.edu/hogg/sr/sr.pdf>
- [2] David Griffiths,  
*Introduction to Elementary Particles*  
John Wiley & Sons, Inc.; ISBN 0-471-60386-4 (1987)
- [3] M. Robinson,  
*Symmetry and the standard model: Mathematics and particle physics*,  
doi:10.1007/978-1-4419-8267-4
- [4] P. B. Pal,  
*Dirac, Majorana and Weyl fermions*  
arXiv:1006.1718 [hep-ph].
- [5] A. Zee,  
*Quantum Field Theory in a Nutshell*  
Princeton University Press; ISBN 0-691-01019-6 (2003)
- [6] Michael E. Peskin and Daniel V. Schroeder,  
*An Introduction to Quantum Field Theory*  
Reading, USA: Addison-Wesley; ISBN 0-201-50397-2 (1995)
- [7] I. J. R. Aitchison and A. J. G. Hey,  
*Gauge theories in particle physics: A practical introduction.*  
*Vol. 1: From relativistic quantum mechanics to QED*,  
Bristol, UK: IOP (2003) 406 p  
*Vol. 2: Non-Abelian gauge theories: QCD and the electroweak theory*,  
Bristol, UK: IOP (2004) 454 p
- [8] J. C. Romao and J. P. Silva,  
*A resource for signs and Feynman diagrams of the Standard Model*  
arXiv:1209.6213 [hep-ph].
- [9] F. Olness and R. Scalise,  
*Regularization, Renormalization, and Dimensional Analysis:*  
*Dimensional Regularization meets Freshman E & M*,  
Am. J. Phys. **79** (2011) 306 [arXiv:0812.3578 [hep-ph]].
- [10] Stefan Pokorsky,  
*Gauge Field Theories*  
Cambridge University Press; ISBN 0-521-47816-2 (2000)
- [11] The particle adventure:  
<http://www.particleadventure.org/>
- [12] F. Jegerlehner,  
*Renormalizing the standard model*,  
Conf. Proc. C **900603** (1990) 476.
- [13] Steven Weinberg,  
*The Quantum Theory of Fields, I and II*  
Cambridge University Press; ISBN 0-521-58555-4 (1995)
- [14] Steven Weinberg,  
*The Quantum Theory of Fields, III*  
Cambridge University Press; ISBN 0-521-66000-9 (2000)
- [15] T. Mohaupt,  
*Introduction to string theory*,  
Lect. Notes Phys. **631** (2003) 173; doi:10.1007/978-3-540-45230-0\_5 [hep-th/0207249].
- [16] W. Siegel,  
*Fields*,  
hep-th/9912205; <http://insti.physics.sunysb.edu/~siegel/plan.html> (2002)

**Homework: Poincaré group**

— due 2018/09/25, 13:00

P.1. Using the generator for Lorentz transformations  $(M_{\alpha\beta})^\mu{}_\nu = -i(\delta^\mu_\alpha g_{\beta\nu} - \delta^\mu_\beta g_{\alpha\nu})$

- (a) On what quantities do these generators act? 0.2 POINTS  
 (b) How would the generator look when it should act onto a scalar? 0.2 POINTS  
 (c) Using  $x^\mu$  as a column vector, write the the six matrices  $(M_{\alpha\beta})^\mu{}_\nu$  for  $\alpha < \beta$ . 0.6 POINTS  
 (d) Check the Lie algebra of the Lorentz group

$$[M_{\alpha\beta}, M_{\gamma\delta}]^\mu{}_\nu = i(g_{\alpha\gamma} M_{\beta\delta} - g_{\beta\gamma} M_{\alpha\delta} - g_{\alpha\delta} M_{\beta\gamma} + g_{\beta\delta} M_{\alpha\gamma})^\mu{}_\nu$$

using the matrices from (c).

1 POINTS

P.2. Using the two Weyl-spinor generators

$$\begin{aligned} (\sigma^{\alpha\beta})_a{}^b &= -\frac{i}{4}(\sigma^\alpha \bar{\sigma}^\beta - \sigma^\beta \bar{\sigma}^\alpha)_a{}^b \\ \text{and } (\bar{\sigma}^{\alpha\beta})^{\dot{a}}{}_{\dot{b}} &= -\frac{i}{4}(\bar{\sigma}^\alpha \sigma^\beta - \bar{\sigma}^\beta \sigma^\alpha)^{\dot{a}}{}_{\dot{b}} \end{aligned}$$

- (a) On what quantities do these two different generators act? 0.1 POINTS  
 (b) In what do these quantities from (a) differ? 0.1 POINTS  
 (c) Writing the two spinors from (a) and (b) as column vectors, write two sets of six matrices for  $(\sigma^{\alpha\beta})_a{}^b$  and  $(\bar{\sigma}^{\alpha\beta})^{\dot{a}}{}_{\dot{b}}$ , each, taking  $\alpha < \beta$ . 0.6 POINTS  
 (d) Check the Lie algebra of the Lorentz group

$$[M^{\alpha\beta}, M^{\gamma\delta}] = i(g^{\alpha\gamma} M^{\beta\delta} - g^{\beta\gamma} M^{\alpha\delta} - g^{\alpha\delta} M^{\beta\gamma} + g^{\beta\delta} M^{\alpha\gamma})$$

using the matrices from (c):  $M^{\alpha\beta} = \sigma^{\alpha\beta}$  or  $M^{\alpha\beta} = \bar{\sigma}^{\alpha\beta}$ .

1 POINTS

- (e) Comment on the differences between  $\sigma^{\alpha\beta}$  and  $\bar{\sigma}^{\alpha\beta}$  and on the relation between boosts and rotations. 0.2 POINTS

**Reading assignment:** David Griffiths, Chapter 7.1 to 7.3, *The Dirac equation, etc.*  
for more interested students: P. B. Pal, *Dirac, Majorana and Weyl fermions*, arXiv:1006.1718 [hep-ph].

**Homework: Spinors**

— due 2018/10/09, 13:00

David Griffiths, Chapter 7, pp. 251-255, 7.2, 7.3, 7.4, 7.6, 7.11, 7.12, 7.13, 7.14, 7.15, 7.22, 7.27

7.2. Show that  $\gamma^0 = \begin{pmatrix} \mathbf{1}_2 & 0 \\ 0 & -\mathbf{1}_2 \end{pmatrix}$ ,  $\gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}$ , satisfy  $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$ . 0.2 POINTS

7.3. Derive the normalization  $N$  (and show that it is the same) for the spinors

$$u^{(1)} = N \begin{pmatrix} 1 \\ 0 \\ \frac{p_z}{E+m} \\ \frac{p_x + ip_y}{E+m} \end{pmatrix} \quad u^{(2)} = N \begin{pmatrix} 0 \\ 1 \\ \frac{p_x - ip_y}{E+m} \\ \frac{-p_z}{E+m} \end{pmatrix}$$

$$v^{(1)} = u^{(4)}(-E, -p) = N \begin{pmatrix} \frac{p_x - ip_y}{E+m} \\ \frac{-p_z}{E+m} \\ 0 \\ 1 \end{pmatrix} \quad v^{(2)} = -u^{(3)}(-E, -p) = -N \begin{pmatrix} \frac{p_z}{E+m} \\ \frac{p_x + ip_y}{E+m} \\ 1 \\ 0 \end{pmatrix}$$

using the normalization condition  $u^{(i)\dagger}u^{(i)} = v^{(i)\dagger}v^{(i)} = 2E$ . 0.2 POINTS

7.4. Show that  $u^{(1)}$  and  $u^{(2)}$  are *orthogonal*, in the sense that  $u^{(1)\dagger}u^{(2)} = 0$ . Likewise, show that  $v^{(1)}$  and  $v^{(2)}$  are orthogonal. Are  $u^{(1)}$  and  $v^{(1)}$  orthogonal? 0.1 POINTS

7.6. If the  $z$ -axis (= 3-axis) points along the direction of motion, calculate the spinors  $u^{(1)}$ ,  $u^{(2)}$ ,  $v^{(1)}$  and  $v^{(2)}$ . Confirm that these are eigenspinors of  $S_z = \frac{\hbar}{2}\gamma^0\gamma^3\gamma^5$ , and find their eigenvalues. 0.3 POINTS

7.11. (a) Starting from  $\mathbf{S}(\Lambda) = a_+ + a_-\gamma^0\gamma^1$  with  $a_\pm = \pm\sqrt{\frac{1}{2}(\gamma \pm 1)}$  and  $\Lambda$  being a Lorentz transformation only along the 1-axis, calculate  $\mathbf{S}^\dagger\mathbf{S}$ , and confirm that it is not the unity matrix. 0.1 POINTS

(b) Show that  $\mathbf{S}^\dagger\gamma^0\mathbf{S} = \gamma^0$ . 0.1 POINTS

(\*) Show that  $\bar{\mathbf{S}}(\Lambda) := \gamma^0\mathbf{S}^\dagger(\Lambda)\gamma^0 = \mathbf{S}^{-1}(\Lambda)$  0.1 POINTS

7.12. Show that  $\bar{\psi}\gamma^5\psi$  is invariant under  $\psi \rightarrow \psi' = S\psi$  with  $S$  from 7.11.a. 0.2 POINTS

7.13. Show that the adjoint spinors  $\bar{u}^{(1,2)}$  and  $\bar{v}^{(1,2)}$  satisfy the equations 0.4 POINTS

$$\bar{u}(\gamma^\mu p_\mu - m) = 0 \quad \text{and} \quad \bar{v}(\gamma^\mu p_\mu + m) = 0 .$$

7.14. Show that the normalisation condition from 7.3., expressed with the adjoint spinors, becomes  $\bar{u}u = -\bar{v}v = 2mc$ . 0.2 POINTS

7.15. Show that  $V^\mu = \bar{\psi}\gamma^\mu\psi$  is a four-vector, by confirming that its components transform according to the Lorentztransformation  $V'^\mu = \Lambda^\mu_\nu V^\nu$  by transforming the spinors. 0.4 POINTS

7.22. Using  $u^{(1)}$ ,  $u^{(2)}$ , and  $v^{(1)}$ ,  $v^{(2)}$ , prove the completeness relations 0.2 POINTS

$$\sum_{s=1,2} u^{(s)}\bar{u}^{(s)} = (\gamma^\mu p_\mu + m) \quad \text{and} \quad \sum_{s=1,2} v^{(s)}\bar{v}^{(s)} = (\gamma^\mu p_\mu - m) .$$

7.27. (a) Show that  $\gamma^0\gamma^{\nu\dagger}\gamma^0 = \gamma^\nu$ , for  $\nu = 0, 1, 2, \text{ or } 3$ . 0.1 POINTS

(b) If  $\Gamma$  is any product of  $\gamma$ -matrices ( $\Gamma = \gamma^a\gamma^b \cdots \gamma^c$ ) show that  $\bar{\Gamma} = \gamma^0\Gamma^\dagger\gamma^0$  is the same product in reverse order,  $\bar{\Gamma} = \gamma^c \cdots \gamma^b\gamma^a$ . 0.2 POINTS

**Homework: Particle kinematics** best done as early as possible — **due 2018/10/23, 13:00**  
David Griffiths, Chapter 3, pp. 100-102, n. 3.14, n. 3.16, n. 3.18, n. 3.19, n. 3.22 and n. 3.22:

3.14. Particle  $A$  (energy  $E$ ) hits particle  $B$  (at rest), producing particles  $C_1, C_2, \dots, C_n$ . Calculate the threshold (i.e. the minimum  $E$ ) for this reaction, in terms of the various particle masses. 0.4 POINTS

3.16. Particle  $A$ , at rest, decays into particles  $B$  and  $C$  ( $A \rightarrow B + C$ ).

(a) Find the energy of the outgoing particles in terms of the various masses.

0.3 POINTS

(b) Find the magnitude of the outgoing momenta.

0.3 POINTS

(c)  $|\vec{p}_B|$  goes to zero when  $m_A = m_B + m_C$ , and "runs imaginary" when  $m_A < m_B + m_C$ . Explain.

0.1 POINTS

3.18. (a) A pion at rest decays into a muon and a neutrino ( $\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$ ). On the average, how far will the muon travel (in vacuum) before disintegrating? **Calculate** the speed of the muon!

0.4 POINTS

(b) The length in the muon track in Figure 1.7 is about 0.6 mm (the photograph has been enlarged). How do you explain this?

0.2 POINTS

3.19. Particle  $A$ , at rest, decays into three or more particles:  $A \rightarrow B + C + D + \dots$ .

(a) Determine the maximum and the minimum energies that particle  $B$  can have in such a decay, in terms of the various masses ( $m_A, m_B, m_C, m_D, \dots$ ).

0.5 POINTS

(b) Find the maximum and minimum electron energies in muon decay,  $\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$ .

0.2 POINTS

3.22. In a two-body scattering event, ( $A + B \rightarrow C + D$ ), it is convenient to introduce the *Mandelstam variables*

$$s = (p_A + p_B)^2 \quad t = (p_A - p_C)^2 \quad u = (p_A - p_D)^2$$

(a) Show that  $s + t + u = m_A^2 + m_B^2 + m_C^2 + m_D^2$ .

0.2 POINTS

The *theoretical* virtue of the Mandelstam variables is that they are Lorentz invariants, with the same value in any inertial system. *Experimentally*, though, the more accessible parameters are energies and scattering angles.

(b) Find the CM energy of A, in terms of  $s, t, u$ , and the masses.

0.2 POINTS

(c) Find the Lab ( $B$  at rest) energy of A.

0.2 POINTS

(d) Find the total CM energy ( $E_{\text{TOT}} = E_A + E_B = E_C + E_D$ ).

0.2 POINTS

3.23. For elastic scattering of identical particles,  $A + A \rightarrow A + A$ , show that the Mandelstam variables (Problem 3.22) become

$$s = 4(|\vec{p}|^2 + m^2) \quad t = -2|\vec{p}|^2(1 - \cos \theta) \quad u = -2|\vec{p}|^2(1 + \cos \theta)$$

where  $\vec{p}$  is the CM momentum of an incident particle, and  $\theta$  is the scattering angle.

0.4 POINTS

**Reading assignment:** A. Zee, Chapter I.7. *Feynman Diagrams* and David Griffiths, Chapter 6.3, *The Feynman Rules for a Toy Theory*

**Homework: Feynman Diagrams** best done soon after 2018/09/25 — **due 2018/11/06, 13:00**  
David Griffiths, Chapter 6, p. 212, n. 6.11 and n. 6.12 (a):

- 6.11. (a) Is  $A \rightarrow B + B$  a possible process in the  $ABC$  theory? 0.5 POINTS  
 (b) Suppose a diagram has  $n_A$  external  $A$  lines,  $n_B$  external  $B$  lines, and  $n_C$  external  $C$  lines. Develop a simple criterion for determining whether it is an allowed reaction. 1.5 POINTS  
 (c) Assuming  $A$  is heavy enough, what are the most likely decay modes, after  $A \rightarrow B + C$ ? Draw a Feynman diagram for each decay. 1 POINTS
- 6.12. (a) Draw all lowest order diagrams for  $A + A \rightarrow A + A$ . (There are six of them.) 1.2 POINTS

**Reading suggestion:** Romao and Silva, *A resource for signs and Feynman diagrams of the Standard Model* [8] or Stefan Pokorsky, *Gauge Field Theories* [10], Appendix C, pp. 563 to 582. You should get the idea, how the parts of Feynman diagrams in the Standard Model look like. You can get these Feynman rules from other sources, too — where ever you like.

**Homework: Processes in the Standard Model**

possible to do it after 2018/09/25

— **due 2018/11/27, 13:00**

Use the Feynman Rules for the Standard Model.

- SM.1 Draw all lowest order diagrams for  $e^- + e^+ \rightarrow t + \bar{t}$ . 1 POINTS
- SM.2 Draw all lowest order diagrams for  $H \rightarrow \gamma + \gamma$ . You will notice that the Higgs particle does not couple directly to the photon. Nevertheless, this channel is one of the most promising channels for the Higgs discovery at the LHC. 2 POINTS
- SM.2 Draw diagrams for the LHC with  $p + p \rightarrow t(\bar{t}) + X$ . Treat the proton as a bunch of different partons, i.e. parts of the proton, like up-quark, down-quark, gluons, sea-quarks, and other virtual particles. And the outcome should be either a top- or an antitop-quark and something, where this something is not specified. In the specific Feynman diagram, this  $X$  will be something specific, but think of all possibilities. 2 POINTS