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ANALYSING THE EXTENDED HIGGS SECTOR OF THE STANDARD MODEL: IN SEARCH OF STABLE *CP*-CONSERVING TWO HIGGS DOUBLET POTENTIALS IN DIFFERENT BASES

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Abbreviations and Notations

2HDM - Two Higgs Doublet Model
2HDMC - Two Higgs Doublet Model Calculator
BSM - Physics Beyond the Standard Model
CKM - Cabibbo–Kobayashi–Maskawa Matrix
EW - Electroweak
FCNC - Flavour Changing Neutral Currents
HB - Higgs Bounds
LH - Left-Handed
PMNS - Pontecorvo–Maki–Nakagawa–Sakata Matrix
QCD - Quantum Chromodynamics
QED - Quantum Electrodynamics
RH - Right-Handed
SM - Standard Model
SSB - Spontaneous Symmetry Breaking
VEV - Vacuum Expectation Value

$s_\xi \equiv \sin\xi$
$c_\xi \equiv \cos\xi$
$t_\xi \equiv \tan\xi$

Note:
Lagrangian $\equiv$ Lagrangian density
Natural units are used: $\hbar = c = 1$

Introduction

In the following thesis basic concepts of the two Higgs doublet model (2HDM) are discussed. During our research we performed both theoretical and numerical analyses of the CP-conserving 2HDM.

A lot of experimental data was accumulated, during the past few decades, confirming validity of the Standard Model (SM). In 2012 the last missing piece of the SM, the Higgs boson-like particle, was observed by ATLAS [1] and CMS [2] collaborations. Based on quantum properties of the new boson, it was later confirmed that in the frame of the SM, it is indeed the Higgs boson. The combined mass measurement, based on the data from LHC Run 1 is $m_H = 125.09 \pm 0.21(\text{stat.})\pm 0.11(\text{syst.})$ GeV [3].

There is still no experimental verification that there exists the only Higgs boson, therefore it is assumed that the Higgs sector of the electroweak (EW) theory might be extended. One of the simplest possible extensions of the SM Higgs sector is the 2HDM. In the 2HDM a second complex $SU(2)$ doublet is added, which leads to eight scalar fields in total. Out of these fields, five different physical Higgs bosons are generated: two neutral CP-even Higgs bosons h and H, a neutral CP-odd Higgs boson A and a pair of charged Higgs bosons $H^{\pm}$.

There is a lot of research going on. A list of restrictions on mass terms can be found in ref. [4]. Currently the most sought and promising channels are $H^{\pm}$ and $H^{\pm\pm}$. The second channel $H^{\pm\pm}$ is not a part of the 2HDM. We cover only the minimal extension of the Higgs sector. We assume that the found Higgs boson is the SM-like and not the heavy one.

The purpose of this thesis is to discuss phenomenological aspects of the CP-conserving 2HDM. The generic basis and the physical mass eigenstates basis were scrutinized. For the numerical analysis we modified the two Higgs doublet model calculator (2HDMC) [5]. Also by the means of Mathematica computation program we performed a check of possible models. In order to get a 2HDM potential, satisfying our needs, we performed both theoretical and numerical analysis. In our research we mainly rely on the following theoretical constraints of the 2HDM:

- stability of the potential
- S matrix tree-level unitarity
- quartic Higgs boson perturbativity

The thesis is organised as follows.

In Sec.1 we briefly introduce the SM. We cover the mathematical aspects of the SM, which are needed for the understanding of the Higgs sector, presented in the next section. In this chapter we emphasis that the extension of the SM Higgs sector might be the solution to some of the SM problems. The textbooks ref. [6–9] were used for this section.

In Sec.2 we derive basic principles of the SM Higgs sector. We also discuss the idea of the spontaneous symmetry breaking (SSB) and the Higgs mechanism. We show how the SM Higgs boson interacts with other bosons and fermions. Decay rates of the SM Higgs boson at the minimal order are presented. We also introduce to the theoretical constraints of the SM Higgs sector. For this section we used the textbooks ref. [9, 10].

In Sec.3 we describe the 2HDM. We cover the most commonly used conversions. These are: the generic basis, Higgs bases and the mass eigenstates basis or the physical one. Due to complexity of mathematics, we only managed to derive transformations between possible 2HDM when vacuum is neutral and the 2HDM potential is CP-conserving. We show these transformations. Next we analyse interactions with other fundamental particles: Higgs-self interactions, interactions with gauge bosons and interactions with fermions. For this section ref. [11] was an invaluable help.

In Sec.4 we derive several constraints of the 2HDM. Due to complexity of mathematics we cover only the CP-conserving case. In this section we discuss stability of the 2HDM scalar potential, S matrix tree-level unitarity, and quartic Higgs bosons perturbativity conditions. Also we cover the Peskin-Takeuchi parameters. We take into account all these constraints in order to analyse the CP-conserving 2HDM potential.

In Sec.5 we present and analyse our results of the CP-conserving 2HDM potential. We use both 2HDMC and our own code for this part. We present our approach and discuss what was done to improve the computing of our Monte-Carlo sampling.

1 The Standard Model

Up till these days the SM is in perfect agreement with experimental data. It proved to be a useful tool in describing particle interactions: electromagnetic, weak, strong as well as classifying all of the known elementary particles. The first step was done by the S. Glashow in 1961, when he succeeded in describing the combined model of the electromagnetic and weak interactions [12]. The second step was to understand how particles acquire mass.

In 1964 three independent groups proposed different approaches on how mass terms can arise in gauge invariant models as a result of spontaneous symmetry breaking (SSB) models. These groups are: F. Englert and R. Brout [13], P. Higgs [14] and G. Guralnik, C. R. Hagen, T. Kibble [15]. Later the Higgs mechanism was incorporated into EW theory by S. Weinberg [16] and A. Salam [17].

In the following chapter a brief introduction to the SM is presented. A more detailed overview can be found in the following textbooks ref. [6–9], which this chapter is based on.

1.1 A Brief Introduction to the Standard Model

The SM describes 17 elementary particles and for this description 19 input parameters are required. These parameters are: 9 fermion masses without neutrinos and the Higgs boson mass, 4 parameters to describe Cabibbo-Kobayashi-Maskawa matrix (CKM), 3 gauge couplings and the vacuum expectation value (VEV). Origin of the most of these parameters is unknown and the SM does not give the answer. Technically saying, the SM describes interactions between the fundamental forces, except for gravity, and elementary particles. This theory is based on quantum field theory.

All of the matter consists of 12 fundamental quantum fields with spin $s = 1/2$, which are elementary particles, called fermions, and 12 corresponding anti-particles. It turns out that these elementary particles can be divided into 2 equal groups, flavours, forming 3 different generations, which are shown in tab.1. The SM fermions are classified according to how they interact.

Table 1: Fermions of the SM.

	Gen. I	Gen. II	Gen. III
Leptons	e ν_e	μ ν_μ	τ ν_τ
Quarks	d u	s c	b t

Leptons can be split into 2 different groups: charged leptons or electron-like leptons (electron e, muon μ, tau τ) and neutrinos (electron neutrino ν_e, muon neutrino ν_μ, tau neutrino ν_τ). Charged leptons participate in EW interactions. Neutrinos do not posses a charge and therefore participate only in weak interaction. The first generation charged particles do not decay. Neutrinos can oscillate between different flavours.

Quarks can be split into 2 different groups: up (up u, charm c, top t) and down (down d, strange s, bottom b) quarks. Quarks in comparison to leptons have a colour charge and participate in strong interactions. Due to colour confinement quarks are strongly bound together. In nature quarks are bound together to form composite particles, called hadrons. Quarks carry electric charge and weak isospin, therefore they participate in EW interactions.

Another group of particles are bosons. Bosons are split into 2 groups based on their spin. If spin is not equal to zero, such bosons are called gauge bosons ($\gamma, W^{\pm}, Z, g$), if spin is equal to zero, then it is a scalar boson. Only one scalar elementary boson was found so far and it is the Higgs boson h. Gauge bosons are defined as force carriers.

Photons γ are responsible for the electromagnetic interactions between electrically charged particles. Photons are described by the theory of quantum electrodynamics (QED).

$W^{\pm}$ and Z bosons are responsible for the weak interactions between different flavour particles. While $W^{\pm}$ interact only with left-handed (LH) particles and right-handed (RH) antiparticles, Z interacts with LH particles and antiparticles.

In total there are 8 gluons g, which are responsible for the strong interaction. The strong interaction is only possible between particles with colour charge. The theory of strong interactions based on the gauge symmetry $SU(3)_C$ is quantum chromodynamics (QCD).

The Standard Model is described by an internal gauge symmetry $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$. In general, these mathematical objects can be thought of as transformations in space-time, under which they retain physical quantities. For example, volume of a sphere does not change if it is rotated by an arbitrary angle or shifted in one of the planes.

There are two types of symmetries: local and global. Under global symmetry object is transformed in the same way at every point in space-time. Conservation laws are required by global symmetries. In contrast, local symmetries depend smoothly on points of the base manifold. This means, that local transformations act on quantity differently but smoothly at every space-time point.

Of particular interest are gauge symmetries. Gauge symmetry is not a physical one. Gauge transformations give rise to gauge fields. Introduction of these fields is the requirement to keep the symmetry and it leads to particle states.

1.2 Quantum Electrodynamics

Electromagnetism is a well-known example of a gauge theory. Moreover, the first interaction which utilized a gauge theory was QED. In QED the invariance of the Lagrangian under the local transformations plays a fundamental role. Photon exchange mediates the electromagnetic interactions. Lagrangian, describing the electromagnetism, is invariant under the gauge group

$U(1)$. The $U(n)$ group is a subgroup of the general linear group $GL(n, \mathbb{C})$. The group $U(1)$ corresponds to the circle group, consisting of all complex numbers with absolute value of one under multiplication.

The Lagrangian for Maxwell's equations in the absence of any sources is:

$$\mathcal{L}_M = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \tag{1.1}$$

where $F_{\mu\nu}$ is the field strength tensor:

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \tag{1.2}$$

and A_μ is a four-vector.

Equation of motion for complex fields ψ:

$$(i\gamma^\mu \partial_\mu - m)\,\psi = 0, \quad \overline{\psi}\,(i\gamma^\mu \partial_\mu + m) = 0 \tag{1.3}$$

which are relativistic Dirac equations for fermions and $\overline{\psi}$ is Dirac adjoint:

$$\overline{\psi} = \psi^\dagger \gamma^0 \tag{1.4}$$

$\psi^\dagger$ is the Hermitian adjoint of the spinor ψ. Introduction of the Dirac adjoint is needed to conserve the Lorentz symmetry. One get these equations (1.3) by plugging the Lagrangian for a free Dirac field ψ into the Euler-Lagrange equation. Lagrangian for the Dirac field ψ with mass m is:

$$\mathcal{L}_D = \overline{\psi}\,(i\gamma^\mu \partial_\mu - m)\,\psi \tag{1.5}$$

or in a matrix notation:

$$\mathcal{L}_D = \overline{\psi}_\alpha\,(i\gamma^\mu \partial_\mu - m)_{\alpha\beta}\,\psi_\beta \tag{1.6}$$

where ψ is a 4-component column vector. The Dirac Lagrangian(1.5) is invariant under the global $U(1)$ gauge transformation:

$$\psi \to e^{iQ\xi}\psi \tag{1.7}$$

where Q is the charge operator, such that:

$$Q \begin{pmatrix} \psi \\ \overline{\psi} \end{pmatrix} = \begin{pmatrix} +\psi \\ -\overline{\psi} \end{pmatrix} \tag{1.8}$$

ξ is an arbitrary real parameter independent of the space-time. Let us consider infinitesimal rotations:

$$e^{-i\xi} = 1 - i\xi + \mathcal{O}(\xi^2) \tag{1.9}$$

Therefore fields ψ change under such transformation:

$$\begin{aligned} \psi &\to \psi + i\xi\psi, \\ \overline{\psi} &\to \overline{\psi} - i\xi\overline{\psi} \end{aligned} \tag{1.10}$$

What happens if the parameter ξ is space-time dependent? As we are interested in gauge transformations it is obvious to ask such question. Under infinitesimal transformation the field ψ transforms as:

$$\begin{aligned} \psi(x^\mu) &\to \psi(x^\mu) + i\xi(x^\mu)\psi(x^\mu), \\ \overline{\psi}(x^\mu) &\to \overline{\psi}(x^\mu) - i\xi(x^\mu)\overline{\psi}(x^\mu) \end{aligned} \tag{1.11}$$

Dirac Lagrangian (1.5) is not invariant under these transformations due to terms:

$$\left\{\overline{\psi}(x^\mu) - i\xi(x^\mu)\overline{\psi}(x^\mu)\right\} i\gamma^\mu\partial_\mu \left\{\psi(x^\mu) + i\xi(x^\mu)\psi(x^\mu)\right\} \tag{1.12}$$

and infinitesimal Lagrangian transformation is:

$$\delta\mathcal{L}_D = -\overline{\psi}(x^\mu)\gamma^\mu\partial_\mu \left(Q\xi(x^\mu)\right)\psi(x^\mu) \tag{1.13}$$

As for now it might seem that the symmetry is broken however this might be fixed by introducing a gauge fixing field A_μ. Under a gauge transformation:

$$-eQA_\mu \to -eQA_\mu + Q\partial_\mu\xi(x^\mu) \tag{1.14}$$

so that:

$$\delta\left(-e\overline{\psi}\gamma_\mu A_\mu\psi\right) = \delta\mathcal{L}_D \tag{1.15}$$

Hence, the Dirac Lagrangian (1.5) can be written down as:

$$\mathcal{L}_D = \overline{\psi}\left(i\gamma^\mu\left(\partial_\mu + ieQA_\mu\right) - m\right)\psi \tag{1.16}$$

where e is the electric charge and A_μ is the photon field. By introducing the covariant derivative:

$$D_\mu = \partial_\mu + ieQA_\mu \tag{1.17}$$

one can simplify the Dirac equation to the form:

$$\mathcal{L}_D = \overline{\psi}\left(i\gamma^\mu D_\mu - m\right)\psi \tag{1.18}$$

The Maxwell Lagrangian (1.1) and the Dirac Lagrangian (1.18) can be combined in order to describe how light and matter interact. The QED Lagrangian is:

$$\mathcal{L}_{QED} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \overline{\psi}\left(i\gamma^{\mu}D_{\mu} - m\right)\psi \tag{1.19}$$

From the QED Lagrangian (1.19) one can note that photon couples directly only to the electrically charged particles. This means, that in the frame of the SM, photons interact directly only with charged leptons, quarks and $W^{\pm}$ bosons.

1.3 The Weak Interaction

The weak interaction is one of the fundamental interactions of Nature. This force is responsible for the stochastic radioactive decay. This process can happen when 2 particles, either elementary or composite, exchange the weak bosons. The weak interaction is responsible for flavour changes of fermions.

Following the success of QED efforts were made to derive a theory for the weak interactions. In 1964 Yang and Mills tried to further extended the QED sector to include local non-Abelian transformations. Three massless vector fields were introduced. This theory was criticized as it required massless particles. Later this problem was solved by implementing the Higgs mechanism.

Considering the group theory it is interesting to study this theory as it violates parity symmetry P and also charge parity symmetry CP. In the mid-1950's Chen Ning Yang and Tsung-Dao Lee suggested that the weak interaction might violate spatial symmetry and in 1957 Chien Shiung Wu and collaborators discovered such violation.

The weak interaction is mediated by the weak bosons, or also called intermediate vector bosons, $W^{\pm}$ and Z^{0}. As it can be seen, there are 2 differently charged W^{+} with $q = +e$ and W^{-} with $q = -e$ bosons. These bosons are each other's antiparticles. Another boson Z^{0} is electrically neutral and is also its own antiparticle. All of the weak bosons are spin-1 particles. Up to date masses are: $m_{W^{\pm}} \approx 80.37$GeV [18] and $m_{Z} \approx 91.19$ GeV [19]. Such high masses limit the the range of the weak force.

Considering spatial symmetry violations and experimental proof, only LH particles are able to interact with charged currents. To describe how particles interact under the weak force a quantum number, weak isospin T_3, was introduced. Under the weak force quarks never decay into the state with the same weak isospin quantum number.

All RH fermions and LH antiparticles have a zero weak isospin value, while LH particles and RH antiparticles have a half-integer weak isospin value. Values of the LH fermions can be found in tab.2. In order to get weak isospin for RH antiparticles one needs to multiple the values in tab.2 by minus one.

Table 2: The weak isospin of the left-handed fermions in the SM.

Generation 1		Generation 2		Generation 3	
e	$-1/2$	μ	$-1/2$	τ	$-1/2$
ν_e	$+1/2$	ν_μ	$+1/2$	ν_τ	$+1/2$
d	$-1/2$	s	$-1/2$	b	$-1/2$
u	$+1/2$	c	$+1/2$	t	$+1/2$

Taking into consideration the weak interaction, particles from tab.1 can be written as LH $SU(2)$ isospin doublets:

$$\begin{aligned} L_L^I &= \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \qquad L_L^{II} = \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L, \qquad L_L^{III} = \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L \\ Q_L^I &= \begin{pmatrix} u \\ d \end{pmatrix}_L, \qquad Q_L^{II} = \begin{pmatrix} c \\ s \end{pmatrix}_L, \qquad Q_L^{III} = \begin{pmatrix} t \\ b \end{pmatrix}_L \end{aligned} \tag{1.20}$$

and RH singlets:

$$\begin{aligned} &e_R, \quad \mu_R, \quad \tau_R, \ (\nu_{eR}, \quad \nu_{\mu R}, \quad \nu_{\tau R}) \\ &d_R, \quad s_R, \quad b_R, \quad u_R, \quad c_R, \quad t_R \end{aligned} \tag{1.21}$$

RH neutrinos are included just to show that there is a space for them in the SM, although there is no experimental proof of their existence. One should note that introduced fields above in general are not mass eigenstates. In order to get mass eigenstates one has to diagonalize mass states. Conjugated fields of (1.20) are:

$$\overline{\Psi_L^i} = \left(\overline{\psi_{\alpha_i}} \ \overline{\psi_{\beta_i}} \right)_L \tag{1.22}$$

where $\psi_{\alpha_i} = \{\nu_i, u_i\}$ and $\psi_{\beta_i} = \{l_i, d_i\}$. We use the following notation for different generations of fermions:

$$\begin{aligned} \nu_i &= \{\nu_e, \ \nu_\mu, \ \nu_\tau\}, \\ l_i &= \{e, \ \mu, \ \tau\}, \\ u_i &= \{u, \ c, \ t\}, \\ d_i &= \{d, \ s, \ b\} \end{aligned} \tag{1.23}$$

1.4 The Electroweak Theory

Taking a look back at the Universe formation time all of the fundamental forces once were unified. The EW force split into the electromagnetic and the weak force during the so-called quark epoch. The EW theory describes the unified interaction of the electromagnetic and the weak forces. There exists the so-called vacuum expectation value (VEV), which is

$v \approx 246.22$ GeV. Unification energy of the EW force is of order of VEV.

From the mathematical point of view it turned out that it is not that simple to unify both the electromagnetic and the weak forces. The hugest obstacle was that while photons are massless, the weak bosons are not. This had to be somehow implemented.

The EW theory is described by the $SU(2)_L \otimes U(1)_Y$ gauge group. The $U(1)$ group was presented earlier, while the other one $SU(n)$ is a special unitary group of degree n. The special unitary group is a subgroup of the unitary group $U(n)$:

$$SU(n) \subset U(n) \subset GL(n, \mathbb{C}) \tag{1.24}$$

$SU(2)$group has the following properties:

$$SU(2) = \left\{ \begin{pmatrix} \alpha & -\bar{\beta} \\ \beta & \bar{\alpha} \end{pmatrix} \; \alpha, \beta \in \mathbb{C};, \; |\alpha|^2 + |\beta|^2 = 1 \right\} \tag{1.25}$$

The Lie algebra is generated by the following matrices:

$$u_1 = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}, \; u_2 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \; u_3 = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix} \tag{1.26}$$

These generators are related to the Pauli matrices by $u_1 = i\sigma_1$, $u_2 = -i\sigma_2$ and $u_3 = i\sigma_3$.

A quantum number describing the EW interactions is the weak hypercharge Y_W. This quantum numer relates the electric charge and the weak isospin. The weak hypercharge is the generator of the $U(1)$ component of the EW gauge group. There exists a specific combination which is conserved:

$$Q = T_3 + \frac{Y_W}{2} \tag{1.27}$$

Considering this relation we get the following possible values of the weak hypercharge, which are given in tab.3.

Table 3: The weak hypercharge in the SM.

Generation 1		Generation 2		Generation 3	
e_L	-1	μ_L	-1	τ_L	-1
e_R	-2	μ_R	-2	τ_R	-2
ν_{e_L}	-1	ν_μ	-1	ν_τ	-1
d_L	$+1/3$	s_L	$+1/3$	b_L	$+1/3$
d_R	$-2/3$	s_R	$-2/3$	b_R	$-2/3$
u_L	$+1/3$	c_L	$+1/3$	t_L	$+1/3$
u_R	$+4/3$	c_R	$+4/3$	t_R	$+4/3$

The SM EW Lagrangian is:

$$\mathcal{L}_{SU(2)\otimes U(1)_Y} = \mathcal{L}_{gauge} + \mathcal{L}_{fermions} + (\mathcal{L}_{Yukawa} + \mathcal{L}_{Higgs}) \tag{1.28}$$

where the part in the brackets will be covered later. As for now we are interested only in the gauge interactions $\mathcal{L}_{gauge}$ and fermionic interactions $\mathcal{L}_{fermions}$.

The gauge term is:

$$\mathcal{L}_{gauge} = -\frac{1}{4}W_i^{\mu\nu}W^i_{\mu\nu} - \frac{1}{4}B^{\mu\nu}B_{\mu\nu} \tag{1.29}$$

where

$$\begin{aligned} W^i_{\mu\nu} &= \partial_\mu W^i_\nu - \partial_\nu W^i_\mu - g\varepsilon_{ijk}W^j_\mu W^k_\nu, \\ B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu \end{aligned} \tag{1.30}$$

This Lagrangian describes interaction between the three W fields and B field. Three W fields correspond to particles $W^\pm$and Z which form the weak triplet and the single hypercharge B field, corresponding to γ.

The fermions kinetic term is:

$$\mathcal{L}_{fermions} = \mathcal{L}_{leptons} + \mathcal{L}_{quarks} \tag{1.31}$$

where

$$\mathcal{L}_{leptons} = i\overline{L^0_i}\gamma^\mu D^L_\mu L^0_i + i\overline{l^0_i}\gamma^\mu D^R_\mu l^0_i + \left(i\overline{\nu^0_i}\gamma^\mu D^R_\mu \nu^0_i\right) \tag{1.32}$$

and

$$\mathcal{L}_{quarks} = i\overline{Q^0_i}\gamma^\mu D^L_\mu Q^0_i + i\overline{u^0_i}\gamma^\mu D^R_\mu u^0_i + i\overline{d^0_i}\gamma^\mu D^R_\mu d^0_i \tag{1.33}$$

where subscript 0 indicates that fermions are in the weak basis and that those are not the mass eigenstates. The covariant derivatives are:

$$\begin{aligned} D^L_\mu L^0_i &= \left(\partial_\mu - \frac{ig}{2}\sigma_i W^i_\mu - \frac{ig'}{2}B_\mu\right)L^0_i, \\ D^L_\mu Q^0_i &= \left(\partial_\mu - \frac{ig}{2}\sigma_i W^i_\mu + \frac{ig'}{6}B_\mu\right)Q^0_i, \\ D^R_\mu l^0_i &= (\partial_\mu - ig'B_\mu)\, l^0_i, \\ D^R_\mu \nu^0_i &= \partial_\mu \nu^0_i, \\ D^R_\mu u^0_i &= \left(\partial_\mu + \frac{ig'2}{3}B_\mu\right)u^0_i, \\ D^R_\mu d^0_i &= \left(\partial_\mu - \frac{ig'}{3}B_\mu\right)d^0_i \end{aligned} \tag{1.34}$$

where g, g' are $SU(2)$ and $U(1)$ couplings respectively.

Possible interactions of the first generation of leptons and the EW bosons at the tree-level are presented in fig.1.

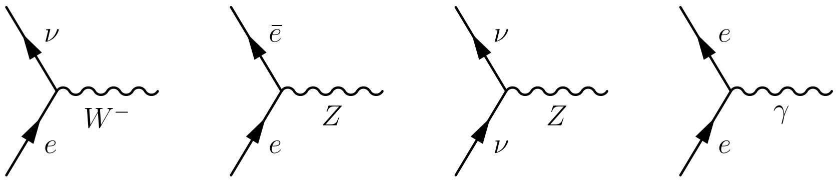

Figure 1: Interactions between leptons and vector bosons.

1.5 A Chiral Theory

The Dirac γ matrices are widely used in the chiral theory. In terms of the Pauli spin matrices, the Dirac matrices are:

$$\begin{aligned} \gamma^0 &= \begin{pmatrix} I_{2\times 2} & 0 \\ 0 & -I_{2\times 2} \end{pmatrix}, \qquad \gamma^\mu = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix} \\ \gamma^5 &= i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} 0 & I_{2\times 2} \\ I_{2\times 2} & 0 \end{pmatrix} \end{aligned} \tag{1.35}$$

where $I_{2\times 2}$ are identity matrices and σ_i are Pauli spin matrices:

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{1.36}$$

Some of the Dirac γ matrices properties are:

$$\begin{aligned} &(\gamma^0)^\dagger = \gamma^0, \qquad (\gamma^5)^\dagger = \gamma^5, \\ &(\gamma^5)^2 = 1, \qquad \{\gamma^5, \gamma^\mu\} = 0, \\ &(\gamma^\mu)^\dagger = \gamma^0\gamma^\mu\gamma^0 = -\gamma \text{ when } \mu \neq 0 \end{aligned} \tag{1.37}$$

The Dirac matrix γ^5 is also called the chirality matrix. The γ^5 matrix is diagonalizable by a unitary matrix U:

$$U\gamma^5 U^\dagger = \hat{\gamma}^5 \tag{1.38}$$

Taking into account that, it is easy to find that the eigenvalues of γ^5 are ± 1. Let us make an assumption that LH ψ_L and RH ψ_R fields, which are eigenfunctions of γ^5, so that:

$$\gamma^5 \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} = \begin{pmatrix} -\psi_L \\ +\psi_R \end{pmatrix} \tag{1.39}$$

A spinor can be separated into the LH and RH chiral components:

$$\psi = \psi_L + \psi_R \tag{1.40}$$

Now assume that there exist such chirality projection operators that:

$$\psi = (P_L + P_R)\,\psi = P_L\psi + P_R\psi = \psi_L + \psi_R \tag{1.41}$$

It follows that:

$$\psi_L = P_L\psi, \quad \text{and} \quad \psi_R = P_R\psi \tag{1.42}$$

It is obvious that the next step would be to find how these projection operators P_L and P_R look like. First of all, from (1.41) we can see that $P_L + P_R = 1$. Secondly, using (1.42) and (1.39):

$$\begin{aligned}\psi_L &= \frac{1}{2}(\psi_L + \psi_L + \psi_R - \psi_R) = \frac{1}{2}\psi - \frac{1}{2}\left(\gamma^5\psi_L + \gamma^5\psi_R\right) = \frac{1-\gamma^5}{2}\psi, \\ \psi_R &= \frac{1}{2}(\psi_L - \psi_L + \psi_R + \psi_R) = \frac{1}{2}\psi + \frac{1}{2}\left(\gamma^5\psi_L + \gamma^5\psi_R\right) = \frac{1+\gamma^5}{2}\psi\end{aligned} \tag{1.43}$$

we find that $P_{L,R}$ projection operators are:

$$P_R = \frac{1+\gamma^5}{2} \qquad \text{and} \qquad P_L = \frac{1-\gamma^5}{2} \tag{1.44}$$

The projection operators satisfy the following properties:

$$\begin{aligned}&P^\dagger_{L,R} = P_{L,R}, \\ &P_L + P_R = \frac{1-\gamma^5+1+\gamma^5}{2} = 1, \\ &P^2_{L,R} = \frac{1 \pm 2\gamma^5 + (\gamma^5)^2}{4} = \frac{1\pm\gamma^5}{2} = P_{L,R}, \\ &[P_L, R_R] = \frac{1}{4}\left(1 - \left(\gamma^5\right)^2 - 1 + \left(\gamma^5\right)^2\right) = 0, \\ &\{P_L, R_R\} = \frac{1}{4}\left(1 - \left(\gamma^5\right)^2 + 1 - \left(\gamma^5\right)^2\right) = 0\end{aligned} \tag{1.45}$$

Let us take a look what happens with the Dirac Lagrangian (1.5) by writing down the different chiral states of the field ψ. We start with this decomposition of ψ:

$$\begin{aligned}\mathcal{L}_D &= \left(\overline{\psi_L} + \overline{\psi_R}\right)(i\gamma^\mu\partial_\mu - m)(\psi_L + \psi_R) \\ &= \overline{\psi_L}(i\gamma^\mu\partial_\mu - m)\psi_L + \overline{\psi_L}(i\gamma^\mu\partial_\mu - m)\psi_R \\ &\quad + \overline{\psi_R}(i\gamma^\mu\partial_\mu - m)\psi_L + \overline{\psi_R}(i\gamma^\mu\partial_\mu - m)\psi_R\end{aligned} \tag{1.46}$$

where LH and RH Dirac adjoint states can be written down as:

$$\begin{aligned}\overline{\psi_L} = \psi_L^\dagger \gamma^0 = \psi^\dagger P_L \gamma^0 = \psi^\dagger \gamma^0 P_R = \overline{\psi} P_R,\\ \overline{\psi_R} = \psi_R^\dagger \gamma^0 = \psi^\dagger P_R \gamma^0 = \psi^\dagger \gamma^0 P_L = \overline{\psi} P_L\end{aligned} \tag{1.47}$$

Therefore transformations are as following:

$$\overline{\psi_L} = \overline{P_L \psi} = \overline{\psi} P_R, \quad \overline{\psi_R} = \overline{P_R \psi} = \overline{\psi} P_L \tag{1.48}$$

Taking into consideration properties of γ matrices (1.37) we get:

$$\overline{\psi}_{\binom{L}{R}} \gamma^\mu \psi_{\binom{R}{L}} = 0 \tag{1.49}$$

The Dirac Lagrangian (1.5) in terms of the chiral fields ψ_L and ψ_R is therefore:

$$\mathcal{L}_D = \overline{\psi_L} \left(i\gamma^\mu \partial_\mu - m\right) \psi_L + \overline{\psi_R} \left(i\gamma^\mu \partial_\mu - m\right) \psi_R \tag{1.50}$$

Hence the LH chiral fields couple only to the LH chiral fields, and the RH chiral fields couple only to the RH chiral fields. Although it does not follow that there are LH and RH distinct particles. Moreover chirality state is a space-time dependent state as fields can fluctuate between chiral states. On the other hand, the conserved quantity is helicity.

The chiral fields ψ_L and ψ_R are also known as Weyl spinors. The Weyl spinor is constructed from 2 independent components. One of the possible options for Weyl spinors is:

$$P_L = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad \text{and} \quad P_R = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \tag{1.51}$$

Thus the 4-component spinor ψ can be written as:

$$\psi = \begin{pmatrix} \chi_R \\ \chi_L \end{pmatrix}, \qquad \psi_L = \begin{pmatrix} 0 \\ \chi_L \end{pmatrix}, \quad \psi_R = \begin{pmatrix} \chi_R \\ 0 \end{pmatrix} \tag{1.52}$$

where $\chi_{L,R}$ are the Weyl 2-component fields.

The Dirac Lagrangian (1.5) in terms of the Weyl fields is:

$$\mathcal{L}_D = \chi_L^\dagger i\overline{\sigma}^\mu \partial_\mu \chi_L + \chi_R^\dagger i\sigma^\mu \partial_\mu \chi_R - m\left(\chi_L^\dagger \chi_R + \chi_R^\dagger \chi_L\right) \tag{1.53}$$

where

$$\sigma_\mu = (I, \overline{\sigma}), \quad \overline{\sigma}^\mu = (I, -\overline{\sigma}) \tag{1.54}$$

The free field Dirac equations of motion are:

$$i\overline{\sigma}^{\mu}\partial_{\mu}\chi_L - m\chi_R = 0, \quad i\sigma^{\mu}\partial_{\mu}\chi_R - m\chi_L = 0 \tag{1.55}$$

1.6 Motivation for Extending the Standard Model

Although it might sound that the SM successfully describes Nature, this is far from being the truth. Indeed the SM is a great mathematical tool, which is capable of answering many of the questions. Acknowledging the fact that the SM is the theory, which describes an approximate observable world it is worth taking a note that there is physics beyond the SM (BSM). There are a lot of physical phenomena in Nature, which the SM can not explain. Some of the problems are:

- theory of gravity
- dark matter and dark energy
- asymmetry of matter-antimatter
- neutrino masses
- anomalous magnetic moment of muon
- strong CP problem

One of the hugest drawbacks of the theory is that it does not explain gravity. A simple addition of the graviton to the SM Lagrangian does not fully correspond to the observable world. Up till now the most popular theories are string theory and loop quantum gravity.

Another drawback is that the SM explains roughly only 5% of the observed energy. The other part of the energy corresponds to dark matter and dark energy, 26% and 69% correspondingly. One of the dark matter candidates can be the 2HDM [20].

The 2HDM might also be the answer to several more SM problems. For example, introduction of the second Higgs doublet leads to several options how fermions can couple to the Higgs doublet. This might be a hint to hierarchy problem, quantum triviality etc.

As it can be seen it is obvious that the SM needs to be extended in order to take into account some of the described problems. We are interested in the Higgs sector of the SM and therefore accept the idea that some of the mentioned problems can be solved by extending the Higgs sector.

2 The Standard Model Higgs Sector

Basic principles of the SM Higgs are presented in this chapter. In order to describe the Higgs mechanism a complex scalar doublet is needed. We start with mathematical formulation of the scalar field theory. Also basic ideas of the SSS and the Higgs mechanism are presented. SSB is one of the fundamental SM ideas, without which there would be several issues. Finally we take a look at how the Higgs boson interact with other particles. This chapter is based on ref. [9, 10].

Gauge boson must be massless in order for the gauge theory to stay unbroken. This is the requirement for the QED and QCD. In order to implement massive gauge bosons $W^{\pm}, Z$ the theory should be extended to introduce symmetry breaking. A simple solution would be to introduce mass terms for the gauge bosons but on the other hand this would violate renormalizability of the theory. Obviously renormalizable theories are more attractive as they solve some infinite problems.

2.1 Scalar Field Theory

Scalar field is such that at every space-time point a scalar value, a physical quantity, is associated. This physical quantity should be Lorentz invariant. Therefore, in terms of the quantum field theory, this should be a spin-zero [1] quantum field. The Higgs field is the only fundamental scalar quantum field which has been observed so far.

Assume that there exists such a field function $\varphi(x^\mu)$, which satisfies $\varphi(x^\mu) = \varphi(x^\mu)^\dagger$. This is a real or Hermitian field. The Hermitian scalar field Lagrangian is:

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu\varphi\right)\left(\partial^\mu\varphi\right) - \frac{1}{2}m^2\varphi^2 - V(\varphi) \tag{2.1}$$

This Lagrangian is invariant under the global $U(1)$ symmetry. The field equation should satisfy the Klein-Gordon equation in a potential:

$$\left(\partial_\mu\partial^\mu + m^2\right)\varphi + \frac{\partial V}{\partial\varphi} = 0 \tag{2.2}$$

Let us now take a look at a complex scalar field $\varphi \neq \varphi^\dagger$. The Lagrangian of a complex scalar field is:

$$\mathcal{L} = (\partial_\mu\varphi)^\dagger(\partial^\mu\varphi) - m^2\varphi^\dagger\varphi - V(\varphi, \varphi^\dagger) \tag{2.3}$$

where the interaction term is:

$$V(\varphi, \varphi^\dagger) = \frac{\lambda}{4}\left(\varphi^\dagger\varphi\right)^2 + \mathrm{non-renormalizable} \tag{2.4}$$

[1] A review of whether the Higgs boson is a spin-zero particle can be found in ref. [21].

In general, it is possible to express a complex scalar field φ in terms of two real fields:

$$\varphi = \frac{1}{\sqrt{2}}\left(\varphi_1 + i\varphi_2\right) \tag{2.5}$$

Hermitian conjugate of the field φ is:

$$\varphi^{\dagger} = \frac{1}{\sqrt{2}}\left(\varphi_1 - i\varphi_2\right) \tag{2.6}$$

One could note that this complex scalar field φ can be treated as two independent scalar fields φ_1 and φ_2. These fields can be viewed as components of a two-dimensional vector:

$$\overline{\varphi} = \varphi_1 \overline{i} + \varphi_2 \overline{j} \tag{2.7}$$

Under the global $U(1)$ transformation:

$$\begin{pmatrix} \varphi_1' \\ \varphi_2' \end{pmatrix} = \begin{pmatrix} c_\theta & s_\theta \\ -s_\theta & c_\theta \end{pmatrix} \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix} \tag{2.8}$$

which describes rotation by an angle θ in the $1-2$ plane. The Lagrangian is invariant under such transformations.

The most general renormalizable Lagrangian is therefore:

$$\mathcal{L} = \frac{1}{2}\left[(\partial_\mu \varphi_1)^2 + (\partial_\mu \varphi_2)^2\right] - \frac{1}{2}m^2\left(\varphi_1^2 + \varphi_2^2\right) - \frac{\lambda}{16}\left(\varphi_1^2 + \varphi_2^2\right)^2 \tag{2.9}$$

2.2 Spontaneous Symmetry Breaking

A symmetry is spontaneously broken if the Lagrangian is not continuously invariant under a symmetry. The lowest energy state, vacuum, does not possess the same symmetry as its Lagrangian. The SSB enables the existence of several vacua states.

Mass terms are not allowed in the Lagrangian for the gauge bosons and fermions as such Lagrangian is not gauge invariant. On the other hand the weak bosons are not massless, therefore there should be some sort of mechanism through which bosons and fermions can acquire mass terms [13–15]. During this spontaneous breaking the renormalizability should be preserved [22].

Let us start with a simple example. Assume that the Higgs field is a real scalar field. Then the Lagrangian is:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - V(\phi) \tag{2.10}$$

where the potential is given by:

$$V(\phi) = \frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda\phi^4 \tag{2.11}$$

This Lagrangian is invariant under $\phi \to -\phi$. In order to bound the potential from below value of the coupling coefficient λ should be positive [2]. There are two different possibilities for the sign of μ^2.

In case of $\mu^2 > 0$ everything is trivial. When the vacuum of the potential satisfies $v = \phi_0 = \text{Const}$, the minimum condition can be written as:

$$v\left(\mu^2 + \lambda v^2\right) = 0 \tag{2.12}$$

Vacuum of such states corresponds to $\phi_0 = 0$. In this case $SU(2) \otimes U(1)$ symmetry is unbroken at the minimum. Such Lagrangian describes a free particle with a mass parameter μ.

In case of $\mu^2 < 0$ the previously mentioned point $v = 0$ is no longer stable. Now we get two vacuum conditions:

$$\phi_0^{1,2} = \pm\sqrt{-\frac{\mu^2}{\lambda}} \tag{2.13}$$

Lagrangian is invariant under $\phi \to -\phi$, therefore there should be no difference between the choice of the vacuum state ϕ_0^1 or ϕ_0^2. This non-zero vacuum value breaks the $SU(2) \otimes U(1)$ symmetry. Turns out that we are interested in this case.

Let us consider fluctuations around the minimum:

$$\phi = v + \phi' \tag{2.14}$$

By requiring this, we introduce an excitation of the field, which is the physical particle. The Lagrangian in terms of these fluctuations is:

$$\begin{aligned} \mathcal{L} &= \frac{1}{2}\left(\partial_\mu(v+\phi')\right)\left(\partial^\mu(v+\phi')\right) - \frac{\mu^2}{2}\left(v+\phi'\right) - \frac{\lambda}{4}\left(v+\phi'\right) \\ &= \frac{1}{2}\left(\partial_\mu\phi'\right)\left(\partial^\mu\phi'\right) - \lambda v^2\phi'^2 - \lambda v\phi'^3 + \frac{1}{4}\lambda v^4 - \frac{1}{4}\lambda\phi'^4 \end{aligned} \tag{2.15}$$

This Lagrangian describes a particle ϕ' with mass:

$$m_{\phi'} = \sqrt{2\lambda v^2} \tag{2.16}$$

[2]In other words this is requirement for the vacuum stability.

In reality the SM Higgs sector is described by a weak isospin doublet:

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix} \tag{2.17}$$

The SM Higgs scalar potential is:

$$V = \frac{1}{2}\mu^2 \left(\sum_{i=1}^{4} \phi_i^2 \right) + \frac{1}{4}\lambda \left(\sum_{i=1}^{4} \phi_i^2 \right)^2 \tag{2.18}$$

The same procedure can be taken for the complex scalar potential as for the real scalar potential. We are now dealing with the four-dimensional space. It is possible to rotate the VEV so that only one component of ϕ_i is aligned with the direction of the VEV [3]. Let us assume that there exists a non-zero VEV:

$$v = \langle 0|\phi_3|0\rangle = \text{Const} \tag{2.19}$$

such that:

$$\phi_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \tag{2.20}$$

With this choice we get the following quantum numbers for the vacuum:

By introducing fluctuations around the VEV we get 4 different fields: one massive and three massless. The massive scalar particle corresponds to the SM Higgs boson. Three massless particles are the so-called Nambu-Goldstone bosons [23, 24] or simply Goldstone bosons.

Goldstone's theorem states that if a generic continuous symmetry is spontaneously broken, then for each broken generator a new massless scalar particle appears. During the spontaneous symmetry breaking of the $SU(2) \otimes U(1)$ group, 3 generators are broken. These 3 Goldstone bosons correspond to the longitudinal polarization components of the weak bosons $W^{\pm}$ and Z. These Goldstone bosons are eaten to give mass terms to the weak bosons.

2.3 The Higgs Mechanism

The main task of the Higgs mechanism is to explain why some particles acquire mass terms and others do not. Without this mechanism no bosons would acquire masses. This mechanism assumes that there exists the Higgs field at every point of the space.

The SM Lagrangian for the Higgs field is:

$$\mathcal{L}_H = (D^\mu \Phi)^\dagger (D_\mu \Phi) - V(\Phi) \tag{2.21}$$

[3] Natural choice would be to choose either ϕ_1 or ϕ_3.

where covariant derivative D_μ is:

$$D_\mu = \partial_\mu + ig\frac{\sigma_i}{2}W_\mu^i + \frac{ig'}{2}B_\mu \tag{2.22}$$

We do not cover the QCD theory here and thus no mathematical approach is provided. In reality one would need to add another term to the (2.22):

$$+i\frac{g_s}{2}G_a^\mu L_a$$

where L_a are Gell-Mann matrices and g_s is the strong $SU(3)_C$ coupling.

The SM Higgs doublet can be parametrised:

$$\Phi = \exp\left(\frac{i}{v}G^i\sigma_i\right)\begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}(v+h) \end{pmatrix} \tag{2.23}$$

where G are the Goldstone bosons. The Goldstone fields are unphysical and thus can be removed by an appropriate $SU(2)$ transformation. In a U-gauge the SM Higgs doublet is:

$$\Phi_U = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}(v+h) \end{pmatrix} \tag{2.24}$$

During the spontaneous breaking, the $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ gauge group is broken down to the gauge group $SU(3)_C \otimes U(1)_Y$. This EW breaking mechanism is the Higgs mechanism, due to which particles obtain masses.

Let us take a look how the gauge group $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ is broken. First of we start with $SU(2)_L$ generators:

$$\begin{aligned}
\sigma_1\Phi &= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}(v+h) \end{pmatrix} &= \begin{pmatrix} \frac{1}{\sqrt{2}}(v+h) \\ 0 \end{pmatrix}, \\
\sigma_2\Phi &= \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}(v+h) \end{pmatrix} &= -i\begin{pmatrix} \frac{1}{\sqrt{2}}(v+h) \\ 0 \end{pmatrix}, \\
\sigma_3\Phi &= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}(v+h) \end{pmatrix} &= -\begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}(v+h) \end{pmatrix}
\end{aligned} \tag{2.25}$$

and $U(1)_Y$ generator:

$$Y\Phi = +1 \times \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}(v+h) \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}(v+h) \end{pmatrix} \tag{2.26}$$

As it can be seen $SU(2)_L \otimes U(1)_Y$ group is broken[4]. Taking this into consideration it turns out that gauge bosons W_i and B acquire a mass term through the Higgs mechanism. The VEV was chosen so that it carries nor electric charge nor colour charge. Therefore a straight conclusion can be made that the $U(1)_{EM}$ and $SU(3)_C$ generators are not broken. Therefore under the Higgs mechanism generators are broken to the form:

$$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \rightarrow SU(3)_C \otimes U(1)_{EM} \tag{2.27}$$

2.4 Interactions with Bosons

First of all we start from the generation of masses. Let us take a look at photons. The $U(1)_{EM}$ generator, corresponding to the electric charge leaves the vacuum invariant:

$$Q\phi_0 = \frac{1}{2}\left(\sigma_3 + Y\right)\phi_0 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} \tag{2.28}$$

Therefore the photon remains massless. As stated earlier the same procedure can be applied and for gluons. It turns out that gluons are also massless.

The procedure of getting masses of the gauge bosons is straightforward. They are identified by substituting the VEV into the kinetic part of the Higgs Lagrangian:

$$\begin{aligned} \left(D^\mu\Phi\right)^\dagger\left(D_\mu\Phi\right) &= |\left(\partial_\mu + ig\frac{\sigma_i}{2}W_\mu^i + \frac{ig'}{2}B_\mu\right)\begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}|^2 \\ &= \frac{v^2}{8}|\left(g\sigma^i W_\mu^i + g'B_\mu\right)\begin{pmatrix} 0 \\ 1 \end{pmatrix}|^2 \\ &= \frac{v^2}{8}|\begin{pmatrix} gW_\mu^1 - igW_\mu^2 \\ -gW_\mu^3 + g'B_\mu \end{pmatrix}|^2 \\ &= \frac{v^2}{8}\left[g^2\left(\left(W_\mu^1\right)^2 + \left(W_\mu^2\right)^2\right) + \left(gW_\mu^3 - g'B_\mu\right)^2\right] \end{aligned} \tag{2.29}$$

To be more precisely the Higgs Lagrangian is:

$$\begin{aligned} \mathcal{L}_{Higgs} = &-\frac{1}{2}\mu^2 h^2 + \frac{v^2}{8}\left[2g^2\left(W_\mu^+ W_\mu^-\right) + \left(g'^2 + g^2\right)Z_\mu^2\right] \\ &+ \mathcal{L}_{other\ interactions} + \mathcal{L}_{fermions} \end{aligned} \tag{2.30}$$

[4]This can be seen as from the fact that all generators are not invariant; product is not equal to zero. Alternation of the quantum numbers T, T_3 and Y conserves some of the generators.

where

$$\begin{aligned}
W_\mu^\pm &= \frac{W_\mu^1 \mp iW_\mu^2}{\sqrt{2}}, \\
Z_\mu &= \frac{gW_\mu^3 - g'B_\mu}{\sqrt{g^2+g'^2}}, \\
A_\mu &= \frac{g'W_\mu^3 + gB_\mu}{\sqrt{g^2+g'^2}}
\end{aligned} \tag{2.31}$$

The mass terms can be obtained from the eq.(2.30). The Higgs boson acquires mass through the self-interaction and from:

$$\left(\frac{vg}{2}\right)^2 W_\mu^+ W_\mu^- \tag{2.32}$$

it easy to identify the mass term of the $W^\pm$:

$$m_W = \frac{gv}{2} \tag{2.33}$$

The remaining gauge term of eq.(2.30) is:

$$\frac{v^2}{8}\begin{pmatrix} W_\mu^3 & b_\mu \end{pmatrix}\begin{pmatrix} g^2 & -gg' \\ -gg' & g'^2 \end{pmatrix}\begin{pmatrix} W^{3\mu} \\ B^\mu \end{pmatrix} \tag{2.34}$$

and one of the eigenvalues is zero. In terms of the physical fields Z_μ and A_μ, after the mass matrix diagonalization, we get:

$$\frac{1}{2}M_Z^2 Z_\mu^2 + \frac{1}{2}M_A^2 A_\mu^2 \tag{2.35}$$

Therefore the remaining mass terms are:

$$\begin{aligned}
m_Z &= \frac{v}{2}\sqrt{g^2+g'^2}, \\
m_A &= 0
\end{aligned} \tag{2.36}$$

On the other hand one can introduce the mixing angle, the Weinberg angle θ_W, which is defined as follows:

$$\begin{aligned}
c_w &= \frac{g}{\sqrt{g^2+g'^2}}, \\
s_w &= \frac{g'}{\sqrt{g^2+g'^2}}, \\
t_w &= \frac{g'}{g}
\end{aligned} \tag{2.37}$$

In terms of the Weinberg angle θ_W, relation between $W^\pm$ and Z bosons masses is:

$$\frac{m_W}{m_Z} = c_w \tag{2.38}$$

and also between W, B, Z, A states:

$$\begin{pmatrix} Z \\ A_\mu \end{pmatrix} = \begin{pmatrix} c_w & -s_w \\ s_w & c_w \end{pmatrix} \begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix} \tag{2.39}$$

at the tree-level.

Let us take a look at what the other terms in eq. (2.30) are. We start from the Higgs boson self interactions:

$$\begin{aligned} \mathcal{L}_{hhh} &= -\lambda v h^3, \\ \mathcal{L}_{hhhh} &= -\frac{\lambda}{4} h^4 \end{aligned} \tag{2.40}$$

Interactions with $W^\pm$ and Z vector bosons are:

$$\begin{aligned} \mathcal{L}_{WWh} &= g m_W W_\mu^- W^{\mu +} h, \\ \mathcal{L}_{WWhh} &= \frac{1}{4} g^2 W_\mu^- W^{\mu +} hh, \\ \mathcal{L}_{ZZhh} &= \frac{1}{8} \left(g^2 + g'^2 \right) Z_\mu Z^\mu hh \end{aligned} \tag{2.41}$$

2.5 Interactions with Fermions

So far we have seen how gauge bosons acquire their mass in the SM . The next step is to take a look at the fermion sector. Everything would be trivial if term $m\bar{\psi}\psi$ was gauge invariant. We will take a look how the Higgs mechanism generates fermions mass terms. We start with the decomposition of the Dirac mass term:

$$\begin{aligned} -\mathcal{L}_{Dirac} =& m_D \overline{\psi} \psi \\ =& m_D \left(\overline{\psi}_L + \overline{\psi}_R \right) \left(\psi_L + \psi_R \right) \\ =& m_D \left(\overline{\psi} P_R + \overline{\psi} P_L \right) \left(P_L \psi + P_R \psi \right) \\ =& m_D \overline{\psi} \left(P_L^2 + 2 P_L P_R + P_R^2 \right) \psi \\ =& m_D \left(\overline{\psi} P_L \psi + \overline{\psi} P_R \psi \right) \\ =& m_D \left(\overline{\psi_L} \psi_R + \overline{\psi_R} \psi_L \right) \end{aligned} \tag{2.42}$$

It turns out that such a decomposition is not gauge invariant as gauge transformations of left-handed and right-handed fields are different. Therefore there should be a way to construct an invariant $SU(3) \otimes SU(2)_L \otimes U(1)_Y$ term for fermions. This can we achieved with help of the Higgs mechanism. Fermions acquire masses through the Higgs mechanism then and only then when the Higgs doublet has a non-zero vacuum value.

There exists such Lagrangian, which describes interactions between the Higgs field and the fermion field. It is called the Yukawa Lagrangian:

$$
\begin{aligned}
-\mathcal{L}_Y &= Y_{ij}\overline{\psi_i}\phi\psi_j \\
&= Y_{ij}\overline{\psi_{L_i}}\phi\psi_{R_j} + Y_{ij}^\dagger\overline{\psi_{R_i}}\phi^\dagger\psi_{R_J} \\
&= Y_{ij}\overline{\psi_{L_i}}\phi\psi_{R_j} + h.c.
\end{aligned} \tag{2.43}
$$

where Y_{ij} are the so-called Yukawa couplings. In total there are 4 particular ones for every fermion type:

$$
Y^u, \quad Y^d, \quad Y^l, \quad Y^\nu \tag{2.44}
$$

where u stands for up-type quarks, d for down-type quarks, l for leptons, ν for neutrinos.

Let us demonstrate how the down-type quarks interact with the Higgs doublet [5] in one generation model:

$$
\begin{aligned}
-\mathcal{L}_Y^{\bar{d}\phi d} &= Y^d\overline{Q_L}\phi d_R + Y^{d\dagger}\overline{Q_L}\phi^\dagger d_R \\
&= Y^d\Bigg[(\overline{u} \;\; \overline{d})_L \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v+h+iG^0) \end{pmatrix} d_R \\
&\qquad + \overline{d}_R\left(G^- \;\; \frac{1}{\sqrt{2}}(v+h-iG^0)\right)\begin{pmatrix} u \\ d \end{pmatrix}_L\Bigg]
\end{aligned} \tag{2.45}
$$

As for now we are interested only in interactions between the Higgs field and the fermion field and thus we can drop down terms proportional to G^+ and G^0 [6]:

$$
\begin{aligned}
-\mathcal{L}_Y^{\bar{d}\phi d} =& \frac{Y^d(v+h)}{\sqrt{2}}\left[\overline{d}_L d_R + \overline{d}_R d_L\right] \\
=& \frac{Y^d v}{\sqrt{2}}\overline{d}d + \frac{Y^d}{\sqrt{2}}h\overline{d}d \\
=& m_d\overline{d}d + \frac{m_d}{v}h\overline{d}d
\end{aligned} \tag{2.46}
$$

where the first term is mass of a fermion and the second is the Higgs boson coupling to fermions in the SM:

$$
g_{h\bar{f}f} = -i\frac{m_f}{v} \tag{2.47}
$$

It follows that Yukawa couplings can be expressed in the following way:

$$
Y^f = \sqrt{2}\frac{m_f}{v} \tag{2.48}
$$

From eq.(2.46) one could only get mass terms for down-type quarks and charged leptons. In order to generate masses for up-type quarks and neutrinos another term in the Lagrangian

[5] For simplicity we assume that Yukawa couplings are real.

[6] This is equivalent to the U-gauge.

should be introduced. By intuition term should be proportional to the inverse of the Higgs doublet. The charge-conjugated Higgs doublet:

$$\tilde{\phi} \equiv -i \left[\phi_i^\dagger \sigma_2\right]^T = i\sigma_2 \phi^* = \begin{pmatrix} \frac{1}{\sqrt{2}} \left(v + h - iG^0\right) \\ G^- \end{pmatrix} \tag{2.49}$$

It is now straightforward we construct the Yukawa Lagrangian for up-type quarks:

$$-\mathcal{L}_Y^{\overline{u}\phi u} = Y^u \overline{Q_L} \tilde{\phi} u_R + Y^{u\,\dagger} \overline{Q_R} \tilde{\phi}^\dagger u_L \tag{2.50}$$

Combining both parts of the Lagrangian for down-type quarks eq.(2.46) and up-type quarks eq.(2.50) we get the full Lagrangian for quarks sector:

$$\begin{aligned} -\mathcal{L}_Y^{quarks} =& Y^d \overline{Q_L} \phi d_R + Y^{d\,\dagger} \overline{Q_R} \phi^\dagger d_L \\ &+ Y^u \overline{Q_L} \tilde{\phi} u_R + Y^{u\,\dagger} \overline{Q_R} \tilde{\phi}^\dagger u_L \end{aligned} \tag{2.51}$$

Note that in the SM the Higgs boson decay into two different generations of fermions at the tree-level is prohibited and thus Yukawa couplings are diagonal. Masses of fermions are not predicted since Yukawa couplings are free parameters. It is still unknown why fermions have such masses. The Higgs mechanism only gives an answer how masses of fermions are generated.

In reality it turns out that the model is not that simple. There are at least 3 generations of fermions. In general, Yukawa couplings are 3×3 complex matrices. Taking this into consideration there arises mixing between different generations. This implies that Yukawa matrices are indeed non diagonal and in principle can be complex. The Yukawa Lagrangian is:

$$-\mathcal{L}_Y = Y_{ij} \overline{\psi^0_{L_i}} \phi \psi^0_{R_j} + Y^\dagger_{ij} \overline{\psi^0_{R_i}} \phi^\dagger \psi^0_{R_J} \tag{2.52}$$

where superscript 0 indicates the weak eigenstates. This means that fields transform according to $SU(2)$ representation. Let us take a look at the U-gauge :

$$-\mathcal{L}_Y = \overline{u^0_{L_i}} \left(\mathcal{M}^u_{ij} + \frac{\mathcal{M}^u_{ij}}{v} h\right) u^0_{R_j} + h.c. + \{d, e, \nu\} \text{ terms} \tag{2.53}$$

where fermion mass matrix $\mathcal{M}^\xi_{ij} = \frac{Y^\xi_{ij} v}{\sqrt{2}}$ is not diagonal provided that Yukawa matrix Y^ξ_{ij} got off-diagonal elements. Moreover it is nor Hermitian nor symmetric. In order to get definite particles masses one needs to diagonalize the mass matrix $\mathcal{M}^\xi_{ij}$. This can be done by introducing unitary transformations V_L and V_R:

$$V^{\xi\dagger}_{\{L,R\}} V^\xi_{\{L,R\}} = \mathbb{I}_{3\times 3} \tag{2.54}$$

Therefore the mass matrix diagonalization is:

$$V_L^{\xi\dagger}\mathcal{M}_{ij}^{\xi}V_R^{\xi}=\hat{\mathcal{M}}^{\xi}=\begin{pmatrix} m_1 & 0 & 0\\ 0 & m_2 & 0\\ 0 & 0 & m_3\end{pmatrix} \tag{2.55}$$

Fermion weak eigenstates can also be expressed as mass eigenstates:

$$\psi_{\{L,R\}_i}=V^{\psi\dagger}_{\{L,R\}_{ij}}\psi^0_{\{L,R\}_j} \tag{2.56}$$

Due to the mismatch of unitarity transformations between the weak and mass eigenstates of different fermion families there exists a mixing [7]. These mixing matrices arise between quarks or leptons. When talking about quarks sector such matrix is called the Cabbibo-Kobayashi-Maskawa (CKM) matrix [25]:

$$V_{CKM}=V_L^uV_L^{d\dagger}=\begin{pmatrix} V_{ud} & V_{us} & V_{ub}\\ V_{cd} & V_{cs} & V_{cb}\\ V_{td} & V_{ts} & V_{tb}\end{pmatrix} \tag{2.57}$$

It can be parametrized in terms of Euler angles and a single phase [26]:

$$V_{CKM}=\begin{pmatrix} c_{\theta_{12}}c_{\theta_{13}} & s_{\theta_{12}}c_{\theta_{13}} & s_{\theta_{13}}e^{-i\delta}\\ -s_{\theta_{12}}c_{\theta_{23}}-c_{\theta_{12}}s_{\theta_{23}}s_{\theta_{13}}e^{i\delta} & c_{\theta_{12}}c_{\theta_{23}}-s_{\theta_{12}}s_{\theta_{23}}s_{\theta_{13}}e^{i\delta} & s_{\theta_{23}}c_{\theta_{13}}\\ s_{\theta_{12}}s_{\theta_{23}}-c_{\theta_{12}}c_{\theta_{23}}s_{\theta_{13}}e^{i\delta} & -c_{\theta_{12}}s_{\theta_{23}}-s_{\theta_{12}}c_{\theta_{23}}s_{\theta_{13}}e^{i\delta} & c_{\theta_{23}}c_{\theta_{13}}\end{pmatrix} \tag{2.58}$$

Unitary matrix describing mixing between leptons is called the Pontecorvo–Maki–Nakagawa–Sakata (PMNS) matrix [27, 28]:

$$V_{PMNS}=V_L^{\nu}V_L^{l\dagger}=\begin{pmatrix} V_{e1} & V_{e2} & V_{e3}\\ V_{\mu1} & V_{\mu2} & V_{\mu3}\\ V_{\tau1} & V_{\tau2} & V_{\tau3}\end{pmatrix} \tag{2.59}$$

Due to the lack of information on neutrino sector the PMNS matrix is still a mystery. It might turn out that the PMNS matrix in not unitary. This depends on the neutrino mass model. It is also possible to parametrize the PMNS matrix as the CKM. There are several approaches and are entirely based on the choice of the neutrino sector. In case of Dirac neutrinos, the PMNS parametrization is mathematically identical to the eq. (2.58). In terms of Majorana neutrinos

[7] In other words, Yukawa matrices cannot be diagonalized simultaneously.

it changes to the form ref. [29]:

$$V_{PMNS} = \begin{pmatrix} c_{\theta_{12}}c_{\theta_{13}} & s_{\theta_{12}}c_{\theta_{13}} & s_{\theta_{13}}e^{-i\delta} \\ -s_{\theta_{12}}c_{\theta_{23}} - c_{\theta_{12}}s_{\theta_{23}}s_{\theta_{13}}e^{i\delta} & c_{\theta_{12}}c_{\theta_{23}} - s_{\theta_{12}}s_{\theta_{23}}s_{\theta_{13}}e^{i\delta} & s_{\theta_{23}}c_{\theta_{13}} \\ s_{\theta_{12}}s_{\theta_{23}} - c_{\theta_{12}}c_{\theta_{23}}s_{\theta_{13}}e^{i\delta} & -c_{\theta_{12}}s_{\theta_{23}} - s_{\theta_{12}}c_{\theta_{23}}s_{\theta_{13}}e^{i\delta} & c_{\theta_{23}}c_{\theta_{13}} \end{pmatrix} \times \begin{pmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{2.60}$$

where α_i are Majorana phases.

2.6 Decay Rates

The Higgs boson cannot be detected directly in the particle detectors. One of the possible solutions is to detect other particles and compare decay widths to get idea about processes happening right after the collision. There are many possible decay processes as the Higgs boson interacts with most of the SM particles.

When talking about decay rates there are several parameters used [30]. First of all there is the decay rate Γ. It shows the probability per unite time that particle will decay. The total decay rate is the sum of all possible decay channels. Lifetime of the particle is proportional to the total decay:

$$\tau = \frac{1}{\Gamma_{total}} \tag{2.61}$$

Another critical parameter when talking about particle decay is the branching ration. Branching ration shows the ratio between different decay modes and the total decay of the particle:

$$Br_i = \frac{\Gamma_i}{\Gamma_{total}} \tag{2.62}$$

Result of the Higgs boson decay into two particles is presented in fig.2.

Now let us take a closer look at the two body Higgs boson decay rates. Mathematical derivation of some of the lowest order decay is presented in ref. [32]. We start with the Higgs decay at the tree-level.

One of the possible Higgs boson decays is into a pair of fermions. The Feynman diagram for the Higgs boson decay into a pair of fermions at the tree-level is presented in fig.3.

The decay width of such process is:

$$\Gamma_{h\to f\bar{f}} = \frac{N_C m_f^2 m_h}{8\pi v^2}\left(1 - \frac{4m_f^2}{m_h^2}\right)^{3/2} \tag{2.63}$$

where $N_c = 1$ for leptons and $N_C = 3$ for quarks. One should always be careful about in

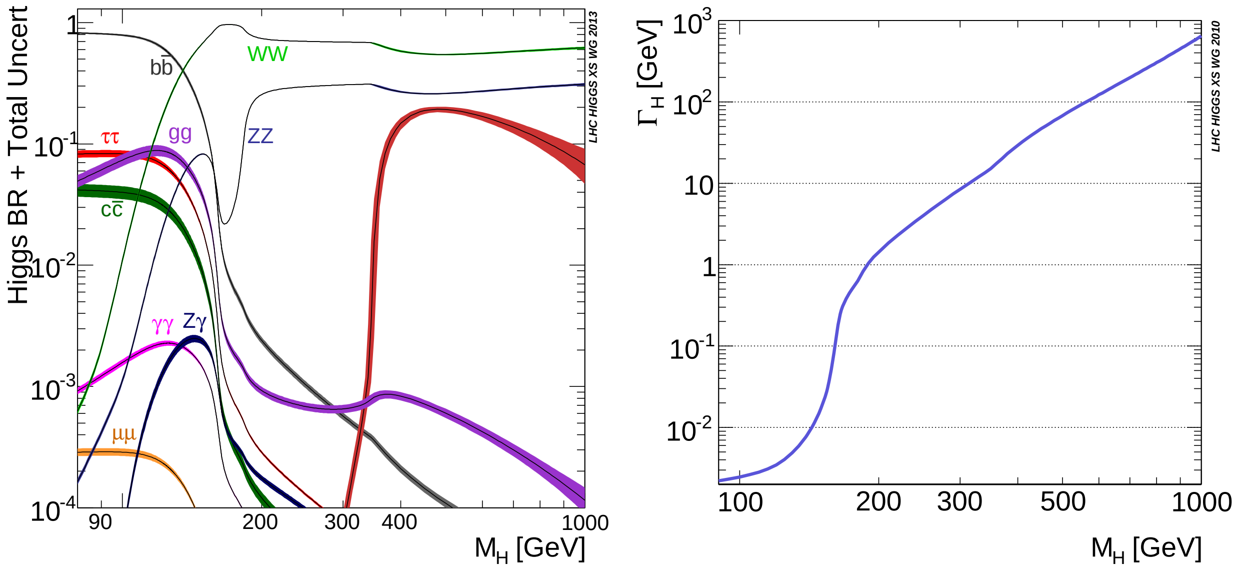

Figure 2: The SM Higgs boson decay branching ratios and the total width [31].

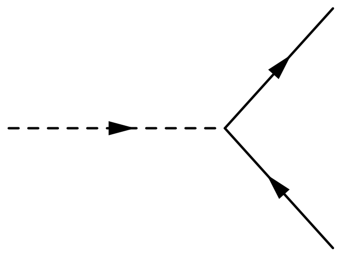

Figure 3: The Feynman diagram of the Standard Model Higgs decay into fermion and antifermion pair.

what renormalization scheme they are. The higher order radiative corrections are also known ref. [33, 34].

Another possible Higgs decay is into a pair of EW bosons. The Feynman diagram for the decay to a pair of weak gauge bosons is shown in fig.4.

Then the decay width is:

$$\Gamma_{h\to VV} = \frac{m_V^4}{4\pi m_h v^2}\sqrt{1-\frac{4m_V^2}{m_h^2}}\left(3+\frac{m_h^4}{4m_V^4}-\frac{m_h^2}{m_V^2}\right) \tag{2.64}$$

In case of the decay $H \to Z\ Z$ there are two indistinguishable outgoing fields. Therefore the decay width should be multiplied by a factor of $1/2$. The Higgs boson is less heavier than two EW bosons. Thus the Higgs boson cannot directly decay into two EW bosons. This problem is solved by introducing the off-shell conditions. Recent analysis can be found in ref. [35].

The Higgs boson can also decay into other massless particles through loop interactions. Gluons and photons do not couple directly to the Higgs boson and thus there are some additional mathematical complications.

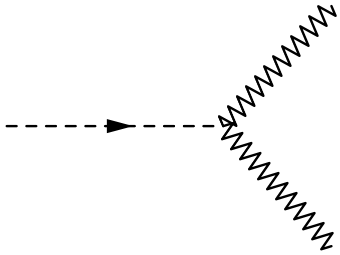

Figure 4: The Feynman diagram of the SM Higgs decay into EW bosons.

First of all we start from the Higgs boson decay into a pair of gluons. The Feynman diagrams can be found in fig. 5.

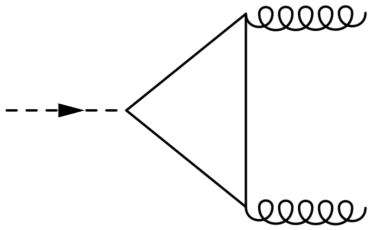

Figure 5: The Feynman diagram of the SM Higgs boson decay into a gluon-gluon pair.

The decay width is:

$$\Gamma_{h \to gg} = \frac{\alpha \alpha_s^2 m^4}{8\pi^2 m_h \sin^2 \theta_W m_W^2} \left|\Delta\right|^2 \tag{2.65}$$

Higher order corrections up to $\mathcal{O}(\alpha_s^3 G_F m_t^2)$ can be found in ref. [36].

The decay of the Higgs boson into a pair of photons is a little bit more complicated. In total there are 26 possible Feynman diagrams. Some of them are presented in fig. 6,7. Contributing amplitudes can be found in ref. [37–39].

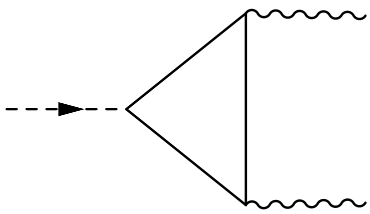

Figure 6: The Feynman diagram of the Standard Model Higgs boson decay into photon-photon pair via a fermion loop.

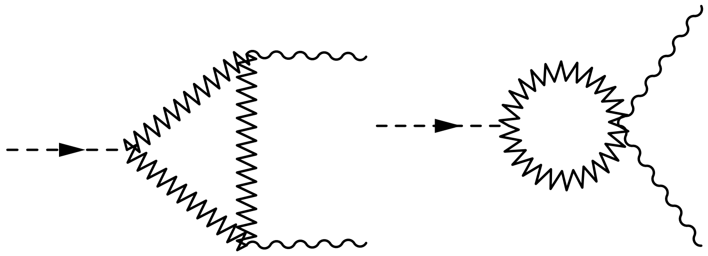

Figure 7: The Feynman diagram of the Standard Model possible Higgs boson decay into a photon-photon pair via W loops.

The total decay rate is:

$$\Gamma\left(h \to \gamma\gamma\right) = |\mathcal{F}|^2 \left(\frac{\alpha}{4\pi}\right)^2 \frac{G_F m_h^3}{8\sqrt{2}\pi} \tag{2.66}$$

where

$$\mathcal{F} = \mathcal{F}_W(\beta_W) + \sum_{\text{fermions}} N_C Q^2 \mathcal{F}_f(\beta_f) \tag{2.67}$$

and corresponding contributions from loops are:

$$\begin{aligned}
&\mathcal{F}_W(\beta) = 2 + 3\beta + 3\beta\left(2 - \beta\right) f(\beta), \\
&\mathcal{F}_f(\beta) = -2\beta\left[1 + \left(1 - \beta\right) f(\beta)\right], \\
&f(\beta) = \begin{cases} \arcsin^2\left(\beta^{-1/2}\right), & \beta \geq 1 \\ -\frac{1}{4}\left[ln\left(\frac{1+\sqrt{1-\beta}}{1-\sqrt{1-\beta}}\right)\right], & \beta < 1 \end{cases}
\end{aligned} \tag{2.68}$$

$\beta_{W,f}$ are W boson and fermion mass relations to the Higgs boson mass:

$$\beta_W = \frac{4m_W^2}{m_h^2}, \qquad \beta_f = \frac{4m_f^2}{m_h^2} \tag{2.69}$$

2.7 Theoretical Constraints

Although the SM Higgs boson mass is a free parameter in the theory, there are several theoretical constraints for the upper and lower bounds on the mass of the Higgs boson. These constraints can be applied if and only if there is no new physics between the EW scale and the higher scale Λ.

When contribution from the lowest order Feynman diagrams are small perturbative methods can be used. The idea of unitarity comes from the elastic scattering [40]:

$$W_L^+ W_L^- \to W_L^+ W_L^- \tag{2.70}$$

Amplitude for this process at the tree-level gets contribution from 7 diagrams:

$$\mathcal{M} = -\frac{g^2}{2m_W^2}\left(m_H^2 + \frac{t}{s} + \frac{s}{t} + m_Z^2\right) + \mathcal{O}(E^{-2}) \tag{2.71}$$

If there was no Higgs field, then such process would diverge quadratically. Therefore due to the Higgs boson it is possible to cancel high energy divergence. Introduction of the scalar field is a must to fix divergence and for theory to remain unitary and renormalisable.

Let us take a closer look at the unitarity bound. We start from the Jacob-Wick expansion for the Lorentz invariant amplitude [41]:

$$\mathcal{M} = 16\pi \sum_j (2j+1)\,\mathcal{M}^j(s) P_j(c_\theta) \tag{2.72}$$

where $\mathcal{M}^j$ is a partial wave amplitude with the total angular momentum j. P_j is the Legendre polynomial. The partial wave amplitude in terms of Mandelstam variables [42]:

$$\mathcal{M}^j(s) = \frac{\sqrt{2}}{4i|\overline{p}|}\left(S^j - 1\right) \tag{2.73}$$

where S^j is a unitarity matrix element and $\overline{p}$ is the momentum of colliding particles in the centre of mass. In the high energy limit:

$$|\overline{p}| \to \frac{1}{2}\sqrt{s} \tag{2.74}$$

The partial wave amplitude is therefore:

$$|\mathbb{I}m(\mathcal{M}^j)| \geq |\mathcal{M}^j|^2 = \left(\mathbb{I}m(\mathcal{M}^j)\right)^2 + \left(\mathbb{R}e(\mathcal{M}^j)\right)^2 \tag{2.75}$$

This leads to the following relation:

$$|\mathcal{M}^j| \leq 1 \tag{2.76}$$

There also exists the stronger constraint [43]:

$$|\mathbb{R}e(\mathcal{M}^j)| \leq \frac{1}{2} \tag{2.77}$$

The partial wave amplitude $\mathcal{M}^j$ is related to the scattering matrix S. Let us now take a look at the upper mass bound of the Higgs boson in the SM. The partial wave amplitude (2.71) can be written as:

$$\mathcal{M} = -g^2\frac{m_h^2}{2m_W^2} \tag{2.78}$$

From the condition (2.76) we get:

$$m_h^2 \leq \frac{4\pi\sqrt{2}}{g_F} \tag{2.79}$$

The leading high energy terms:

$$\mathcal{L} = -\frac{g^2}{32}\frac{m_h^2}{m_W^2}\left(2W^-W^+ + Z^2 + h^2\right)^2 \tag{2.80}$$

The scattering matrix of such physical process is:

$$\begin{pmatrix} 4 & \sqrt{2} & \sqrt{2} & 0 \\ \sqrt{2} & 3 & 1 & 0 \\ \sqrt{2} & 1 & 3 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix} \tag{2.81}$$

Eigenvalues of this matrix are: $\{6, 2, 2, 2\}$. There is no angular dependence in this process. Only the first partial wave amplitude is present. Taking that into consideration:

$$m_h < \sqrt{\frac{8\pi\sqrt{2}}{3G_F}} \approx 1008 \text{ GeV} \tag{2.82}$$

and the stronger restriction is:

$$m_h < \sqrt{\frac{4\pi\sqrt{2}}{3G_F}} \approx 713 \text{ GeV} \tag{2.83}$$

In case the mass of the Higgs boson would be over these values, this would mean that perturbation theory is not valid and in such model the theory is not renormalisable.

Another constraint comes from the Veltman parameter ρ, which shows the relative strength of the neutral and charged current weak interactions [44]:

$$\rho = \frac{m_W^2}{m_Z^2 c_{\theta_W}} \tag{2.84}$$

Experimentally $\rho \approx 1$ in the SM at the tree-level [4]. Taking a step forward, the general expression for the tree-level ρ with N multiplets is [45]:

$$\rho = \frac{\sum_{i=1}^{N}\left[T_i\left(T_i+1\right) - \frac{Y_i^2}{4}\right] v_i}{\frac{1}{2}\sum_{i=1}^{N} Y_i^2 v_i} \tag{2.85}$$

Also there exist the Peskin–Takeuchi parameters [46,47]. In total there are three measurable parameters [8]: S, T and U. These parameters show possible new physics contribution to EW radiative corrections; the vacuum polarization diagrams that contribute to four fermion scattering processes.

There are several restrictions. First of all, there should be no EW gauge bosons apart from $W^{\pm}$, Z, γ. Secondly, the energy scale at which the new physics appears is assumed to be larger than the EW scale. Finally, it is assumed that the new physics couplings to light fermions are suppressed.

The Peskin-Takeuchi parameters are defined in a such way that at a reference point, that is in the SM, they are zero. Parameters of the oblique corrections are the self energies of $W^{\pm}$, Z, γ. Self energy of the vacuum polarisation amplitude:

$$\Pi_{ij}(q^2) = \Pi_{ij}(0) + q^2\Pi'_{ij}(0) \tag{2.86}$$

where $\{i, j\} \in \{W^{\pm},\ Z,\ \gamma\}$. There are 4 possible options for self energies: Π_{WW}, Π_{ZZ}, $\Pi_{\gamma\gamma}$ and $\Pi_{Z\gamma}$.

The Peskin-Takeuchin parameters are defined in the following way:

$$\begin{aligned}
\alpha S &= 4s_w^2 c_w^2 \left[\Pi'_{ZZ}(0) - \frac{c_w^2 - s_w^2}{s_w c_w}\Pi'_{Z\gamma}(0) - \Pi'_{\gamma\gamma}(0)\right], \\
\alpha T &= \frac{\Pi_{WW}(0)}{m_W^2} - \frac{\Pi_{ZZ}(0)}{m_Z^2}, \\
\alpha U &= 4s_w^2 \left[\Pi'_{WW}(0) - c_w^2\Pi'_{ZZ}(0) - 2s_w c_w \Pi'_{Z\gamma}(0) - s_w^2\Pi'_{\gamma\gamma}(0)\right]
\end{aligned} \tag{2.87}$$

If the scale of new physics is close to the EW one then above equations are no longer valid.

Let us take a closer look at what is the physical meaning of these S, T and U parameters. The S parameter shows the possible extension of the fermion sector. It measures the symmetry between the number of different chirality fermions that carry weak isospin. The T parameter is dependant on the difference between the loop corrections of the W vacuum polarisation function and the Z vacuum polarisation function. Therefore it measures discrepancy of the total weak isospin of a new model. Both S and T parameters are really sensitive to the mass of the Higgs boson [9]. The last discussed parameter U gets a really small contribution from new physics. This is because U parameter is a dimension-eight operator.

[8] There exist more parameters but they are very rarely used.

[9] Provided there is no new physics close to the EW scale, the SM is a good description up to energies around 1 TeV

3 The Two Higgs Doublet Model

In this chapter we move on from the SM Higgs sector to the simplest gauge invariant 2HDM. The Higgs sector is still covered in mysteries and thus it is assumed to be non minimal.

In the 2HDM a second Higgs doublet with the same quantum numbers as the first one is added. There are no restrictions between possible choice of $SU(2)_L$ doublet scalar fields $\Phi_a(a = 1, 2)$. The only restriction is that this combination should be orthonormal. Due to multiple possibilities of basis choices there arises a problem of invariant parameters. All physically observable parameters must be basis independent.

In the following section, the main tool for the analysis is ref. [11]. It is a great guide to the extended Higgs sector.

3.1 Different Two Higgs Doublet Models

We start by introducing the second Higgs doublet. In the two-Higgs-doublet model there are two identical complex doublets of $SU(2)_L$ symmetry with the same quantum numbers:

$$\Phi_i = \begin{pmatrix} \phi_i^+ \\ \phi_i^0 \end{pmatrix} \tag{3.1}$$

for $i = 1, 2$ which is the Higgs flavour index. The corresponding conjugated fields also obey $SU(2)_L$:

$$\tilde{\Phi}_i = i\sigma_2 \Phi_i^* = \begin{pmatrix} \phi_i^{0*} \\ -\phi_i \end{pmatrix} \tag{3.2}$$

In the 2HDM there are two VEV. The second Higgs doublet can also acquire the VEV:

$$\langle \Phi_1 \rangle = \frac{v_1}{\sqrt{2}}, \qquad \langle \Phi_2 \rangle = \frac{v_2}{\sqrt{2}} e^{i\theta} \tag{3.3}$$

and the total VEV should be a combination of both:

$$v = \sqrt{v_1^2 + v_2^2} \tag{3.4}$$

where $v \approx 246.22$ GeV.

The two-Higgs-doublet model Lagrangian is

$$\mathcal{L} = \mathcal{L}_{scalar} + \mathcal{L}_{kinetic} + \mathcal{L}_{GF} + \mathcal{L}_Y \tag{3.5}$$

Unlike the SM Higgs sector there are different possible combinations when talking about the 2HDM. Let us construct the most general renormalizable 2HDM potential. For this talk we will be using the gauge invariant operators:

$$\begin{aligned}
&\Phi_{11} \equiv \Phi_1^\dagger \Phi_1, \qquad \Phi_{22} \equiv \Phi_2^\dagger \Phi_2, \\
&\Phi_{\mathbb{R}e} \equiv \frac{1}{2}\left(\Phi_1^\dagger \Phi_2 + \Phi_2^\dagger \Phi_1\right), \\
&\Phi_{\mathbb{I}m} \equiv -\frac{i}{2}\left(\Phi_1^\dagger \Phi_2 - \Phi_2^\dagger \Phi_1\right)
\end{aligned} \tag{3.6}$$

It was explained earlier that in order to have gauge invariant renormalizable scalar potential it needs to be of form:

$$\mathcal{L}_{2HDM} = \mathcal{L}_{2HDM}^{\phi^2} + \mathcal{L}_{2HDM}^{\phi^4} \tag{3.7}$$

By permuting operators (3.6) it is possible to get the following combinations for cubic potential:

$$-\mathcal{L}_{2HDM}^{\phi^2} = \mu_1 \Phi_{11} + \mu_2 \Phi_{22} + \mu_3 \Phi_{\mathbb{R}e} + \mu_4 \Phi_{\mathbb{I}m} \tag{3.8}$$

and quartic potential:

$$\begin{aligned}
\mathcal{L}_{2HDM}^{\phi^4} =& \xi_1 \Phi_{11}^2 + \xi_2 \Phi_{22}^2 + \xi_3 \Phi_{\mathbb{R}e}^2 + \xi_4 \Phi_{\mathbb{I}m}^2 \\
&+ \xi_5 \Phi_{11}\Phi_{22} + \xi_6 \Phi_{11}\Phi_{\mathbb{R}e} + \xi_7 \Phi_{11}\Phi_{\mathbb{I}m} \\
&+ \xi_8 \Phi_{22}\Phi_{\mathbb{R}e} + \xi_9 \Phi_{22}\Phi_{\mathbb{I}m} + \xi_{10} \Phi_{\mathbb{R}e}\Phi_{\mathbb{I}m}
\end{aligned} \tag{3.9}$$

where μ_i and ξ_i are coupling coefficients [10]. As it can be seen this Lagrangian is more complex than the SM Higgs Lagrangian. In total there are fourteen free parameters.

Assuming that the potential is invariant under a charge conjugation then doublets transform in the following way:

$$\Phi_i^\dagger \Phi_j \rightarrow e^{i(\alpha_j - \alpha_i)} \Phi_j^\dagger \Phi_i \tag{3.10}$$

choosing $\alpha_i = \alpha_j$ results in:

$$\begin{aligned}
-\mathcal{L}_{2HDM}^{\phi^2} =& \mu_1 \Phi_{11} + \mu_2 \Phi_{22} + \mu_3 \Phi_{\mathbb{R}e}, \\
\mathcal{L}_{2HDM}^{\phi^4} =& \xi_1 \Phi_{11}^2 + \xi_2 \Phi_{22}^2 + \xi_3 \Phi_{\mathbb{R}e}^2 + \xi_4 \Phi_{\mathbb{I}m}^2 \\
&+ \xi_5 \Phi_{11}\Phi_{22} + \xi_6 \Phi_{11}\Phi_{\mathbb{R}e} + \xi_8 \Phi_{22}\Phi_{\mathbb{R}e}
\end{aligned} \tag{3.11}$$

A CP-invariant Lagrangian can be achieved under the $\mathcal{Z}_2$ symmetry. This implies that:

$$\Phi_1 \rightarrow \Phi_1, \quad \Phi_2 \rightarrow -\Phi_2 \tag{3.12}$$

[10] Possible choice of these couplings will be discussed in the next chapter.

Therefore the 2HDM Lagrangian is:

$$\begin{aligned} -\mathcal{L}_{2HDM}^{\phi^2} =& \mu_1\Phi_{11} + \mu_2\Phi_{22}, \\ \mathcal{L}_{2HDM}^{\phi^4} =& \xi_1\Phi_{11}^2 + \xi_2\Phi_{22}^2 + \xi_3\Phi_{\mathbb{R}e}^2 + \xi_4\Phi_{\mathbb{I}m}^2 + \xi_5\Phi_{11}\Phi_{22} \end{aligned} \tag{3.13}$$

The Higgs bosons masses depend on the parameters μ_i, ξ_i and also two angles. Theses angles are α and β. α is the mixing angle between CP-even Higgs bosons and depends on the potential choice. This mixing angle is found from the diagonalization process:

$$\begin{aligned} \begin{pmatrix} m_H^2 & 0 \\ 0 & m_h^2 \end{pmatrix} &= \begin{pmatrix} c_\alpha & s_\alpha \\ -s_\alpha & c_\alpha \end{pmatrix} \begin{pmatrix} \mathcal{M}_{11}^2 & \mathcal{M}_{12}^2 \\ \mathcal{M}_{21}^2 & \mathcal{M}_{22}^2 \end{pmatrix} \begin{pmatrix} c_\alpha & -s_\alpha \\ s_\alpha & c_\alpha \end{pmatrix} \\ &= \begin{pmatrix} \mathcal{M}_{11}^2 c_\alpha^2 + 2\mathcal{M}_{12}^2 c_\alpha s_\alpha + \mathcal{M}_{22}^2 s_\alpha^2 & \mathcal{M}_{12}^2\left(c_\alpha^2 - s_\alpha^2\right) + \left(\mathcal{M}_{22}^2 - \mathcal{M}_{11}^2\right) s_\alpha c_\alpha \\ \mathcal{M}_{12}^2\left(c_\alpha^2 - s_\alpha^2\right) + \left(\mathcal{M}_{22}^2 - \mathcal{M}_{11}^2\right) s_\alpha c_\alpha & \mathcal{M}_{11}^2 s_\alpha^2 - 2\mathcal{M}_{12}^2 c_\alpha s_\alpha + \mathcal{M}_{22}^2 c_\alpha^2 \end{pmatrix} \end{aligned} \tag{3.14}$$

Squared masses of the CP-even Higgs bosons are:

$$m_{H,h}^2 = \frac{1}{2}\left[\mathcal{M}_{11}^2 + \mathcal{M}_{22}^2 \pm \sqrt{\left(\mathcal{M}_{11}^2 - \mathcal{M}_{22}^2\right)^2 + 4\left(\mathcal{M}_{12}^2\right)^2}\right] \tag{3.15}$$

The mixing angle α can acquire values:

$$|\alpha| \leq \frac{1}{2}\pi \tag{3.16}$$

It is defined by setting off-diagonal elements of the squared mass matrix to zero. If $m_H = m_h$ then there is no mixing between CP-even Higgs bosons and thus α cannot be defined.

β relates the two VEV values:

$$t_\beta = \frac{\langle\Phi_2\rangle}{\langle\Phi_1\rangle} \tag{3.17}$$

It follows that VEVs are related:

$$\frac{v_2}{v_1} = \frac{s_\beta v}{c_\beta v} \tag{3.18}$$

In turns out that β is not a physical parameter:

$$\begin{aligned} t'_\beta = \frac{\frac{1}{2}\langle 2\Phi'_2\rangle}{\frac{1}{2}\langle 2\Phi'_1\rangle} &= \frac{\langle\ U_{21}\Phi_1 + U_{22}\Phi_2\rangle}{\langle\ U_{11}\Phi_1 + U_{12}\Phi_2\rangle} = \frac{U_{21}\langle\Phi_1\rangle + U_{22}\langle\Phi_2\rangle}{U_{11}\langle\Phi_1\rangle + U_{12}\langle\Phi_2\rangle} \\ &= \frac{U_{21} + U_{22}t_\beta}{U_{11} + U_{12}t_\beta} \end{aligned} \tag{3.19}$$

In case when both VEV values are real, this is the CP-conserving case, the Higgs sector consists of the following particles, two CP-even scalars h and H, CP-odd pseudo-scalar A, two charged Higgs bosons $H^\pm$ and the Goldstone bosons $G^\pm$ and G. Gauge eigenstates are transformed into the mass eigenstates in the following way:

$$
\begin{aligned}
&\begin{pmatrix} c_\beta & s_\beta \\ -s_\beta & c_\beta \end{pmatrix} \begin{pmatrix} \phi_1^+ \\ \phi_2^+ \end{pmatrix} = \begin{pmatrix} G^+ \\ H^+ \end{pmatrix}, \quad \begin{pmatrix} G^- \\ H^- \end{pmatrix} = \begin{pmatrix} (G^+)^\dagger \\ (H^+)^\dagger \end{pmatrix}, \\
&\begin{pmatrix} c_\beta & s_\beta \\ -s_\beta & c_\beta \end{pmatrix} \begin{pmatrix} \sqrt{2}\mathbb{I}m\,(\phi_1^0) \\ \sqrt{2}\mathbb{I}m\,(\phi_2^0) \end{pmatrix} = \begin{pmatrix} G \\ A \end{pmatrix}, \\
&\begin{pmatrix} c_\alpha & s_\alpha \\ -s_\alpha & c_\alpha \end{pmatrix} \begin{pmatrix} \sqrt{2}\mathbb{R}e\,(\phi_1^0 - v_1) \\ \sqrt{2}\mathbb{R}e\,(\phi_2^0 - v_2) \end{pmatrix} = \begin{pmatrix} H \\ h \end{pmatrix}
\end{aligned}
\tag{3.20}
$$

3.2 Generic Basis

One of the main features of the two-Higgs-doublet model is the choice of the Higgs scalar potential. We start with the generic basis. Following ref. [48] in the generic basis the most general gauge-invariant Higgs scalar potential can be written down in the following form:

$$
\begin{aligned}
V = {} & m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - m_{12}^2 \Phi_1^\dagger \Phi_2 - m_{12}^{2\,*} \Phi_2^\dagger \Phi_1 \\
& + \frac{1}{2}\lambda_1 \left(\Phi_1^\dagger \Phi_1\right)^2 + \frac{1}{2}\lambda_2 \left(\Phi_2^\dagger \Phi_2\right)^2 \\
& + \lambda_3 \left(\Phi_1^\dagger \Phi_1\right)\left(\Phi_2^\dagger \Phi_2\right) + \lambda_4 \left(\Phi_1^\dagger \Phi_2\right)\left(\Phi_2^\dagger \Phi_1\right) \\
& + \left\{ \frac{1}{2}\lambda_5 \left(\Phi_1^\dagger \Phi_2\right)^2 + \left[\lambda_6 \Phi_1^\dagger \Phi_1 + \lambda_7 \Phi_2^\dagger \Phi_2\right] \Phi_1^\dagger \Phi_2 + h.c. \right\}
\end{aligned}
\tag{3.21}
$$

where m_{11}^2,m_{22}^2 and $\lambda_1,\ \lambda_2,\ \lambda_3,\ \lambda_4$ are real parameters and $m_{12}^2,\ \lambda_5,\ \lambda_6,\ \lambda_7$ are complex.

It is always possible to redefine the fields by an arbitrary transformation $\Phi_i \to B_{ij}\Phi_j$. Matrix B_{ij} consists of eight real parameters which corresponds to eight possible fields.

Applying a global $U(2)$ transformation to doublets yields:

$$
\Phi_i \to U_{i\bar{j}}\Phi_i, \qquad \Phi_{\bar{i}}^\dagger \to \Phi_{\bar{j}}^\dagger U_{j\bar{i}}^\dagger \tag{3.22}
$$

From the definition of the unitary group the following properties arise:

$$
U_{j\bar{i}}^\dagger U_{i\bar{k}} = \delta_{j\bar{k}}, \qquad \det(U) = e^{i\chi} \tag{3.23}
$$

In this index convention $U_{i\bar{j}} \equiv U_i{}^j$. Summation is only allowed over barred and unbarred index pairs. Complex conjugation is equivalent to replacing indexes with barred ones. $U(2)$ invariant quantities are basis independent. They do not depend on the possible choice of the $\Phi_1 - \Phi_2$ basis.

Applying $U(2)$ transformations we get:

$$\begin{aligned} V &= m_{11}^2\Phi_1^\dagger\Phi_1 + m_{22}^2\Phi_2^\dagger\Phi_2 - m_{12}^2\Phi_1^\dagger\Phi_2 - m_{12}^{2\,*}\Phi_2^\dagger\Phi_1 + f(\lambda) \\ &= m_{11}^2\left(\Phi_2^\dagger U_{21}^\dagger U_{12}\Phi_2\right) + m_{22}^2\left(\Phi_1^\dagger U_{12}^\dagger U_{21}\Phi_1\right) \\ &\quad - m_{12}^2\left(\Phi_2^\dagger U_{21}^\dagger U_{21}\Phi_1\right) - m_{12}^{2\,*}\left(\Phi_1^\dagger U_{12}^\dagger U_{12}\Phi_2\right) + f(\lambda) \\ &\equiv Y_{11}\left(\Phi_1^\dagger\Phi_1\right) + Y_{22}\left(\Phi_2^\dagger\Phi_2\right) + Y_{12}\left(\Phi_1^\dagger\Phi_2\right) + Y_{21}\left(\Phi_2^\dagger\Phi_1\right) + f'(\lambda) \end{aligned} \tag{3.24}$$

Hermicity of V implies that:

$$Y_{i\bar{j}} = \left(Y_{j\bar{i}}\right)^*, \qquad Z_{i\bar{j}k\bar{l}} = \left(Z_{j\bar{i}l\bar{k}}\right)^* \tag{3.25}$$

After applying the corresponding transformations the scalar Higgs potential can be written in a $U(2)$ covariant form:

$$V = Y_{i\bar{j}}\Phi_{\bar{i}}^\dagger\Phi_j + \frac{1}{2}Z_{i\bar{j}k\bar{l}}\left(\Phi_{\bar{i}}^\dagger\Phi_j\right)\left(\Phi_{\bar{k}}^\dagger\Phi_l\right) \tag{3.26}$$

where coupling coefficients are:

$$\begin{aligned} Y_{11} &= m_{11}^2, & Y_{12} &= -m_{12}^2, \\ Y_{21} &= -m_{12}^{2\,*}, & Y_{22} &= m_{22}^2, \\ Z_{1111} &= \lambda_1, & Z_{2222} &= \lambda_2, \\ Z_{1122} &= Z_{2211} = \lambda_3, & Z_{1212} &= Z_{2112} = \lambda_4, \\ Z_{1212} &= \lambda_5, & Z_{2121} &= \lambda_5^*, \\ Z_{1112} &= Z_{1211} = \lambda_6, & Z_{1121} &= Z_{2111} = \lambda_6^*, \\ Z_{2212} &= Z_{1222} = \lambda_7, & Z_{2221} &= Z_{2122} = \lambda_7^* \end{aligned} \tag{3.27}$$

$Y_{i\bar{j}}$ has dimension of mass squared while $Z_{i\bar{j}k\bar{l}}$ is dimensionless. This can be seen from looking at the generic potential eq.(3.21).

The Higgs potential tensors transform covariantly under a $U(2)$ transformation:

$$\begin{aligned} Y_{i\bar{j}} &\to U_{i\bar{k}}Y_{k\bar{l}}U_{l\bar{j}}^\dagger, \\ Z_{i\bar{j}k\bar{l}} &\to U_{i\bar{m}}U_{n\bar{l}}^\dagger U_{k\bar{o}}U_{p\bar{l}}^\dagger Z_{m\bar{n}o\bar{p}} \end{aligned} \tag{3.28}$$

Also it is possible to change indexes with help of the Kronecker delta:

$$\delta_{j\bar{i}}Z_{i\bar{j}k\bar{l}} = Z_{i\bar{i}k\bar{l}} \tag{3.29}$$

Global transformations of Φ_i leaves the functional form of V invariant, while only coefficients depend on the ones choice of the 2HDM.

EW symmetry is broken as the minimum of the Higgs scalar potential occurs non zero vacuum expectation value. Minimum conditions of the scalar potential can be written as:

$$\left.\frac{\partial V}{\partial \Phi_i}\right|_{\Phi_j=\langle\Phi_j\rangle} = 0 \tag{3.30}$$

This condition yields the vacuum expectation values of the two doublets $\langle\Phi_1\rangle, \langle\Phi_2\rangle$. If the mass matrix constructed from the Higgs scalar potential squared mass parameters m_{ij}^2 got at least one negative eigenvalue then the scalar fields vacuum expectation value becomes non zero. It is possible to write the VEV:

$$\langle\Phi_1\rangle = \frac{v}{\sqrt{2}}\begin{pmatrix} 0 \\ \hat{v}_1 \end{pmatrix}, \quad \langle\Phi_2\rangle = \frac{v}{\sqrt{2}}\begin{pmatrix} u \\ \hat{v}_2 e^{i\xi} \end{pmatrix} \tag{3.31}$$

where both vacuum expectation values v_1, v_2 are real and positive, the arbitrary angle acquires values $0 \le \xi < 2\pi$.

In case of $u \neq 0$ vacuum is called charged. Depending on the parameters of the Higgs scalar potential it describes minimum of the potential or saddle point. $U(1)_{EM}$ symmetry is spontaneously broken. In case of $u = 0$, $U(1)_{EM}$ symmetry is preserved. Only this condition is treated here.

Both scalar doublets preserve vacuum invariance under the electromagnetic gauge symmetry $U(1)_{EM}$. Therefore the vacuum expectation values of the Higgs fields Φ_1 and Φ_2 must be aligned:

$$\langle\Phi_a\rangle = \frac{v}{\sqrt{2}}\begin{pmatrix} 0 \\ \hat{v}_a \end{pmatrix}, \quad \begin{aligned} \hat{v}_1 &= e^{i\eta}\cos\beta, \\ \hat{v}_2 &= e^{i\eta}\sin\beta e^{i\xi}, \\ v^2 &= v_1^2 + v_2^2 \end{aligned} \tag{3.32}$$

where $0 \le \beta \le \pi/2$, $v \equiv {}^{2m_W}/_{g} = 246$ GeV, $\hat{v}_a$ $\left(v_a = {}^{v\hat{v}_a}/\sqrt{2},\ \ \hat{v}_a^\dagger = (\hat{v}^\dagger)_{\bar{a}}\right)$ is a unit norm vector and phase η is arbitrary which can be removed with help of $U(1)_Y$ group transformation.

In the generic potential eq.(3.21) coefficients m and λ are all $U(1)_Y$ group invariants but not under $SU(2)$ symmetry while $\hat{v}$ preserves invariance under $U(2)$. Also it is always possible to rephase Φ_2 in a such way that $\xi = 0$.

In order to get the minimum condition for the scalar potential we take the derivative of potential V with respect to Φ_j [49]:

$$\left.\frac{\partial V}{\partial \Phi_j}\right|_{\langle\Phi_i^0\rangle = {}^{v_i}/\sqrt{2}} = 0 \tag{3.33}$$

The minimum condition is:

$$\begin{aligned}\frac{\partial V}{\partial \Phi_m} &= Y_{i\bar{j}}\left(\frac{\partial \Phi^\dagger_{\bar{i}}}{\partial \Phi_m}\Phi_j + \Phi^\dagger_{\bar{i}}\frac{\partial \Phi_j}{\partial \Phi_m}\right) \\ &+ \frac{1}{2}Z_{i\bar{j}k\bar{l}}\left(\frac{\partial\left(\Phi^\dagger_{\bar{i}}\Phi_j\right)}{\partial \Phi_m}\Phi^\dagger_{\bar{k}}\Phi_l + \Phi^\dagger_{\bar{i}}\Phi_j\frac{\partial\left(\Phi^\dagger_{\bar{k}}\Phi_l\right)}{\partial \Phi_m}\right) \\ &= \left[\frac{\partial \Phi^\dagger_{\bar{i}}}{\partial \Phi_m} = 0, \quad \frac{\partial \Phi_i}{\partial \Phi_m} = \delta_{i\bar{m}}\right] \\ &= Y_{i\bar{j}}\Phi^\dagger_{\bar{i}}\delta_{j\bar{m}} + \frac{1}{2}Z_{i\bar{j}k\bar{l}}\left(\Phi^\dagger_{\bar{i}}\delta_{j\bar{m}}\Phi^\dagger_{\bar{k}}\Phi_l + \Phi^\dagger_{\bar{i}}\Phi_j\Phi^\dagger_{\bar{k}}\delta_{l\bar{m}}\right)\end{aligned} \tag{3.34}$$

After inserting the corresponding vacuum expectation values ($\langle\Phi_i\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ \hat{v}_i\end{pmatrix}$) and using the following equations:

$$Y_{a\bar{e}}\Phi^\dagger_{\bar{a}}\big|_{\Phi_a=0} \to \hat{v}^*_{\bar{a}}, \qquad Z_{a\bar{b}c\bar{d}} = Z_{c\bar{d}a\bar{b}} \tag{3.35}$$

we get the minimum condition:

$$v\hat{v}^*_{\bar{i}}Y_{i\bar{j}} + \frac{1}{2}v^3 Z_{i\bar{j}k\bar{l}}\hat{v}^*_{\bar{i}}\hat{v}^*_{\bar{k}}\hat{v}_l = 0 \tag{3.36}$$

The gauge transformations $SU(2)_L \otimes U(1)_Y$ leaves the scalar potential invariant, therefore Y and Z are also invariant under $U(1)$ transformations, although $\hat{v}$ changes by an overall phase. Only $SU(2)$ transformations are the ones responsible for a change of basis and all $U(2)$ transformations change basis. Considering eq.(3.22) we can straightforward transform $\hat{v}$ under $U(2)$ group:

$$\hat{v}_i \to U_{i\bar{j}}\hat{v}_j \tag{3.37}$$

It is also possible to construct a hermitian matrix $V_{i\bar{j}}$:

$$V_{i\bar{j}} = \hat{v}_i\hat{v}^*_{\bar{j}} \tag{3.38}$$

The matrix $V_{i\bar{j}}$ is hermitian and therefore it possesses two eigenvectors of unit norm. It is also possible to rewrite the minimum condition eq.(3.36) with help of eq.(3.38):

$$\mathrm{tr}(VY) + \frac{1}{2}v^2 Z_{i\bar{j}k\bar{l}}V_{j\bar{i}}V_{l\bar{k}} = 0 \tag{3.39}$$

It is crucial to note that considering the above minimum condition we go from two degrees of freedom to only one. Looking at the picture of the Higgs potential this implies that from explicitly describing the whole potential we simplify to finding only the vacuum value.

From the minimum conditions it is possible to derive:

$$\begin{aligned} m_{11}^2 &= m_{12}^2 e^{i\xi} t_\beta - \frac{1}{2} v^2 \left[\lambda_1 c_\beta^2 + \left(\lambda_3 + \lambda_4 + \lambda_5 e^{2i\xi}\right) s_\beta^2 \right. \\ &\qquad\qquad \left. + \left(2\lambda_6 e^{i\xi} + \lambda_6^* e^{-i\xi}\right) s_\beta c_\beta + \lambda_7 s_\beta^2 t_\beta e^{i\xi}\right], \\ m_{22}^2 &= \left(m_{12}^2 e^{i\xi}\right)^* t_\beta^{-1} - \frac{1}{2} v^2 \left[\lambda_2 s_\beta^2 + \left(\lambda_3 + \lambda_4 + \lambda_5^* e^{-2i\xi}\right) c_\beta^2 \right. \\ &\qquad\qquad \left. + \lambda_6^* c_\beta^2 t_\beta^{-1} e^{-i\xi} + \left(\lambda_7 e^{i\xi} + 2\lambda_7^* e^{-i\xi}\right) s_\beta c_\beta\right] \end{aligned} \tag{3.40}$$

Both mass squared parameters m_{11}^2, m_{22}^2 are real and therefore:

$$\mathbb{I}m(m_{12}^2 e^{i\xi}) = \frac{1}{2} v^2 \left[\mathbb{I}m(\lambda_5 e^{2i\xi}) s_\beta c_\beta + \mathbb{I}m(\lambda_6 e^{i\xi}) c_\beta^2 + \mathbb{I}m(\lambda_7 e^{i\xi}) s_\beta^2\right] \tag{3.41}$$

which can be used to determine phase ξ.

Let us introduce a second vector of unit norm orthogonal to $\hat{v}_i$. Orthogonality implies that:

$$\hat{v}_{\bar{j}}^* w_j = 0 \tag{3.42}$$

The second unit form vector is:

$$\begin{aligned} &\hat{w}_j = \hat{v}_{\bar{i}}^* \epsilon_{ij}, \\ &\hat{w}_1 = -e^{-i\eta} s_\beta e^{-i\xi}, \quad \hat{w}_2 = e^{-i\eta} c_\beta \end{aligned} \tag{3.43}$$

Applying a $U(2)$ transformation to the $\hat{w}$ yields:

$$\hat{w}_i = \det(U)^{-1} U_{i\bar{j}} \hat{w}_j = e^{-i\chi} U_{i\bar{j}} \hat{w}_j \tag{3.44}$$

where $\hat{w}_i$ is pseudo-vector with respect to $U(2)$ transformation. Inverting eq.(3.43) results in:

$$\hat{w}_i = -\epsilon_{ij} \hat{v}_{\bar{j}}^* \qquad \text{or} \qquad \hat{v}_{\bar{i}}^* = \epsilon_{\bar{i}\bar{j}} w_j \tag{3.45}$$

We made an assumption that $\hat{v} = (1, 0)$ and due to orthogonality $\hat{w} = (0, \pm 1)$ is also known up to the direction phase. Therefore it follows that $\hat{v}\hat{v}^T + \hat{w}\hat{w}^T = \mathbb{I}_2$. It is possible to write down a proper second-rank tensor $W_{a\bar{b}}$ as:

$$W_{a\bar{b}} = \delta_{a\bar{b}} - V_{a\bar{b}} \tag{3.46}$$

We are interested in the CP-conserving potential. Thus all of the parameters are real.

After rotating Φ_2 and setting $\xi = 0$:

$$
\begin{aligned}
m_{11}^2 &= m_{12}^2 t_\beta - \frac{1}{2} v^2 \left[\lambda_1 c_\beta^2 + \lambda_{345} s_\beta^2 + 3\lambda_6 s_\beta c_\beta + \lambda_7 s_\beta^2 t_\beta\right], \\
m_{22}^2 &= m_{12}^2 t_\beta^{-1} - \frac{1}{2} v^2 \left[\lambda_2 s_\beta^2 + \lambda_{345} c_\beta^2 + \lambda_6 c_\beta^2 t_\beta^{-1} + 3\lambda_7 s_\beta c_\beta\right]
\end{aligned}
\tag{3.47}
$$

where $\lambda_{345} = \lambda_3 + \lambda_4 + \lambda_5$.

Now let us take a look at how Higgs boson masses look like in terms of the generic basis. $m_{H^\pm}$ and m_A are:

$$
\begin{aligned}
m_A^2 &= \frac{m_{12}^2}{s_\beta c_\beta} - \frac{v^2}{2} \left(2\lambda_5 + \lambda_6 t_\beta + \lambda_7 t_\beta\right) \text{ when } t_\beta > 0, \\
m_A^2 &= m_{22}^2 + \frac{1}{2} v^2 \left(\lambda_3 + \lambda_4 - \lambda_5\right) \text{ otherwise}, \\
m_{H^\pm}^2 &= m_A^2 + \frac{v^2}{2} \left(\lambda_5 - \lambda_4\right)
\end{aligned}
\tag{3.48}
$$

The CP-even states mix and therefore the mass matrix $\mathcal{M}$ is given by:

$$
\begin{aligned}
\mathcal{M}^2 =& m_A^2 \begin{pmatrix} s_\beta^2 & -s_\beta c_\beta \\ -s_\beta c_\beta & c_\beta^2 \end{pmatrix} + \\
& v^2 \begin{pmatrix} \lambda_1 c_\beta^2 + 2\lambda_6 s_\beta c_\beta + \lambda_5 s_\beta^2 & \left(\lambda_3 + \lambda_4\right) s_\beta c_\beta + \lambda_6 c_\beta^2 + \lambda_7 s_\beta^2 \\ \left(\lambda_3 + \lambda_4\right) s_\beta c_\beta + \lambda_6 c_\beta^2 + \lambda_7 s_\beta^2 & \lambda_2 s_\beta^2 + 2\lambda_7 s_\beta c_\beta + \lambda_5 c_\beta^2 \end{pmatrix}
\end{aligned}
\tag{3.49}
$$

from where we get:

$$
\begin{aligned}
\mathcal{M}_{11}^2 &= m_A^2 s_\beta^2 + v^2 \left(\lambda_1 c_\beta^2 + 2\lambda_6 s_\beta c_\beta + \lambda_5 s_\beta^2\right), \\
\mathcal{M}_{12}^2 &= -m_A^2 s_\beta c_\beta + v^2 \left(\left(\lambda_3 + \lambda_4\right) s_\beta c_\beta + \lambda_6 c_\beta^2 + \lambda_7 s_\beta^2\right), \\
\mathcal{M}_{22}^2 &= m_A^2 c_\beta^2 + v^2 \left(\lambda_2 s_\beta^2 + 2\lambda_7 s_\beta c_\beta + \lambda_5 c_\beta^2\right)
\end{aligned}
\tag{3.50}
$$

Diagonalization of the mass matrix $\mathcal{M}$ is possible, as stated earlier, by performing a rotation:

$$
\begin{pmatrix} m_H^2 & 0 \\ 0 & m_h^2 \end{pmatrix} = \mathcal{R}(\alpha) \mathcal{M}^2 \mathcal{R}^T(\alpha)
\tag{3.51}
$$

The mass eigenvalues are:

$$
m_{H,h}^2 = \frac{1}{2} \left[\mathcal{M}_{11}^2 + \mathcal{M}_{22}^2 \pm \sqrt{\left(\mathcal{M}_{11}^2 - \mathcal{M}_{22}^2\right)^2 + 4\left(\mathcal{M}_{12}^2\right)^2}\right]
\tag{3.52}
$$

3.3 Higgs Bases

One of the possible uniquely selected basis are Higgs bases. In this basis only one neutral component gets VEV. By applying a $U(1)_Y \times U(1)$ transformations it is possible to preserve the vacuum conditions. Under $U(1)$ group the Higgs doublets are rotated in the following way:

$$\Phi_1' \to e^{i\chi}\Phi_1', \quad \Phi_2' \to e^{-i\chi}\Phi_2' \tag{3.53}$$

which is parametrized by χ.

We begin in a generic $\Phi_1 - \Phi_2$ basis. This one is not uniquely defined. The newly defined Higgs-doublet fields are:

$$\begin{aligned} H_1 &= \begin{pmatrix} H_1^+ \\ H_1^0 \end{pmatrix} = \hat{v}_{\bar{i}}^* \Phi_i = \ c_\beta \Phi_1 + s_\beta e^{-i\xi}\Phi_2, \\ H_2 &= \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix} = \hat{w}_{\bar{i}}^* \Phi_i = -s_\beta e^{i\xi}\Phi_1 + c_\beta \Phi_2 \end{aligned} \tag{3.54}$$

or in a matrix form:

$$\begin{pmatrix} H_1 \\ H_2 \end{pmatrix} = \begin{pmatrix} c_\beta & s_\beta e^{-i\xi} \\ -s_\beta e^{i\xi} & c_\beta \end{pmatrix} \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix} \tag{3.55}$$

In order to transform from the generic basis to Higgs bases, we introduce the following matrix:

$$\hat{U} = \begin{pmatrix} \hat{v}_1^* & \hat{v}_2^* \\ \hat{w}_1^* & \hat{w}_2^* \end{pmatrix} = \begin{pmatrix} \hat{v}_1^* & \hat{v}_2^* \\ -\hat{v}_2 & \hat{v}_1 \end{pmatrix} \tag{3.56}$$

Note that in the above matrix we used $\hat{w}_{\bar{i}}^* = \epsilon_{\bar{j}\bar{i}}\hat{v}_j$, which can be derived from eq.(3.45). On the other hand it is possible to rewrite the newly defined fields as:

$$\begin{gathered} \Phi_i = H_1 \hat{v}_i + H_2 \hat{v}_j^* \epsilon_{ji}, \\ \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix} = \begin{pmatrix} c_\beta & -s_\beta e^{-i\xi} \\ s_\beta e^{i\xi} & c_\beta \end{pmatrix} \begin{pmatrix} H_1 \\ H_2 \end{pmatrix} \end{gathered} \tag{3.57}$$

Considering that $\hat{v}_{\bar{i}}^* \hat{v}_i = 1$ and $\hat{v}_{\bar{i}}^* \hat{w}_i = 1$, vacuum expectation values in Higgs bases are:

$$\langle H_1^0 \rangle = \frac{v}{\sqrt{2}}, \qquad \langle H_2^0 \rangle = 0 \tag{3.58}$$

The field H_1 is invariant, while H_2 is pseudo-invariant under $U(2)$ group transformation:

$$H_2' = e^{i\chi} H_2 \tag{3.59}$$

It is possible to construct a unitary matrix U_D with respect to which Higgs bases transforms from H_i to H_i':

$$U_D = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\chi} \end{pmatrix} \tag{3.60}$$

To get the scalar potential in Higgs bases we simply insert eq.(3.57) into the generic potential eq.(3.26):

$$\begin{aligned} V = {} & Y_1 H_1^\dagger H_1 + Y_2 H_2^\dagger H_2 + Y_3 H_1^* H_2 + Y_3^\dagger H_2^\dagger H_1 \\ & + \frac{1}{2} Z_1 \left(H_1^\dagger H_1 \right)^2 + \frac{1}{2} Z_2 \left(H_2^\dagger H_2 \right)^2 \\ & + Z_3 \left(H_1^\dagger H_1 \right) \left(H_2^\dagger H_2 \right) + Z_4 \left(H_1^\dagger H_2 \right) \left(H_2^\dagger H_1 \right) \\ & + \left\{ \frac{1}{2} Z_5 \left(H_1^\dagger H_2 \right)^2 + \left[Z_6 \left(H_1^\dagger H_1 \right)^2 + Z_7 \left(H_2^\dagger H_2 \right)^2 \right] \left(H_1^\dagger H_2 \right) + h.c. \right\} \end{aligned} \tag{3.61}$$

In Higgs bases scalar potential Y_1, Y_2 and $Z_{1,2,3,4}$ are all $U(2)$ invariants and real, while Y_3 and $Z_{5,6,7}$ are pseudo-invariants [11] and are complex. It is possible to express coefficients Y and Z in terms of the previously defined second rank tensors $V_{i\bar{j}}$, $W_{i\bar{j}}$ and unit norms $\hat{v}$,$\hat{w}$:

$$\begin{aligned} Y_1 &= Y_{i\bar{j}} V_{j\bar{i}}, & Y_2 &= Y_{i\bar{j}} W_{j\bar{i}}, \\ Z_1 &= Z_{i\bar{j}k\bar{l}} V_{j\bar{i}} V_{l\bar{k}}, & Z_2 &= Z_{i\bar{j}k\bar{l}} W_{j\bar{i}} W_{k\bar{l}}, \\ Z_3 &= Z_{i\bar{j}k\bar{l}} V_{j\bar{i}} W_{l\bar{k}}, & Z_4 &= Z_{i\bar{j}k\bar{l}} V_{j\bar{k}} W_{l\bar{i}}, \\ Y_3 &= Y_{i\bar{j}} \hat{v}_{\bar{i}}^* \hat{w}_j, & Z_5 &= Z_{i\bar{j}k\bar{l}} \hat{v}_{\bar{i}}^* \hat{w}_j \hat{v}_{\bar{k}}^* \hat{w}_l, \\ Z_6 &= Z_{i\bar{j}k\bar{l}} \hat{v}_{\bar{i}}^* \hat{v}_j \hat{v}_{\bar{k}}^* \hat{w}_l, & Z_7 &= Z_{i\bar{j}k\bar{l}} \hat{v}_{\bar{i}}^* \hat{w}_j \hat{w}_{\bar{k}}^* \hat{w}_l \end{aligned} \tag{3.62}$$

Using eq.(3.44) it is possible to transform pseudo-invariants as:

$$\begin{gathered} [Y_3, Z_6, Z_7] \to e^{-i\chi} [Y_3, Z_6, Z_7], \\ Z_5 \to e^{-2i\chi} Z_5 \end{gathered} \tag{3.63}$$

As an example, let us write down the coefficient Y_1 in terms of $\hat{v}$:

$$\begin{aligned} Y_1 &= Y_{i\bar{j}} \hat{v}_j \hat{v}_{\bar{i}}^* = Y_{11} \hat{v}_1 \hat{v}_1^* + Y_{22} \hat{v}_2 \hat{v}_2^* + Y_{12} \left\{ \hat{v}_1 \hat{v}_2^* + \hat{v}_2 \hat{v}_1^* \right\} \\ &= m_{11}^2 c_\beta^2 + m_{22}^2 s_\beta^2 - \mathbb{R}e \left(m_{12}^2 e^{i\xi} \right) s_{2\beta} \end{aligned} \tag{3.64}$$

Following the same procedure in the generic basis invariant and pseudo-invariant coefficients

[11] invariant under $SU(2)$ but not under $U(2)$

can be written down as:

$$
\begin{aligned}
Y_1 &= m_{11}^2 c_\beta^2 + m_{22}^2 s_\beta^2 - \mathbb{R}e\left(m_{12}^2 e^{i\xi}\right) s_{2\beta}, \\
Y_2 &= m_{11}^2 s_\beta^2 + m_{22}^2 c_\beta^2 + \mathbb{R}e\left(m_{12}^2 e^{i\xi}\right) s_{2\beta}, \\
Y_3 e^{i\xi} &= \frac{1}{2}\left(m_{22}^2 - m_{11}^2\right) s_{2\beta} - \mathbb{R}e\left(m_{12}^2 e^{i\xi}\right) c_{2\beta} - i\mathbb{I}m\left(m_{12}^2 e^{i\xi}\right)
\end{aligned}
\tag{3.65}
$$

and

$$
\begin{aligned}
Z_1 &= \lambda_1 c_\beta^4 + \lambda_2 s_\beta^4 + \frac{1}{2}\lambda_{345} s_{2\beta}^2 + 2 s_{2\beta}\left[c_\beta^2 \mathbb{R}e(\lambda_6 e^{i\xi}) + s_\beta^2 \mathbb{R}e(\lambda_7 e^{i\xi})\right], \\
Z_2 &= \lambda_1 s_\beta^4 + \lambda_2 c_\beta^4 + \frac{1}{2}\lambda_{345} s_{2\beta}^2 - 2 s_{2\beta}\left[s_\beta^2 \mathbb{R}e(\lambda_6 e^{i\xi}) + c_\beta^2 \mathbb{R}e(\lambda_7 e^{i\xi})\right], \\
Z_3 &= \frac{1}{4} s_{2\beta}^2\left[\lambda_1 + \lambda_2 - 2\lambda_{345}\right] + \lambda_3 - s_{2\beta} c_{2\beta} \mathbb{R}e\left[(\lambda_6 - \lambda_7) e^{i\xi}\right], \\
Z_4 &= \frac{1}{4} s_{2\beta}^2\left[\lambda_1 + \lambda_2 - 2\lambda_{345}\right] + \lambda_4 - s_{2\beta} c_{2\beta} \mathbb{R}e\left[(\lambda_6 - \lambda_7) e^{i\xi}\right], \\
Z_5 e^{2i\xi} &= \frac{1}{4} s_{2\beta}^2\left[\lambda_1 + \lambda_2 - 2\lambda_{345}\right] + \mathbb{R}e(\lambda_5 e^{2i\xi}) + i c_{2\beta} \mathbb{I}m(\lambda_5 e^{2i\xi}), \\
&\quad - s_{2\beta} c_{2\beta} \mathbb{R}e\left[(\lambda_6 - \lambda_7) e^{i\xi}\right] - i s_{2\beta} \mathbb{I}m\left[(\lambda_6 - \lambda_7) e^{i\xi}\right] \\
Z_6 e^{i\xi} &= -\frac{1}{2} s_{2\beta}\left[\lambda_1 c_\beta^2 - \lambda_2 s_\beta^2 - \lambda_{345} c_{2\beta} - i\mathbb{I}m(\lambda_5 e^{2i\xi})\right] + c_\beta c_{3\beta} \mathbb{R}e(\lambda_6 e^{i\xi}) \\
&\quad + s_\beta s_{3\beta} \mathbb{R}e(\lambda_7 e^{i\xi}) + i c_\beta^2 \mathbb{I}m(\lambda_6 e^{i\xi}) + i s_\beta^2 \mathbb{I}m(\lambda_7 e^{i\xi}), \\
Z_7 e^{i\xi} &= -\frac{1}{2} s_{2\beta}\left[\lambda_1 s_\beta^2 - \lambda_2 c_\beta^2 + \lambda_{345} c_{2\beta} + i\mathbb{I}m(\lambda_5 e^{2i\xi})\right] + s_\beta s_{3\beta} \mathbb{R}e(\lambda_6 e^{i\xi}) \\
&\quad + c_\beta c_{3\beta} \mathbb{R}e(\lambda_7 e^{i\xi}) + i s_\beta^2 \mathbb{I}m(\lambda_6 e^{i\xi}) + i c_\beta^2 \mathbb{I}m(\lambda_7 e^{i\xi})
\end{aligned}
\tag{3.66}
$$

where $\lambda_{345} = \lambda_3 + \lambda_4 + \mathbb{R}e(\lambda_5 e^{2i\xi})$.

With help of the minimum conditions eq.(3.47) and not forgetting that in Higgs bases $t_\beta = 0$ it is possible to transform real parameters in the following way:

$$
\begin{aligned}
Y_1 &= -\frac{1}{2} v^2 Z_1, \\
Y_2 &= m_{12}^2 s_{2\beta} - \frac{1}{2} v^2\left[(\lambda_1 + \lambda_2) c_\beta^2 s_\beta^2 + \lambda_{345}(s_\beta^4 + c_\beta^4) + (\lambda_6 + \lambda_7)\frac{1}{2} s_{4\beta}\right], \\
Y_3 &= -\frac{1}{2} v^2 Z_6
\end{aligned}
\tag{3.67}
$$

and

$$
\begin{aligned}
Z_1 &= \lambda_1 c_\beta^4 + \lambda_2 s_\beta^4 + \frac{1}{2}\lambda_{345} s_{2\beta}^2 + 2 s_{2\beta}\left[c_\beta^2\lambda_6 + s_\beta^2\lambda_7\right],\\
Z_2 &= \lambda_1 s_\beta^4 + \lambda_2 c_\beta^4 + \frac{1}{2}\lambda_{345} s_{2\beta}^2 - 2 s_{2\beta}\left[s_\beta^2\lambda_6 + c_\beta^2\lambda_7\right],\\
Z_3 &= \frac{1}{4}s_{2\beta}^2\left[\lambda_1 + \lambda_2 - 2\lambda_{345}\right] + \lambda_3 - s_{2\beta}c_{2\beta}(\lambda_6 - \lambda_7),\\
Z_4 &= \frac{1}{4}s_{2\beta}^2\left[\lambda_1 + \lambda_2 - 2\lambda_{345}\right] + \lambda_4 - s_{2\beta}c_{2\beta}(\lambda_6 - \lambda_7),\\
Z_5 &= \frac{1}{4}s_{2\beta}^2\left[\lambda_1 + \lambda_2 - 2\lambda_{345}\right] + \lambda_5 - s_{2\beta}c_{2\beta}(\lambda_6 - \lambda_7),\\
Z_6 &= -\frac{1}{2}s_{2\beta}\left[\lambda_1 c_\beta^2 - \lambda_2 s_\beta^2 - \lambda_{345}c_{2\beta}\right] + c_\beta c_{3\beta}\lambda_6 + s_\beta s_{3\beta}\lambda_7,\\
Z_7 &= -\frac{1}{2}s_{2\beta}\left[\lambda_1 s_\beta^2 - \lambda_2 c_\beta^2 + \lambda_{345}c_{2\beta}\right] + s_\beta s_{3\beta}\lambda_6 + c_\beta c_{3\beta}\lambda_7
\end{aligned}
\tag{3.68}
$$

where $\lambda_{345} = \lambda_3 + \lambda_4 + \lambda_5$.

In case of the CP-conserving case, that is when:

$$
\begin{aligned}
&\left\{m_{12}^2,\ \lambda_5\right\} \in \mathbb{R}e,\\
&\lambda_6 = \lambda_7 = 0
\end{aligned}
\tag{3.69}
$$

It is possible to relate the t_β parameter:

$$
t_{2\beta} = \frac{2\left(Z_6 + Z_7\right)}{Z_2 - Z_1}
\tag{3.70}
$$

The Higgs bosons masses are:

$$
\begin{aligned}
m_h^2 &= \left(Z_1 + Z_6 t_{\beta-\alpha}^{-1}\right)v^2,\\
m_H^2 &= Y_2 + \left(\frac{1}{2}Z_{345} - Z_6 t_{\beta-\alpha}^{-1}\right)v^2,\\
m_A^2 &= Y_2 + \frac{1}{2}\left(Z_3 + Z_4 - Z_5\right)v^2,\\
m_{H^\pm}^2 &= Y_2 + \frac{1}{2}Z_3 v^2
\end{aligned}
\tag{3.71}
$$

Derivation of the Higgs boson masses is presented in the next section.

3.4 The Mass Eigenstates Basis

In Higgs bases it is possible to write down the doublets in terms of the physical fields:

$$\begin{aligned} H_1 &= \begin{pmatrix} G^{\pm} \\ \frac{1}{\sqrt{2}}\left(v + \varphi_1^0 + iG^0\right) \end{pmatrix}, \\ H_2 &= \begin{pmatrix} H^{\pm} \\ \frac{1}{\sqrt{2}}\left(\varphi_2^0 + ia^0\right) \end{pmatrix} \end{aligned} \tag{3.72}$$

where $G^{\pm}$ are the charged Goldstone fields, G^0 is a CP-odd neutral one and $H^{\pm}$ are the charged Higgs fields, φ_1^0 is a CP-even neutral Higgs scalar field, φ_2^0 and a^0 are indefinite CP states. In case of the CP violation all Higgs scalar fields are mixed together in order to produce three possible neutral physical Higgs bosons h, H, A of indefinite CP quantum numbers. In case of non CP-violation three scalar fields are absorbed to form non-distinguishable Higgs bosons. The neutral Goldstone boson is always CP definite.

After writing down explicitly the Higgs potential it is possible to split it into several parts $V = V_{const} + V_{linear} + V_{quadratic} + V_{3,4}$. Earlier we have seen that linear scalar potential part V_{linear} determines the vacuum conditions. As for now we are interested in the generation of masses. The quadratic scalar potential terms $V_{quadratic}$ should be looked into to determine the mass eigenstates.

It is possible to split $G^{\pm}$ and $H^{\pm}$ parts into real and imaginary:

$$\begin{aligned} G^{\pm} &= \frac{1}{\sqrt{2}}\left(G_r^{+} \mp iG_i^{+}\right), \\ H^{\pm} &= \frac{1}{\sqrt{2}}\left(H_r^{+} \mp iH_i^{+}\right) \end{aligned} \tag{3.73}$$

Rewriting eq.(3.72) in terms of eq.(3.73) and re expressing scalar terms yields:

$$\begin{aligned} H_1 &= \frac{1}{\sqrt{2}} \begin{pmatrix} G_r^{+} \mp iG_i^{+} \\ v + \varphi_1^0 + iG^0 \end{pmatrix}, \\ H_2 &= \frac{1}{\sqrt{2}} \begin{pmatrix} H_r^{+} \mp iH_i^{+} \\ \varphi_2^0 + ia^0 \end{pmatrix} \end{aligned} \tag{3.74}$$

The Higgs scalar potential is now expressed in terms of eight real variables. Inserting eq.(3.74) into the minimum conditions eq.(3.36) we get the mass matrix. In case of the minimum condition all fields are set to zero. The scalar potential minimum conditions eq.(3.67) are also considered. After the minimization we get the following results:

$$\begin{aligned} \left.\frac{\partial V}{\partial G_r^+}\right| = \left.\frac{\partial V}{\partial G_i^+}\right| = \left.\frac{\partial V}{\partial G^0}\right| &= 0, \\ \left.\frac{\partial V}{\partial H_r^+}\right| = \left.\frac{\partial V}{\partial H_i^+}\right| &= 0, \\ \left.\frac{\partial V}{\partial \varphi_1^0}\right| = v\left(Y_1 + \frac{v^2}{2}Z_1\right) &= 0, \\ \left.\frac{\partial V}{\partial \varphi_2^0}\right| = v\left(\mathbb{R}e(Y_3) + \frac{v^2}{2}\mathbb{R}e(Z_6)\right) &= 0, \\ \left.\frac{\partial V}{\partial a^0}\right| = v\left(\mathbb{I}m(Y_3) + \frac{v^2}{2}\mathbb{I}m(Z_6)\right) &= 0 \end{aligned} \tag{3.75}$$

The Higgs boson squared mass matrix is obtained by applying the second derivative to the scalar potential minimization conditions eq.(3.75):

$$\mathcal{M}^2_{G,H} = \begin{pmatrix} \mathcal{M}^2_G & 0 & 0 \\ 0 & \mathcal{M}^2_{\varphi_1^0,\varphi_2^0,a^0} & 0 \\ 0 & 0 & \mathcal{M}^2_{H^\pm} \end{pmatrix} \tag{3.76}$$

$\mathcal{M}^2_G$ is a three by three matrix in which all of the elements are zeroes due to the fact that the Goldstone bosons are massless. Three neutral scalar fields mix together to form the neutral Higgs squared mass matrix $\mathcal{M}^2_{\varphi_1^0,\varphi_2^0,a^0}$:

$$v^2 \begin{pmatrix} Z_1 & \mathbb{R}e(Z_6) & -\mathbb{I}m(Z_6) \\ \mathbb{R}e(Z_6) & \frac{1}{2}\left[Z_3 + Z_4 + \mathbb{R}e(Z_5)\right] + \frac{Y_2}{v^2} & -\frac{1}{2}\mathbb{I}m(Z_5) \\ -\mathbb{I}m(Z_6) & -\frac{1}{2}\mathbb{I}m(Z_5) & \frac{1}{2}\left[Z_3 + Z_4 - \mathbb{R}e(Z_5)\right] + \frac{Y_2}{v^2} \end{pmatrix} \tag{3.77}$$

The last component of matrix (3.76) is the charged Higgs squared mass matrix $\mathcal{M}^2_{H^\pm}$:

$$\begin{pmatrix} Y_2 + \frac{1}{2}v^2 Z_3 & 0 \\ 0 & Y_2 + \frac{1}{2}v^2 Z_3 \end{pmatrix} \tag{3.78}$$

Therefore the charged Higgs boson mass is given by:

$$m^2_{H^\pm} = Y_2 + \frac{1}{2}Z_3 v^2 \tag{3.79}$$

One of the possibilities to diagonalize the Higgs square mass matrix $\mathcal{M}$ is by the mean of orthogonal transformation:

$$\mathcal{R}\mathcal{M}\mathcal{R}^T = \mathcal{M}_D = \mathrm{diag}\left(m_1^2, m_2^2, m_3^2\right) \tag{3.80}$$

where $\mathcal{R}\mathcal{R}^T = \mathcal{I}$ and the corresponding mass eigenvalues are m_i^2. $\mathcal{R}$ are Euler's rotation matrices:

$$\begin{aligned}\mathcal{R} &= \mathcal{R}_{12}\mathcal{R}_{13}\mathcal{R}_{23} \\ &= \begin{pmatrix} c_{12} & -s_{12} & 0 \\ s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_{13} & 0 & -s_{13} \\ 0 & 1 & 0 \\ s_{13} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & -s_{23} \\ 0 & s_{23} & c_{23} \end{pmatrix} \\ &= \begin{pmatrix} c_{12}c_{13} & -s_{12}c_{23} - c_{12}s_{13}s_{23} & s_{12}s_{23} - c_{12}c_{23}s_{13} \\ s_{12}c_{13} & c_{12}c_{23} - s_{12}s_{13}s_{23} & -c_{12}s_{23} - s_{12}s_{13}c_{23} \\ s_{13} & c_{13}s_{23} & c_{13}c_{23} \end{pmatrix}\end{aligned} \tag{3.81}$$

Applying the corresponding limitations on the angles $-\pi \leq \theta_{12}, \theta_{23} < \pi$ and $|\theta_{13}| \leq \pi/2$ covers the whole $SO(3)$ group.

Three neutral fields $\varphi_1^0, \varphi_2^0, a^0$ are mixed together to produce three possible neutral Higgs bosons. Using the introduced parametrization the neutral Higgs bosons mass eigenstates can be written in terms of:

$$\begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix} = \mathcal{R} \begin{pmatrix} \varphi_1^0 \\ \varphi_2^0 \\ a^0 \end{pmatrix} \tag{3.82}$$

In this convention, for simplicity, it is worth denoting masses in the following way $m_i \leq m_{i+1}$. The mass eigenstates are $U(2)$ invariants.

Following ref. [50] the following unitary matrix is introduced:

$$W = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & -i/\sqrt{2} & i/\sqrt{2} \end{pmatrix} \tag{3.83}$$

Considering the fact that W is unitary, it possible to rewrite the diagonalized squared mass matrix $\mathcal{M}_D$ in the following way:

$$(\mathcal{R}\mathcal{W})(\mathcal{W}^\dagger \mathcal{M}\mathcal{W})(\mathcal{R}\mathcal{W})^\dagger = \mathcal{M}_D \tag{3.84}$$

Explicitly writing down the transformations:

$$\mathcal{W}^\dagger \mathcal{M}\mathcal{W} = v^2 \begin{pmatrix} Z_1 & \frac{1}{\sqrt{2}}Z_6 & \frac{1}{\sqrt{2}}Z_6^* \\ \frac{1}{\sqrt{2}}Z_6^* & \frac{1}{2}(Z_3 + Z_4) + Y_2/v^2 & \frac{1}{2}Z_5^* \\ \frac{1}{\sqrt{2}}Z_6 & \frac{1}{2}Z_5 & \frac{1}{2}(Z_3 + Z_4) + Y_2/v^2 \end{pmatrix} \tag{3.85}$$

$$\mathcal{RW} = \begin{pmatrix} q_{11} & \frac{1}{\sqrt{2}} q_{12}^* e^{i\theta_{23}} & \frac{1}{\sqrt{2}} q_{12} e^{-i\theta_{23}} \\ q_{21} & \frac{1}{\sqrt{2}} q_{22}^* e^{i\theta_{23}} & \frac{1}{\sqrt{2}} q_{22} e^{-i\theta_{23}} \\ q_{31} & \frac{1}{\sqrt{2}} q_{32}^* e^{i\theta_{23}} & \frac{1}{\sqrt{2}} q_{32} e^{-i\theta_{23}} \end{pmatrix},$$
$$(\mathcal{RW})^\dagger = \begin{pmatrix} q_{11} & q_{21} & q_{31} \\ \frac{1}{\sqrt{2}} q_{12} e^{-i\theta_{23}} & \frac{1}{\sqrt{2}} q_{22} e^{-i\theta_{23}} & \frac{1}{\sqrt{2}} q_{32} e^{-i\theta_{23}} \\ \frac{1}{\sqrt{2}} q_{12}^* e^{i\theta_{23}} & \frac{1}{\sqrt{2}} q_{22}^* e^{i\theta_{23}} & \frac{1}{\sqrt{2}} q_{32}^* e^{i\theta_{23}} \end{pmatrix} \tag{3.86}$$

where coefficients q_{ij} are:

$$\begin{aligned} q_{11} &= c_{13} c_{12}, & q_{12} &= -s_{12} - i c_{12} s_{13}, \\ q_{21} &= c_{13} s_{12}, & q_{22} &= c_{12} - i s_{12} s_{13}, \\ q_{31} &= s_{13}, & q_{32} &= i c_{13} \end{aligned} \tag{3.87}$$

The determinant of $\mathcal{RW}$ can be evaluated:

$$\begin{aligned} \det \mathcal{RW} &= \frac{1}{2} \left\{ q_{11} \left(q_{22}^* q_{32} - q_{22} q_{32}^* \right) + q_{21} \left(q_{12}^* q_{32} - q_{12} q_{32}^* \right) \right. \\ &\qquad \left. + q_{31} \left(q_{12}^* q_{22} - q_{12} q_{22}^* \right) \right\} \\ &= \frac{1}{2} \sum_{j,k,l=1}^{3} \varepsilon_{jkl} q_{j1} \mathbb{I}m \left(q_{k2}^* q_{l2} \right) = 1 \end{aligned} \tag{3.88}$$

Therefore the following properties arise:

$$\mathbb{R}e \left(q_{k1} q_{l1}^* + q_{k2} q_{l2}^* \right) = \delta_{kl} \tag{3.89}$$

$$\begin{aligned} \sum_{k=1}^{3} |q_{k1}|^2 = \frac{1}{2} \sum_{k=1}^{3} |q_{k2}|^2 = 1, \\ \sum_{k=1}^{3} q_{k2}^2 = \sum_{k=1}^{3} q_{k1} q_{k2} = 0 \end{aligned} \tag{3.90}$$

As one can see, the matrix $(\mathcal{RW})^\dagger$ consists only of coefficients Y and Z. The following $U(2)$ transformations are applied:

$$q_{kl} \to q_{kl}, \qquad e^{i\theta_{23}} \to e^{-i\chi} e^{i\theta_{23}} \tag{3.91}$$

It is possible to introduce another diagonalizing matrix $\tilde{\mathcal{R}} = \mathcal{R}_{12} \mathcal{R}_{13}$ which only depends on angles θ_{12} and θ_{13} by setting $\theta_{23} = 0$. This matrix is:

$$\begin{aligned}
\tilde{\mathcal{R}} &= \mathcal{R}_{12}\mathcal{R}_{13}\mathcal{R}_{23} \\
&= \begin{pmatrix} c_{12} & -s_{12} & 0 \\ s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_{13} & 0 & -s_{13} \\ 0 & 1 & 0 \\ s_{13} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \\
&= \begin{pmatrix} c_{12}c_{13} & -s_{12} & -c_{12}s_{13} \\ c_{13}s_{12} & c_{12} & -s_{12}s_{13} \\ s_{13} & 0 & c_{13} \end{pmatrix}
\end{aligned} \tag{3.92}$$

The diagonal neutral Higgs squared-mass matrix is then:

$$\begin{gathered}
\tilde{\mathcal{R}}\ \mathrm{diag}\left(m_1^2, m_2^2, m_3^2\right) \tilde{\mathcal{R}}^T = \\
\begin{pmatrix} m_3^2 s_{13}^2 + (m_2^2 s_{12}^2 + m_1^2 c_{12}^2) c_{13}^2 & (m_2^2 - m_1^2) s_{12} c_{12} c_{13} & (m_3^2 - (m_2^2 s_{12}^2 + m_1^2 c_{12}^2)) s_{13} c_{13} \\ (m_2^2 - m_1^2) s_{12} c_{12} c_{13} & m_2^2 c_{12}^2 + m_1^2 s_{12}^2 & -(m_2^2 - m_1^2) s_{12} c_{12} s_{13} \\ (m_3^2 - (m_2^2 s_{12}^2 + m_1^2 c_{12}^2)) s_{13} c_{13} & -(m_2^2 - m_1^2) s_{12} c_{12} s_{13} & m_3^2 c_{13}^2 - (m_2^2 s_{12}^2 + m_1^2 c_{12}^2) s_{13}^2 \end{pmatrix}
\end{gathered} \tag{3.93}$$

By comparing eq.(3.77) and eq.(3.93) we get the following equations:

$$\begin{aligned}
v^2 \mathbb{R}e(Z_5) &= (m_2^2 c_{12}^2 + m_1^2 s_{12}^2) - m_3^2 c_{13}^2 + (m_2^2 s_{12}^2 + m_1^2 c_{12}^2) s_{13}^2, \\
v^2 \mathbb{I}m(Z_5) &= 2(m_2^2 - m_1^2) s_{12} c_{12} s_{13}, \\
v^2 \mathbb{R}e(Z_6) &= (m_2^2 - m_1^2) s_{12} c_{12} c_{13}, \\
v^2 \mathbb{I}m(Z_6) &= -(m_3^2 - (m_2^2 s_{12}^2 + m_1^2 c_{12}^2)) s_{13} c_{13}, \\
v^2 Z_1 &= m_3^2 s_{13}^2 + (m_2^2 s_{12}^2 + m_1^2 c_{12}^2) c_{13}^2, \\
v^2 Z_4 &= (m_2^2 c_{12}^2 + m_1^2 s_{12}^2) + m_3^2 c_{13}^2 - (m_2^2 s_{12}^2 + m_1^2 c_{12}^2) s_{13}^2 - 2m_{H^\pm}^2
\end{aligned} \tag{3.94}$$

In our case we are interested in the real parameters only. Therefore $\mathbb{I}m(Z_5)$ and $\mathbb{I}m(Z_6)$ are zero. This is possible by setting $s_{13} = 0$. By assumption the following limits were set $-\pi \leq \theta_{13} < \pi$. This yields the only possible result: $\theta_{13} = -\pi$. The diagonal neutral Higgs squared-mass matrix in the real basis becomes:

$$\begin{pmatrix} (m_2^2 s_{12}^2 + m_1^2 c_{12}^2) & (m_1^2 - m_2^2) s_{12} c_{12} & 0 \\ (m_1^2 - m_2^2) s_{12} c_{12} & (m_2^2 c_{12}^2 + m_1^2 s_{12}^2) & 0 \\ 0 & 0 & m_3^2 \end{pmatrix} \tag{3.95}$$

where only m_1 and m_2 are now mixed.

Another possibility is to write down mass eigenstates in terms of the generic fields [50].

First of all neutral fields are shifted by:

$$\bar{\Phi}_i^0 \equiv \Phi_i^0 - \frac{v\hat{v}_i}{\sqrt{2}} \tag{3.96}$$

Therefore the mixing between non charged sector is:

$$h_k = \frac{1}{\sqrt{2}}\left[\bar{\Phi}_{\bar{i}}^{0\dagger}\left(q_{k1}\hat{v}_i + q_{k2}\hat{w}_i e^{-i\theta_{23}}\right) + \left(q_{k1}^*\hat{v}_{\bar{i}}^* + q_{k2}^*\hat{w}_{\bar{i}}^* e^{i\theta_{23}}\right)\bar{\Phi}_i^0\right] \tag{3.97}$$

here coefficients q_{kj} are the neutral angles θ_{12} and θ_{13} mixing parameters. They are presented in tab.4.

Table 4: The $U(2)$ invariants q_{kl} as functions of mixing angles θ_{12} and θ_{23}

k	q_{k1}	q_{k2}
1	$c_{12}c_{13}$	$-s_{12} - ic_{12}s_{13}$
2	$s_{12}c_{13}$	$c_{12} - is_{12}s_{13}$
3	s_{13}	ic_{13}
4	i	0

Inverting eq.(3.97) yields the doublet:

$$\Phi_i = \begin{pmatrix} G^+\hat{v}_i + H^+\hat{w}_i \\ \frac{1}{\sqrt{2}}\left(\hat{v}_i + q_{k1}\hat{v}_i + q_{k2}e^{-i\theta_{23}}\hat{w}_i\right)h_k \end{pmatrix} \tag{3.98}$$

This is $U(2)$-covariant form in terms of the Higgs mass eigenstate.

Also we include transformations from the Higgs mass eigenstates basis to the generic. In case of the CP-conserving 2HDM:

$$\begin{aligned}
\lambda_1 &= \left(m_H^2 c_\alpha^2 + m_h^2 s_\alpha^2 - m_{12}^2 t_\beta\right)\frac{1}{v^2 c_\beta^2} - \frac{3}{2}\lambda_6 t_\beta + \frac{1}{2}\lambda_7 t_\beta^3, \\
\lambda_2 &= \left(m_H^2 s_\alpha^2 + m_h^2 c_\alpha^2 - m_{12}^2 t_\beta^{-1}\right)\frac{1}{v^2 s_\beta^2} + \frac{1}{2}\lambda_6 t_\beta^{-3} - \frac{3}{2}\lambda_7 t_\beta^{-3}, \\
\lambda_3 &= \left((m_H^2 - m_h^2)s_\alpha c_\alpha + 2m_{H^\pm}^2 s_\beta c_\beta - m_{12}^2\right)\frac{1}{v^2 s_\beta c_\beta} - \frac{1}{2}\lambda_6 t_\beta^{-1} - \frac{1}{2}\lambda_7 t_\beta, \\
\lambda_4 &= \left((m_A^2 - 2m_{H^\pm}^2)s_\beta c_\beta + m_{12}^2\right)\frac{1}{v^2 s_\beta c_\beta} - \frac{1}{2}\lambda_6 t_\beta^{-1} - \frac{1}{2}\lambda_7 t_\beta, \\
\lambda_5 &= \left(m_{12}^2 - m_A^2 s_\beta c_\beta\right)\frac{1}{v^2 s_\beta c_\beta} - \frac{1}{2}\lambda_6 t_\beta^{-1} - \frac{1}{2}\lambda_7 t_\beta, \\
m_{22}^2 &= -\frac{1}{2}\left(m_h^2 c_\alpha s_{\beta\alpha} + m_H^2 c_\alpha c_{\beta\alpha}\right) + m_{12}^2 t_\beta^{-1}
\end{aligned} \tag{3.99}$$

In case of the non CP-conserving case we would get several additional parameters, which cannot be uniquely defined: λ_6 and λ_7.

3.5 Interactions with Bosons

We use Haber notation [50] eq. (3.98) to construct the Lagrangian. The product of two Higgs doublets is:

$$\begin{aligned}
\Phi_{\bar{i}}^{\dagger}\Phi_j =& \frac{1}{2}v^2 V_{j\bar{i}} + v h_k \left[V_{j\bar{i}} \mathbb{R}e(q_{k1}) + \frac{1}{2}\left(\hat{v}_j \hat{w}_{\bar{i}}^* q_{k2}^* e^{i\theta_{23}} + \hat{v}_{\bar{i}}^* \hat{w}_j q_{k2} e^{-i\theta_{23}} \right) \right] \\
&+ \frac{1}{2} h_j h_k \left[V_{j\bar{i}} \mathbb{R}e(q_{j1}^* q_{k1}) + W_{j\bar{i}} \mathbb{R}e(q_{j2}^* q_{k2}) + \hat{v}_j \hat{w}_{\bar{i}}^* q_{j2}^* q_{k1} e^{i\theta_{23}} + \hat{v}_{\bar{i}}^* \hat{w}_j q_{j1}^* q_{k2} e^{-i\theta_{23}} \right] \\
&+ G^+ G^- V_{j\bar{i}} + H^+ H^- W_{j\bar{i}} + G^- H^+ \hat{v}_{\bar{i}}^* \hat{w}_j + G^+ H^- \hat{w}_{\bar{i}}^* \hat{v}_j
\end{aligned} \tag{3.100}$$

Of particular interest are cubic couplings:

$$\begin{aligned}
V_3 =& \frac{1}{2} v h_j h_k h_l \left[q_{j1} q_{k1}^* \mathbb{R}e(q_{l1}) Z_1 + q_{j2} q_{k2}^* \mathbb{R}e(q_{l1})(Z_3 + Z_4) + \mathbb{R}e(q_{j1}^* q_{k2} q_{l2} Z_2 e^{-2i\theta_{23}}) \right. \\
& \left. \mathbb{R}e\left(\left[2q_{j1} + q_{j1}^* \right] q_{k1}^* q_{l2} Z_6 e^{-i\theta_{23}} \right) + \mathbb{R}e\left(q_{j2}^* q_{k2} q_{l2} Z_7 e^{-i\theta_{23}} \right) \right] \\
&+ v h_k G^+ G^- \left[\mathbb{R}e(q_{k1}) Z_1 + \mathbb{R}e\left(q_{k2} Z_6 e^{-i\theta_{23}} \right) \right] \\
&+ v h_k H^+ H^- \left[\mathbb{R}e(q_{k1}) Z_3 + \mathbb{R}e\left(q_{k2} Z_7 e^{-i\theta_{23}} \right) \right] \\
&+ \frac{1}{2} v h_k \left[G^- H^+ e^{i\theta_{23}} \left(q_{k2}^* Z_4 + q_{k2} e^{-2i\theta_{23}} Z_5 + 2\mathbb{R}e(q_{k1}) Z_6 e^{-i\theta_{23}} \right) + h.c. \right]
\end{aligned} \tag{3.101}$$

and quartic:

$$\begin{aligned}
V_4 =& \frac{1}{8} h_j h_k h_l h_m \Big[q_{j1} q_{k1} q_{l1}^* q_{m1}^* Z_1 + q_{j2} q_{k2} q_{l2}^* q_{m2}^* Z_2 \\
&+ 2 q_{j1} q_{k1}^* q_{l2} q_{m2}^* \left(Z_3 + Z_4 \right) + 2\mathbb{R}e\left(q_{j1}^* q_{k1}^* q_{l2} q_{m2} Z_5 e^{-2i\theta_{23}} \right) \\
&+ 4\mathbb{R}e\left(q_{j1} q_{k1}^* q_{l1}^* q_{m2} Z_6 e^{-i\theta_{23}} \right) + 4\mathbb{R}e\left(q_{j1}^* q_{k2} q_{l2} q_{m2}^* Z_7 e^{-i\theta_{23}} \right) \Big] \\
&+ \frac{1}{2} h_j h_k G^+ G^- \Big[q_{j1} q_{k1}^* Z_1 + q_{j2} q_{k2}^* Z_3 + 2\mathbb{R}e\left(q_{j1} q_{k2} Z_6 e^{-i\theta_{23}} \right) \Big] \\
&+ \frac{1}{2} h_j h_k H^+ H^- \Big[q_{j2} q_{k2}^* Z_2 + q_{j1} q_{k1}^* Z_3 + 2\mathbb{R}e\left(q_{j1} q_{k2} Z_7 e^{-i\theta_{23}} \right) \Big] \\
&+ \frac{1}{2} h_j h_k \Big\{ G^- H^+ e^{i\theta_{23}} \Big[q_{j1} q_{k2}^* Z_4 + q_{j1}^* q_{k2} Z_5 e^{-2i\theta_{23}} + q_{j1} q_{k1}^* Z_6 e^{-i\theta_{23}} \\
&+ q_{j2} q_{k2}^* Z_7 e^{-i\theta_{23}} \Big] + h.c. \Big\} + \frac{1}{2} Z_1 G^+ G^- G^+ G^- + \frac{1}{2} Z_2 H^+ H^- H^+ H^- \\
&+ \left(Z_3 + Z_4 \right) G^+ G^- H^+ H^- + \frac{1}{2} Z_5 H^+ H^+ G^- G^- + \frac{1}{2} Z_5^* H^- H^- G^+ G^+ \\
&+ G^+ G^- \left(Z_6 H^+ G^- + Z_6^* H^- G^+ \right) + H^+ H^- \left(Z_7 H^+ G^- + Z_7^* H^- G^+ \right)
\end{aligned} \tag{3.102}$$

From eq. (3.101) and eq. (3.102) one can easily identify the Higgs boson interactions in Higgs bases. Although interactions are more simple in Higgs bases but most of the time one is expected to work in the generic basis [12]. Therefore we calculated the Higgs bosons interaction terms in

[12] The simple answer is that in Higgs bases, the second 2HDM doublet does not acquire VEV.

the generic basis. WE dropped down terms proportional to the Goldstone bosons as in our case they carry no interest. We start from the trilinear couplings. First of all, the CP-even Higgs bosons are:

$$\begin{aligned}
g_{hhh} &= 3v\left(-c_\beta s_\alpha^3\lambda_1 + s_\beta c_\alpha^3\lambda_2 - \frac{1}{2}s_{2\alpha}c_{\beta+\alpha}\lambda_{345}\right), \\
g_{Hhh} &= v\left(3c_\beta c_\alpha s_\alpha^2\lambda_1 + 2s_\beta s_\alpha c_\alpha^2\lambda_2 + \left[\left(1-3s_\alpha^2\right)c_{\beta+\alpha} - s_\beta s_\alpha\right]\lambda_{345}\right), \\
g_{HHh} &= v\left(-3c_\beta s_\alpha c_\alpha^2\lambda_1 + 2s_\beta c_\alpha s_\alpha^2\lambda_2 + \left[\left(1-3s_\alpha^2\right)s_{\beta+\alpha} + c_\beta s_\alpha\right]\lambda_{345}\right), \\
g_{HHH} &= 3v\left(c_\beta c_\alpha^3\lambda_1 + s_\beta s_\alpha^3\lambda_2 + \frac{1}{2}s_{2\alpha}s_{\beta+\alpha}\lambda_{345}\right)
\end{aligned} \tag{3.103}$$

Secondly, the CP-odd involving coupling are:

$$\begin{aligned}
g_{AAh} &= v\left(-c_\beta s_\beta^2 s_\alpha\lambda_1 + s_\beta c_\beta^2 c_\alpha\lambda_2 + \left[s_\beta^3 c_\alpha - c_\beta^3 s_\alpha\right]\lambda_{345} - 2s_{\beta-\alpha}\lambda_5\right), \\
g_{AAH} &= v\left(c_\beta s_\beta^2 c_\alpha\lambda_1 + s_\beta c_\beta^2 s_\alpha\lambda_2 + \left[c_\beta^3 c_\alpha + s_\beta^3 s_\alpha\right]\lambda_{345} - 2c_{\beta-\alpha}\lambda_5\right)
\end{aligned} \tag{3.104}$$

Finally, the charged sector yields the following result:

$$\begin{aligned}
g_{hH^\pm H^\mp} &= \frac{s_{2\beta}v}{2}\left(s_\beta s_\alpha\lambda_1 - c_\beta c_\alpha\lambda_2 + c_{\beta+\alpha}\lambda_{345}\right) - s_{\beta-\alpha}\lambda_3, \\
g_{HH^\pm H^\mp} &= \frac{s_{2\beta}v}{2}\left(s_\beta c_\alpha\lambda_1 + c_\beta s_\alpha\lambda_2 - s_{\beta+\alpha}\lambda_{345}\right) + c_{\beta-\alpha}\lambda_3
\end{aligned} \tag{3.105}$$

The following interactions are forbidden:

$$g_{AAA} = g_{Ahh} = g_{AHH} = g_{AHh} = g_{aH^\pm H^\mp} = 0 \tag{3.106}$$

Now we turn to quartic interactions. From the CP-even sector we get:

$$\begin{aligned}
g_{hhhh} &= 3\left(s_\alpha^4\lambda_1 + c_\alpha^4\lambda_2 + \frac{1}{2}s_{2\alpha}^2\lambda_{345}\right), \\
g_{hhhH} &= \frac{3}{2}s_{2\alpha}\left(-s_\alpha^2\lambda_1 + c_\alpha^2\lambda_2 - c_{2\alpha}\lambda_{345}\right), \\
g_{hhHH} &= \frac{3}{4}s_{2\alpha}^2\left(\lambda_1+\lambda_2\right) + \left(1 - \frac{3}{2}s_{2\alpha}^2\right)\lambda_{345}, \\
g_{hHHH} &= \frac{3}{2}s_{2\alpha}\left(-c_\alpha^2\lambda_1 + s_\alpha^2\lambda_2 + c_{2\alpha}\lambda_{345}\right), \\
g_{HHHH} &= 3\left(c_\alpha^4\lambda_1 + s_\alpha^4\lambda_2 + \frac{1}{2}s_{2\alpha}^2\lambda_{345}\right)
\end{aligned} \tag{3.107}$$

CP-odd quartic couplings:

$$\begin{aligned}
g_{hhAA} &= s_\beta^2 s_\alpha^2 \lambda_1 + c_\beta^2 c_\alpha^2 \lambda_2 + \left(c_\beta^2 s_\alpha^2 + s_\beta^2 c_\alpha^2\right) \lambda_{345} - \left(1 - c_{2(\beta-\alpha)}\right) \lambda_5, \\
g_{hHAA} &= \frac{1}{2} s_{2\alpha} \left(-s_\beta^2 \lambda_1 + c_\beta^2 \lambda_2 - c_{2\beta} \lambda_{345}\right) - s_{2(\beta-\alpha)} \lambda_5, \\
g_{HHAA} &= s_\beta^2 c_\alpha^2 \lambda_1 + c_\beta^2 s_\alpha^2 \lambda_2 + \left(c_\beta^2 c_\alpha^2 + s_\beta^2 s_\alpha^2\right) \lambda_{345} - \left(1 + c_{2(\beta-\alpha)}\right) \lambda_5, \\
g_{AAAA} &= 3 \left(s_\beta^4 \lambda_1 + c_\beta^4 \lambda_2 + \frac{1}{2} s_{2\beta}^2 \lambda_{345} \right)
\end{aligned} \tag{3.108}$$

Charged Higgs quartic couplings:

$$\begin{aligned}
g_{hhH^\pm H^\mp} &= s_\beta^2 s_\alpha^2 \lambda_1 + c_\beta^2 c_\alpha^2 \lambda_2 + \frac{1}{2} \left(1 - c_{2(\beta-\alpha)}\right) \lambda_3 + \frac{1}{2} s_{2\beta} s_{2\alpha} \lambda_{345}, \\
g_{hHH^\pm H^\mp} &= \frac{1}{2} s_{2\alpha} \left(-s_\beta^2 \lambda_1 + c_\beta^2 \lambda_2\right) + \frac{1}{2} s_{2(\beta-\alpha)} \lambda_3 - \frac{1}{2} s_{2\beta} c_{2\alpha} \lambda_{345}, \\
g_{HHH^\pm H^\mp} &= s_\beta^2 c_\alpha^2 \lambda_1 + c_\beta^2 s_\alpha^2 \lambda_2 + \frac{1}{2} \left(1 + c_{2(\beta-\alpha)}\right) \lambda_3 - \frac{1}{2} s_{2\beta} s_{2\alpha} \lambda_{345}, \\
g_{AAH^\pm H^\mp} &= s_\beta^4 \lambda_1 + c_\beta^4 \lambda_2 + \frac{1}{2} s_{2\beta}^2 \lambda_{345}, \\
g_{H^\pm H^\mp H^\pm H^\mp} &= 2 \left(s_\beta^4 \lambda_1 + c_\beta^4 \lambda_2 + \frac{1}{2} s_{2\beta}^2 \lambda_{345} \right)
\end{aligned} \tag{3.109}$$

The following interactions are forbidden:

$$g_{hAAA} = g_{HAAA} = g_{hhhA} = g_{hhHA} = g_{hHHA} = g_{HHHA} = g_{hAH^\pm H^\mp} = g_{HAH^\pm H^\mp} = 0 \tag{3.110}$$

Finally we discuss interactions between Higgs bosons and gauge bosons. The procedure is the same as for the SM Higgs sector. These type of couplings arise from the kinematic terms. The covariant derivative in the 2HDM is:

$$D_\mu \Phi_i = \begin{pmatrix} \partial_\mu \Phi_i^+ + \left[\frac{ig}{c_W} \left(\frac{1}{2} - s_W^2 \right) Z_\mu + ieA_\mu \right] \Phi_i^+ + \frac{ig}{\sqrt{2}} W_\mu^+ \Phi_i^0 \\ \partial_\mu \Phi_i^0 - \frac{ig}{2c_W} Z_\mu \Phi_i^0 + \frac{ig}{\sqrt{2}} W_\mu^- \Phi_i^+ \end{pmatrix} \tag{3.111}$$

By inserting this into the kinematic part of the 2HDM Lagrangian we get the following interaction terms:

$$\begin{aligned}
\mathcal{L}_{VVH} = &\left(g m_W W_\mu^+ W^{\mu -} + \frac{g}{2c_W} m_Z Z_\mu Z^\mu \right) \mathbb{R}e\left(q_{k1}\right) h_k \\
&+ e m_W A^\mu \left(W_\mu^+ G^- + W_\mu^- G^+ \right) - g m_Z s_W^2 Z^\mu \left(W_\mu^+ G^- + W_\mu^- G^+ \right)
\end{aligned} \tag{3.112}$$

$$\begin{aligned}
\mathcal{L}_{VVHH} =& \left[\frac{1}{4}g^2 W_\mu^+ W^{\mu-} + \frac{g^2}{8c_W^2} Z_\mu Z^\mu\right] \mathbb{R}e\left(q_{j1}^* q_{k1} + q_{j2}^* q_{k2}\right) h_j h_k \\
&+ \left[\frac{1}{2}g^2 W_\mu^+ W^{\mu-} + e^2 A_\mu A^\mu + \frac{g^2}{c_W^2}\left(\frac{1}{2} - s_W^2\right)^2 Z_\mu Z^\mu \right. \\
&\left. + \frac{2ge}{c_W}\left(\frac{1}{2} - s_W^2\right) A_\mu Z^\mu\right] \left(G^+ G^- + H^+ H^-\right) \\
&+ \left\{\left(\frac{1}{2} eg A^\mu W_\mu^+ - \frac{g^2 s_W^2}{2c_W} Z^\mu W_\mu^+\right)\left(q_{k1} G^- + q_{k2} e^{-i\theta_{23}} H^-\right) h_k + h.c.\right\}
\end{aligned} \tag{3.113}$$

$$\begin{aligned}
\mathcal{L}_{VHH} =& \frac{g}{4c_W} \mathbb{I}m\left(q_{j1} q_{k1}^* + q_{j2} q_{k2}^*\right) Z^\mu h_j \overleftrightarrow{\partial}_\mu h_k \\
&- \frac{1}{2} d\left\{iW_\mu^+\left[q_{k1} G^- \overleftrightarrow{\partial}_\mu h_k + q_{k2} e^{-i\theta_{23}} H^- \overleftrightarrow{\partial}_\mu h_k\right] + h.c.\right\} \\
&+ \left[ieA^\mu + \frac{ig}{c_W}\left(\frac{1}{2} - s_W^2\right) Z^\mu\right]\left(G^+ \overleftrightarrow{\partial}_\mu G^- + H^+ \overleftrightarrow{\partial}_\mu H^-\right)
\end{aligned} \tag{3.114}$$

3.6 Interactions with Fermions

In contrast to the SM Yukawa Lagrangian, the 2HDM Yukawa Lagrangian brings some interesting properties. The main difference is that in the most general Yukawa Lagrangian the flavour changing neutral currents arise (FCNC) at the tree-level [51–53]. Not much is know about FCNC from experimental point of view and thus in most theories there are restrictions on the Higgs-fermions couplings. Nevertheless FCNC effects are phenomenologically accepted.

Another interesting property of the 2HDM is that both doublets can acquire the VEV. This would be the key to the problem of fermion masses. It is still unknown why there are three generations and masses of different families do not coincide: $m_t/m_b \approx 41$. One of the possible solutions is that up quarks couple to one doublet while the down type quarks couple to another doublet.

The most general gauge invariant 2HDM Yukawa Lagrangian without the neutrino sector is [54]:

$$\begin{aligned}
-\mathcal{L}_Y =& \kappa_{ij}^{U,0} \overline{Q_{iL}^0} \tilde{\Phi}_1 U_{jR}^0 + \kappa_{ij}^{D,0} \overline{Q_{il}^0} \Phi_1 D_{jR}^0 + \kappa_{ij}^{E,0} \overline{l_{il}^0} \Phi_1 E_{jR}^0 \\
&+ \rho_{ij}^{U,0} \overline{Q_{iL}^0} \tilde{\Phi}_1 U_{jR}^0 + \rho_{ij}^{D,0} \overline{Q_{il}^0} \Phi_2 D_{jR}^0 + \rho_{ij}^{E,0} \overline{l_{il}^0} \Phi_2 E_{jR}^0 + h.c.
\end{aligned} \tag{3.115}$$

where $\tilde{\Phi}_i = i\sigma_2 \Phi_i$ and $\kappa,\ \rho$ are Yukawa matrices, which are arbitrary. As in the SM Yukawa Lagrangian, the superscript 0 shows that the fermion fields are in the weak basis. We restrict the further talk only to the quarks sector. One can get the lepton sector straightforward from the quark sector.

There are several mechanisms which can suppress FCNC. The most widely accepted one is by Glashow and Weinberg [51]. They implemented a discrete symmetry that automatically forbids FCNC processes. Another mechanism was proposed by Cheng and Sher [55]. This mechanism fixes the values of Yukawa matrices by squares of corresponding masses. We will treat only the first mechanism, proposed by Glashow and Weinberg.

Yukawa matrices κ, ρ cannot be diagonalized simultaneously. In order to suppress the FCNC there should be a way to get rid of one of the Yukawa matrices. This can be achieved by the following discrete symmetry:

$$\begin{aligned}&\Phi_1 \to \Phi_1 \text{ and } \Phi_2 \to -\Phi_2, \\ &D_{jR} \to \pm D_{jR} \text{ and } U_{jR} \to -U_{jR}\end{aligned} \tag{3.116}$$

By demanding this we get two cases. If we add the leptonic sector to the model then there are four cases. Let us take a look at this two cases. The first one is the 2HDM type I. In this case Φ_1 decouples from the Yukawa sector. This implies that only one doublet gives masses to up and down quarks. In contrast, in the Type II 2HDM Φ_1 couples to the down sector while Φ_2 couples to the up sector.

The corresponding 2HDM Type I sector is:

$$-\mathcal{L}_Y = \xi_{ij}^{U,0}\overline{Q_{iL}^0}\tilde{\Phi}_2 U_{jR}^0 + \xi_{ij}^{D,0}\overline{Q_{iL}^0}\Phi_2 D_{jR} + \text{leptonic sector} + h.c. \tag{3.117}$$

In terms of the mass eigenstates:

$$\begin{aligned}-\mathcal{L}_Y =& \frac{g}{2m_W s_\beta}\overline{D}\hat{\mathcal{M}}_D D\left(s_\alpha H + c_\alpha h\right) + \frac{ig}{2m_W t_\beta}\overline{D}\hat{\mathcal{M}}_D\gamma_5 DA \\ &+ \frac{g}{2m_W s_\beta}\overline{U}\hat{\mathcal{M}}_U U\left(s_\alpha H + c_\alpha h\right) - \frac{ig}{2m_W t_\beta}\overline{U}\hat{\mathcal{M}}_U\gamma_5 UA \\ &+ \frac{g}{\sqrt{2}m_W t_\beta}\overline{U}\left(V_{CKM}\hat{\mathcal{M}}_D P_R - \hat{\mathcal{M}}_U V_{CKM} P_L\right) DH^+ \\ &+ \text{leptonic sector} + h.c.\end{aligned} \tag{3.118}$$

In the case of 2HDM Type II:

$$-\mathcal{L}_Y = \kappa_{ij}^{D,0}\overline{Q_{iL}^0}\Phi_1 D_{jR}^0 + \xi_{ij}^{U,0}\overline{Q_{iL}^0}\tilde{\Phi}_2 U_{jR} + \text{leptonic sector} + h.c. \tag{3.119}$$

In terms of the mass eigenstates:

$$\begin{aligned}-\mathcal{L}_Y =& \frac{g}{2m_W c_\beta}\overline{D}\hat{\mathcal{M}}_D D\left(c_\alpha H - s_\alpha h\right) - \frac{igt_\beta}{2m_W}\overline{D}\hat{\mathcal{M}}_D\gamma_5 DA \\ &+ \frac{g}{2m_W s_\beta}\overline{U}\hat{\mathcal{M}}_U U\left(s_\alpha H + c_\alpha h\right) - \frac{ig}{2m_W t_\beta}\overline{U}\hat{\mathcal{M}}_U\gamma_5 UA \\ &- \frac{g}{\sqrt{2}m_W}\overline{U}\left(t_\beta V_{CKM}\hat{\mathcal{M}}_D P_R + t_\beta^{-1}\hat{\mathcal{M}}_U V_{CKM} P_L\right) DH^+ \\ &+ \text{leptonic sector} + h.c.\end{aligned} \tag{3.120}$$

This type of Yukawa Lagrangian is required in the MSSM.

In case of the most general type of the 2HDM, the type III Yukawa Lagrangian eq. (3.115), all fermions can couple to both doublets unless some of the Yukawa matrices elements are zero. It is convenient to make a rotation of the doublets in such a way that only one of the doublets acquire VEV. Therefore we assume that only Φ_1 acquires non zero VEV. In terms of the mass eigenstates it is:

$$\begin{aligned}-\mathcal{L}_Y =& \frac{g}{2m_W}\overline{D}\hat{\mathcal{M}}_D D\left(c_\alpha H - s_\alpha h\right) + \frac{1}{\sqrt{2}}\overline{D}\rho^D D\left(s_\alpha H + c_\alpha h\right)\\ &+ \frac{g}{2m_W}\overline{U}\hat{\mathcal{M}}_U U\left(c_\alpha H - s_\alpha h\right) + \frac{1}{\sqrt{2}}\overline{U}\rho^U U\left(s_\alpha H + c_\alpha h\right)\\ &+ \frac{i}{\sqrt{2}}\overline{D}\rho^D\gamma_5 D A - \frac{i}{\sqrt{2}}\overline{U}\rho^U\gamma_5 U A\\ &+ \overline{U}\left(V_{CKM}\rho^D P_R - \rho^U V_{CKM} P_L\right) D H^+ + \text{leptonic sector} + h.c.\end{aligned} \tag{3.121}$$

To be more precisely, the 2HDM Yukawa Lagrangian in the Higgs mass eigenstates basis, in the basis independent form is [49]:

$$\begin{aligned}-\mathcal{L}_Y =& \frac{1}{\sqrt{2}}\overline{D}\left(\kappa^D s_{\beta-\alpha} + \rho^D c_{\beta-\alpha}\right) D h + \frac{1}{\sqrt{2}}\overline{D}\left(\kappa^D c_{\beta-\alpha} - \rho^D s_{\beta-\alpha}\right) D H\\ &+ \frac{1}{\sqrt{2}}\overline{U}\left(\kappa^U s_{\beta-\alpha} + \rho^U c_{\beta-\alpha}\right) U h + \frac{1}{\sqrt{2}}\overline{U}\left(\kappa^U c_{\beta-\alpha} - \rho^U s_{\beta-\alpha}\right) U H\\ &+ \frac{i}{\sqrt{2}}\overline{D}\rho^D\gamma_5 D A - \frac{i}{\sqrt{2}}\overline{U}\rho^U\gamma_5 U A\\ &+ \overline{U}\left(V_{CKM}\rho^D P_R - \rho^U V_{CKM} P_L\right) D H^+ + \text{leptonic sector} + h.c.\end{aligned} \tag{3.122}$$

On the other hand, if both doublets acquire VEV the type III 2HDM Yukawa Lagrangian changes. Assume that in order to convert this Lagrangian from the weak basis into the mass eigenstates basis the following transformations are needed:

$$\begin{aligned}D_{L,R} &= (V^D_{L,R})D^0_{L,R},\\ U_{L,R} &= (V^U_{L,R})U^0_{L,R}\end{aligned} \tag{3.123}$$

After the parametrization the mass matrices are:

$$\begin{aligned}\hat{\mathcal{M}}_D &= V^D_L\left(\frac{v_1}{\sqrt{2}}\tilde{\kappa}^{D,0} + \frac{v_2}{\sqrt{2}}\tilde{\rho}^{D,0}\right)V^{D\dagger}_R,\\ \hat{\mathcal{M}}_U &= V^U_L\left(\frac{v_1}{\sqrt{2}}\tilde{\kappa}^{U,0} + \frac{v_2}{\sqrt{2}}\tilde{\rho}^{U,0}\right)V^{U\dagger}_R\end{aligned} \tag{3.124}$$

There are two ways of getting the Yukawa Lagrangian. Either expressing $\tilde{\kappa}$ or $\tilde{\rho}$.

4 Constraints of the Two Higgs Doublet Model

So far we have discussed possible choices of the 2HDM basis and interactions terms with bosons and fermions. In order for model to be valid there are several theoretical constraints. In the following section the 2HDM scalar potential constraints are described. We cover only several of them, the ones we used to perform analysis of the CP-conversing 2HDM. We cover stability of the potential, S-matrix unitarity and quartic Higgs bosons perturbativity. Also we take a look at how Peskin-Takeuchi parameters are formulated in the 2HDM.

4.1 Stability of the Potential

First of all potential needs to be stable. For a stable vacuum, the 2HDM scalar potential has to be bounded from below. To fulfil this condition, the potential must be positive in field space along large values of the fields. Therefore the scalar potential has to be bounded from below. The Higgs potential (3.21) must be positive in all field space directions for asymptotically large values of the fields ref. [56]. The perform derivation of the stability condition in the generic basis.

It is possible to rewrite the 2HDM scalar potential (3.21) by applying the following parametrisation:

$$|\Phi_1| = rc_\gamma, \qquad |\Phi_2| = rs_\gamma, \qquad \frac{\Phi_2^\dagger\Phi_1}{|\Phi_1||\Phi_2|} = \rho e^{i\theta} \tag{4.1}$$

where $\gamma \in [0, {}^{\pi}/_{2}],\ \rho \in [0,1],\ \theta \in [0, 2\pi)$. After the parametrisation the scalar potential (3.21) can be written as:

$$V = r^4 V_4 + r^2 V_2 \tag{4.2}$$

where the quartic part with respect to (4.1) is:

$$\begin{aligned} V_4 =& \frac{1}{2}\lambda_1 c_\gamma^4 + \frac{1}{2}\lambda_2 s_\gamma^4 + \lambda_3 c_\gamma^2 s_\gamma^2 + \lambda_4 \rho^2 c_\gamma^2 s_\gamma^2 \\ &+ \lambda_5 \rho^2 c_\gamma^2 s_\gamma^2 c_{2\theta} + \left[\lambda_6 c_\gamma^2 + \lambda_7 s_\gamma^2\right] 2\rho c_\gamma s_\gamma c_\theta \end{aligned} \tag{4.3}$$

V_4 is positive in all directions when the following requirements are met:

$$\lambda_1 > 0, \quad \lambda_2 > 0, \quad \lambda_3 > -\sqrt{\lambda_1\lambda_2} \tag{4.4}$$

In the CP-conserving case there is an additional condition:

$$\lambda_3 + \lambda_4 - |\lambda_5| > -\sqrt{\lambda_1\lambda_2} \tag{4.5}$$

These conditions can be derived by sending some of the directions of the potential to infinity. For example, in order to get constraint of λ_1, $r^2 c_\gamma s_\gamma$ value is set to ∞. The above conditions are sufficient to ensure the EW vacuum stability.

4.2 S-Matrix Tree-Level Unitarity and Perturbativity Conditions

The unitarity constraint comes from the idea, that the scalar-scalar scattering amplitudes at the tree-level must respect unitarity. This leads to an upper bound of the potential parameters [57].

There are several possible $\xi_1\xi_2 \to \xi_3\xi_4$ scattering processes in the high energy limit. Only the scalar interactions contribute to the S matrix partial wave amplitude. There are several different possible channels. For simplicity the S matrix is split into several components as most elements are zeros. This splitting is based on a total charge of the scattering process. There are 14 neutral weak eigenstate channels:

$$\begin{aligned}
&|w_i^+ w_i^- \rangle\,,\ |w_1^+ w_2^- \rangle\,,\ |w_2^+ w_1^- \rangle\,,\ \frac{1}{\sqrt{2}}|z_i z_i \rangle\,,\ \frac{1}{\sqrt{2}}|h_i h_i \rangle\,,\\
&|h_i z_i \rangle\,,\ |z_1 z_2 \rangle\,,\ |h_1 h_2 \rangle\,,\ |h_1 z_2 \rangle\,,\ |h_2 z_1 \rangle
\end{aligned} \tag{4.6}$$

8 singly charged scattering processes:

$$\begin{aligned}
&|w_i^+ z_i \rangle\,,\ |w_1^+ z_2 \rangle\,,\ |w_2^+ z_1 \rangle\,,\\
&|w_i^+ h_i \rangle\,,\ |w_1^+ h_2 \rangle\,,\ |w_2^+ h_1 \rangle
\end{aligned} \tag{4.7}$$

3 doubly charged scattering processes:

$$\frac{1}{\sqrt{2}}|w_1^+ w_1^+ \rangle\,,\ \frac{1}{\sqrt{2}}|w_2^+ w_2^+ \rangle\,,\ |w_1^+ w_2^+ \rangle \tag{4.8}$$

It turns out that all of these scattering processes can be written down with only 4 S matrices. We get the following S matrices:

$$\begin{aligned}
S_{(0,0)} &= \begin{pmatrix} 3\lambda_1 & 2\lambda_3+\lambda_4 & 3\lambda_6 & 3\lambda_6^* \\ 2\lambda_3+\lambda_4 & 3\lambda_2 & 3\lambda_7 & 3\lambda_7^* \\ 3\lambda_6^* & 3\lambda_7^* & \lambda_3+2\lambda_4 & 3\lambda_5^* \\ 3\lambda_6 & 3\lambda_7 & 3\lambda_5 & \lambda_3+2\lambda_4 \end{pmatrix}, \quad S_{(2,0)} = \lambda_3 - \lambda_4,\\
S_{(0,1)} &= \begin{pmatrix} \lambda_1 & \lambda_4 & \lambda_6 & \lambda_6^* \\ \lambda_4 & \lambda_2 & \lambda_7 & \lambda_7^* \\ \lambda_6^* & \lambda_7^* & \lambda_3 & \lambda_5^* \\ \lambda_6 & \lambda_7 & \lambda_5 & \lambda_3 \end{pmatrix}, \quad S_{(2,1)} = \begin{pmatrix} \lambda_1 & \lambda_5 & \sqrt{2}\lambda_6 \\ \lambda_5^* & \lambda_2 & \sqrt{2}\lambda_7^* \\ \sqrt{2}\lambda_6^* & \sqrt{2}\lambda_7 & \lambda_3+\lambda_4 \end{pmatrix}
\end{aligned} \tag{4.9}$$

In a CP-conserving case they are simplified to:

$$
\begin{aligned}
S_{(2,1)} &= \begin{pmatrix} \lambda_1 & \lambda_5 & 0 \\ \lambda_5 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3+\lambda_4 \end{pmatrix}, \\
S_{(2,0)} &= \lambda_3 - \lambda_4, \\
S_{(0,1)} &= \begin{pmatrix} \lambda_1 & \lambda_4 & 0 & 0 \\ \lambda_4 & \lambda_2 & 0 & 0 \\ 0 & 0 & \lambda_3 & \lambda_5 \\ 0 & 0 & \lambda_5 & \lambda_3 \end{pmatrix}, \\
S_{(0,0)} &= \begin{pmatrix} 3\lambda_1 & 2\lambda_3+\lambda_4 & 0 & 0 \\ 2\lambda_3+\lambda_4 & 3\lambda_2 & 0 & 0 \\ 0 & 0 & \lambda_3+2\lambda_4 & 3\lambda_5 \\ 0 & 0 & 3\lambda_5 & \lambda_3+2\lambda_4 \end{pmatrix}
\end{aligned}
\tag{4.10}
$$

Eigenvalues of the corresponding S matrices are:

$$
\begin{aligned}
&\Lambda^{even}_{21\pm} = \frac{1}{2}\left(\lambda_1+\lambda_2 \pm \sqrt{(\lambda_1-\lambda_2)^2 + 4|\lambda_5|^2}\right), \qquad \Lambda^{odd}_{21} = \lambda_3+\lambda_4, \\
&\Lambda^{odd}_{20} = \lambda_3 - \lambda_4, \\
&\Lambda^{even}_{01\pm} = \frac{1}{2}\left(\lambda_1+\lambda_2 \pm \sqrt{(\lambda_1-\lambda_2)^2 + 4\lambda_4^2}\right), \qquad \Lambda^{odd}_{01\pm} = \lambda_3 \pm |\lambda_5|, \\
&\Lambda^{even}_{00\pm} = \frac{1}{2}\left(3\lambda_1+3\lambda_2 \pm \sqrt{9\left(\lambda_1-\lambda_2\right)^2 + 4\left(2\lambda_3+\lambda_4\right)^2}\right), \qquad \Lambda^{odd}_{00\pm} = \lambda_3+2\lambda_4 \pm 3|\lambda_5|
\end{aligned}
\tag{4.11}
$$

In a non partial-wave expansion we limited the eigenvalues of the scattering matrices by 16π. A note should be taken that if the 2HDM tree-level amplitude grows with energy then unitarity condition is violated at higher energies.

Also there exist quartic Higgs perturbativity constraints. These constraints can be applied by demanding that quartic Higgs couplings fulfil:

$$
|g_{h_i h_j h_k h_l}| \leq 4\pi \tag{4.12}
$$

where quartic Higgs couplings are determined in eq. (3.107-3.109). By applying the perturbativity conditions we restrict the value of t_β.

4.3 Peskin-Takeuchi Parameters

In this section we cover 2HDM scalar potential constraints by applying Peskin-Takeuchi parameters. We only check S, T and U parameters. We inspect these values directly without correlation coefficients between these parameters. Analysis was performed in the following linear approximation form [58]:

$$\begin{aligned}
\alpha T &\equiv \frac{A_{WW}(0)}{m_W^2} - \frac{A_{ZZ}(0)}{m_Z^2}, \\
\frac{g^2}{16\pi} S &\equiv c_W^2 \left[F_{ZZ}\left(m_Z^2\right) - F_{\gamma\gamma}\left(m_Z^2\right) + \left(\frac{2s_W^2 - 1}{s_W c_W}\right) F_{Z\gamma}\left(m_Z^2\right) \right], \\
\frac{g^2}{16\pi}(S+U) &\equiv F_{WW}(m_W^2) - F_{\gamma\gamma}(m_W^2) - \frac{c_W}{s_W} F_{Z\gamma}(m_W^2)
\end{aligned} \tag{4.13}$$

Due to interactions of the 2HDM Higgs bosons with gauge bosons there arise additional contribution. For simplicity we use the following two-point loop functions [59]:

$$\begin{aligned}
\mathcal{B}_{22}(q^2;\ m_1^2,\ m_2^2) &\equiv B_{22}(q^2;\ m_1^2,\ m^2) - B_{22}(0;\ m_1^2,\ m^2), \\
\mathcal{B}_0(q^2;\ m_1^2,\ m_2^2) &\equiv B_0(q^2;\ m_1^2,\ m^2) - B_0(0;\ m_1^2,\ m^2)
\end{aligned} \tag{4.14}$$

where

$$\begin{aligned}
B_{22}(q^2;\ m_1^2,\ m_2^2) &= \frac{1}{4}(\Delta+1)\left[m_1^2 + m_2^2 - \frac{1}{3}q^2\right] - \frac{1}{2}\int_0^1 dx X \ln(X - i\epsilon), \\
B_0(q^2;\ m_1^2,\ m_2^2) &= \Delta - \int_0^1 dx \ln(X - i\epsilon)
\end{aligned} \tag{4.15}$$

In d space time dimension coefficients X and Δ are:

$$\begin{aligned}
X &\equiv m_1^2 x + m_2^2(1-x) - q^2 x(1-x), \\
\Delta &\equiv \frac{2}{4-d} + \ln(4\pi) - \gamma
\end{aligned} \tag{4.16}$$

We skip the derivation of the Peskin-Takeuchi parameters. Explicit derivation of S, T and U parameters is presented in ref. [54].

Next we discuss contribution from the 2HDM CP-conserving case to the Peskin-Takeuchi parameters. The 2HDM contribution to T is:

$$\begin{aligned}
T =& \frac{1}{16\pi m_W^2 s_W^2}\Big\{|q_{k2}|^2 F(m_{H^\pm}^2,\ m_k^2) - q_{21}^2 F(m_1^2,\ m_3^2) \\
&- q_{11}^2 F(m_2^2,\ m_3^2) - q_{31}^2 F(m_1^2,\ m_2^2) \\
&+ q_{k1}^2\left[F(m_W^2,\ m_k^2) - F(m_Z^2,\ m_k^2)\right] \\
&+ 4m_W^2 B_0(0;\ m_W^2,\ m_\phi^2) - 4m_Z^2 B_0(0;\ m_Z^2,\ m_\phi^2) \\
&- 4q_{k1}^2\left[m_W^2 B_0(0;\ m_W^2,\ m_k^2) - m_Z^2 B_0(0;\ m_Z^2,\ m_k^2)\right] \\
&+ F(m_Z^2,\ m_\phi^2) - F(m_W^2,\ m_\phi^2)\Big\}
\end{aligned} \tag{4.17}$$

where

$$F(m_1^2,\ m_2^2) \equiv \frac{1}{2}(m_1^2 + m_2^2) - \frac{m_1^2 m_2^2}{m_1^2 - m_2^2} \ln(\frac{m_1^2}{m_2^2}) \tag{4.18}$$

In case of $m_i = m_j$ we get $F(m^2,\ m^2) = 0$. This can be simplified to:

$$\begin{aligned} T =& \frac{g^2}{64\pi^2 m_W^2} \Bigg\{ \sum_{k=1}^{3} |C_k|^2 F(m_{H^\pm}^2, m_{H_k}^2) - \sum_{k=1}^{2} |C_k|^2 F(m_{H_k}^2, m_A^2) \\ & + 3 \sum_{k=1}^{2} |C_{3-k}|^2 \left[F(m_Z^2, m_{H_k}^2) - F(m_W^2, m_{H_k}^2) \right] \\ & - 3 \left[F(m_Z^2, m_h^2) - F(m_W^2, m_h^2) \right] \Bigg\} \end{aligned} \tag{4.19}$$

where

$$C = \{c_{\beta-\alpha}, s_{\beta-\alpha}, 1\} \tag{4.20}$$

The 2HDM contribution to S is:

$$\begin{aligned} S =& \frac{1}{\pi m_Z^2} \big[q_{k1}^2 \mathcal{B}_{22}(m_Z^2;\ m_Z^2,\ m_k^2) - \mathcal{B}_{22}(m_Z^2;\ m_Z^2,\ m_\phi^2) \\ & - m_Z^2 q_{k1}^2 \mathcal{B}_0(m_Z^2;\ m_Z^2,\ m_k^2) + m_Z^2 \mathcal{B}_0(m_Z^2;\ m_Z^2,\ m_\phi^2) \\ & + q_{11}^2 \mathcal{B}_{22}(m_Z^2;\ m_2^2,\ m_3^2) + q_{21}^2 \mathcal{B}_{22}(m_Z^2;\ m_1^2,\ m_3^2) \\ & + q_{31}^2 \mathcal{B}_{22}(m_Z^2;\ m_1^2,\ m_2^2) - \mathcal{B}_{22}(m_Z^2;\ m_{H^\pm}^2,\ m_{H^\pm}^2) \end{aligned} \tag{4.21}$$

In the 2HDM U is defined in the following way:

$$\begin{aligned} S + U =& \frac{1}{\pi m_W^2} \big[- q_{k1}^2 m_W^2 \mathcal{B}_0(m_W^2;\ m_W^2,\ m_k^2) + m_W^2 \mathcal{B}_0(m_W^2;\ m_W^2,\ m_\phi^2) \\ & - \mathcal{B}_{22}(m_W^2;\ m_W^2,\ m_\phi^2) + q_{k1}^2 \mathcal{B}_{22}(m_W^2;\ m_W^2,\ m_k^2) \\ & + |q_{k2}|^2 \mathcal{B}_{22}(m_W^2;\ m_{H^\pm}^2,\ m_k^2) - 2\mathcal{B}_{22}(m_W^2;\ m_{H^\pm}^2,\ m_{H^\pm}^2) \big] \end{aligned} \tag{4.22}$$

S and U parameters can also be simplified. Before that additional functions should be introduced. This functions relates masses of two Higgs bosons above the scale Q, which corresponds to masses m_W or m_Z:

$$\begin{aligned} G(x, y, Q) =& -\frac{16}{3} + 5\frac{x+y}{Q} - 2\frac{(x-y)^2}{Q^2} \\ & + \frac{3}{Q} \left[\frac{x^2+y^2}{x-y} - \frac{x^2-y^2}{Q} + \frac{(x-y)^3}{3Q^2} \right] \ln\frac{x}{y} + \frac{r}{Q^3} f(t, r) \end{aligned} \tag{4.23}$$

where $t \equiv x + y - Q,\ r \equiv Q^2 - 2Q(x+y) + (x-y)^2$ and $f(t,r)$ is:

$$f(t,r) = \begin{cases} \sqrt{r}\ln\left|\frac{t-\sqrt{r}}{t+\sqrt{r}}\right| & r > 0 \\ 0 & r = 0 \\ 2\sqrt{-r}\arctan\frac{\sqrt{-r}}{t} & r < 0 \end{cases} \tag{4.24}$$

Another function is:

$$\hat{G}(x,Q) = G(x,Q,Q) + \left[\frac{x}{Q} - \frac{x+Q}{x-Q} - 1\right]\ln\frac{x}{Q} + \frac{1}{Q}f(t,r) - 24 \tag{4.25}$$

In terms of functions $G(x,y,Q)$ and $\hat{G}(x,Q)$, the S parameter is:

$$\begin{aligned} S =& \frac{g^2}{384\pi^2 c_W^2}\Bigg\{[s_W^2 - c_W^2]^2 G(m_{H^\pm}^2, m_{H^\pm}^2, m_Z^2) + \sum_{k=1}^{2}|C_k|^2 G(m_k^2, m_A^2, m_Z^2) \\ & - 2\ln m_{H^\pm}^2 + \sum_{k=1}^{3}\ln m_{H_k}^2 - \ln m_h^2 \\ & + \sum_{k=1}^{2}|C_{3-k}|^2\hat{G}(m_{H_k}^2, m_Z^2) - \hat{G}(m_h^2, m_Z^2)\Bigg\} \end{aligned} \tag{4.26}$$

and U is:

$$\begin{aligned} U =& \frac{g^2}{384\pi^2}\Bigg\{\sum_{k=1}^{3}|C_k|^2 G(m_{H^\pm}^2, m_{H_k}^2, m_W^2) - [s_W^2 - c_W^2]^2 G(m_{H^\pm}^2, m_{H^\pm}^2, m_Z^2) \\ & - \sum_{k=1}^{2}|C_k|^2 G(m_{H_k}^2, m_A^2, m_Z^2) \\ & + \sum_{k=1}^{2}|C_{3-k}|^2\left[\hat{G}(m_{H_k}^2, m_W^2) - \hat{G}(m_{H_k}^2, m_Z^2)\right] \\ & - \hat{G}(m_h^2, m_W^2) + \hat{G}(m_h^2, m_Z^2)\Bigg\} \end{aligned} \tag{4.27}$$

In our research we used the limits presented by the Gfitter group [60]. At the reference point $m_h = 126$ GeV and $m_t = 173$ GeV they get:

$$\begin{aligned} S &= 0.05 \pm 0.11, \\ T &= 0.09 \pm 0.13, \\ U &= 0.01 \pm 0.11 \end{aligned} \tag{4.28}$$

5 Analysis of the CP-Conserving 2HDM Potential

By taking into consideration everything written in the last two chapters we performed a numerical analyses of the CP-conserving 2HDM. For the research we used the 2HDMC. Also by the means of Mathematica computation program we performed a check of possible models. First of all, we performed a test run of the 2HDMC in the generic basis. For this task we modified the 2HDMC. A random number generator (RNG) was implemented along with changing the input format to a convenient form and fixing several bugs. Theoretical and numerical results were compared to get an idea of possible limits of the 2HDM, which afterwards were used to improve the efficiency of computing and reduce the number of input parameters. We discuss our approach to numerical analyses of the CP-conserving 2HDM and check theoretical and numerical results.

5.1 Our Approach to the Analysis of the CP-Conserving 2HDM Potential

As stated earlier, the main tool for the analysis of the CP-conserving 2HDM potential was the 2HDMC. First of all we performed a test run by generating 10^7 random CP-Conserving 2HDM scalar potentials. It turned out that only 3 427 points were valid in terms of the 2HDM potential stability, tree-level unitarity and quartic Higgs perturbativity. We call this test the UPS check.

This test brought up several problems. First of all, such research is time consuming. On average we managed to get around 7 points per hour [13] after the UPS check. Another problem is that data takes a lot of space. A default single output is 360 bytes long. Therefore modifications of the 2HDMC were a must.

First of all, the 2HDMC code was modified to check the model only if parameters satisfy the UPS check. Prior to that all possible parameters were calculated at once. This simple modification reduced computing time by a lot. Secondly, if all of the UPS check conditions are satisfied result is saved.

At this point a possible further reduction is possible. From the theoretical analysis of

[13] For the relevance all computing was CPU based and performed on i7-6700k 4.6 GHZ under full load.

eq.(4.4, 4.5, 4.11, 4.12) we found that:

$$\begin{aligned} 0 <&\lambda_1 < 4\pi, \\ 0 <&\lambda_2 < 4\pi, \\ -4\pi <&\lambda_3 < 8\pi, \\ -\frac{32}{3}\pi <&\lambda_4 < \frac{32}{3}\pi, \\ -\frac{24}{5}\pi <&\lambda_5 < \frac{24}{5}\pi \end{aligned} \tag{5.1}$$

Due to mathematical complexity we failed to perform a more in-depth analysis. One of the possible solutions is to fix $\lambda_5 = 0$ and therefore it becomes possible to analyse both 2HDM potential stability and S matrix unitarity. On the other hand, if we account for the perturbativity condition, we get two additional parameters and therefore this implies that there are two additional degrees of freedom. Obviously this arises even more problems. We compared theoretical λ_{1-5} values eq.(5.1) with the ones we got from the numerical analysis fig.8:

$$\begin{aligned} 0.01 <&\lambda_1 < 12.13, \\ 0.01 <&\lambda_2 < 10.91, \\ -6.95 <&\lambda_3 < 24.73, \\ -23.8 <&\lambda_4 < 11.75, \\ -12.1 <&\lambda_5 < 8.61 \end{aligned} \tag{5.2}$$

Which are in consistency with the theoretical eq.(5.1) ones. Also we include coupling values in Higgs bases:

$$\begin{aligned} 0.26 <&Z_1 < 4.23, \\ 0.01 <&Z_2 < 4.19, \\ -1.16 <&Z_3 < 24.68, \\ -23.71 <&Z_4 < 11.37, \\ -12.33 <&Z_5 < 7.42, \\ -4.34 <&Z_6 < 4.25, \\ -4.42 <&Z_7 < 4.37 \end{aligned} \tag{5.3}$$

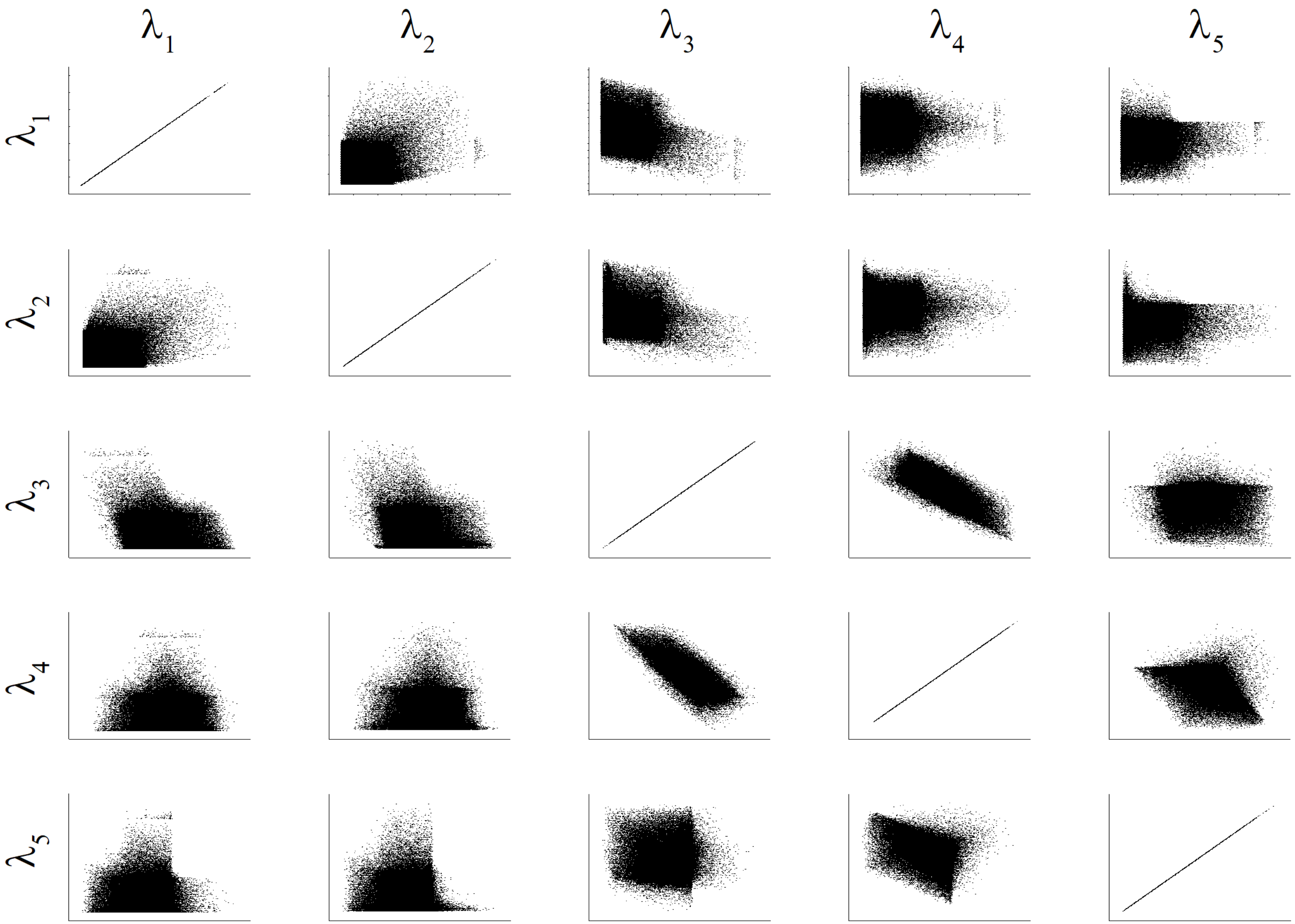

Figure 8: Relation between different coupling coeffiecients λ_{1-5} after the UPS check.

At this point we switched from using the generic basis as an input to the mass eigenstates basis. The input is the following:

$$\begin{aligned} 3 \text{ physical masses} : \; & m_H \, [130; 1200] \, \text{GeV}, \\ & m_A \, [92; 1200] \, \text{GeV}, \\ & m_{H^\pm} \, [300; 1200] \, \text{GeV}. \\ 2 \text{ mixing angles} : \; & \alpha \, [-{}^{\pi}\!/_{2}; \; {}^{\pi}\!/_{2}] \, , \\ & t_\beta \, (0; \; 100] \end{aligned}$$

We assume that CP-even 2HDM sector is degenerate and thus the lower mass limit for the CP-even heavy Higgs is at least $m_H = 130$ GeV. Current experimental combined lower mass limits for m_A and $m_{H^\pm}$ were taken from ref. [4]. We set the upper limit for all of the Higgs bosons to 1200 GeV. We assume that this interval is enough for searches at the LHC in the following years.

We chose the mass eigenstates basis as it is the only basis where one of the parameters is fixed. It is the SM Higgs boson mass. Value of the SM Higgs is randomly generated based on a ref. [3]. The mass of the SM Higgs boson turned out to be uniformly distributed. This can be seen by taking a look at fig.9.

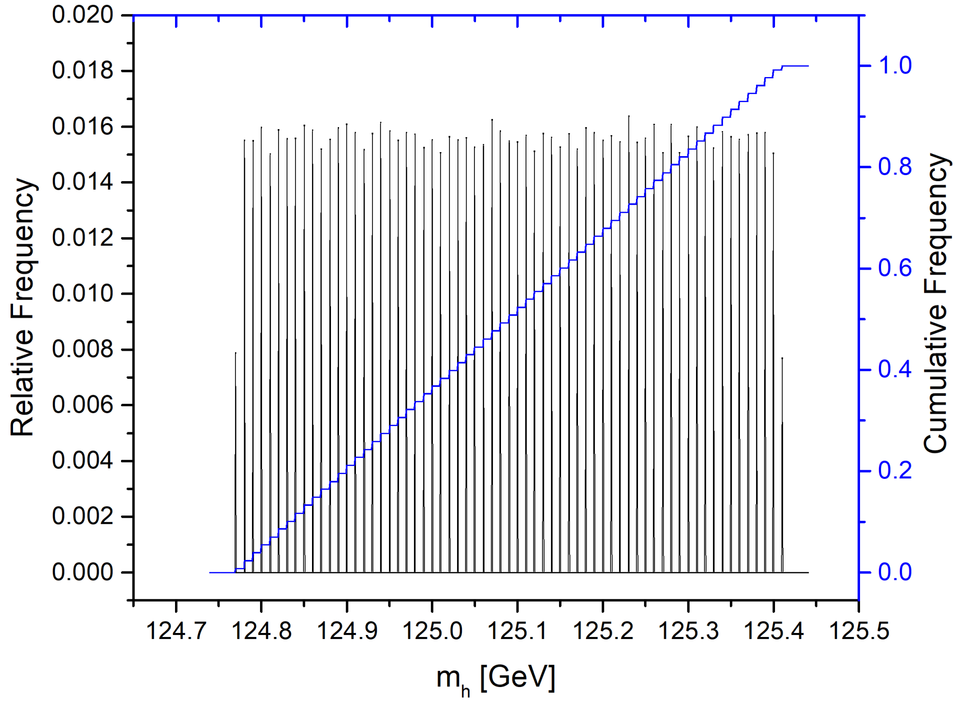

Figure 9: Frequency distribution of the SM-like Higgs boson h mass after the UPS check.

Based on determination of couplings λ_{1-5} and our choice of the possible Higgs boson mass values, the range for the value of the coupling coefficient m_{12}^2 is determined to give a valid parameter point. Afterwards m_{12}^2 is randomly chosen in the allowed range. Result is presented in fig.10.

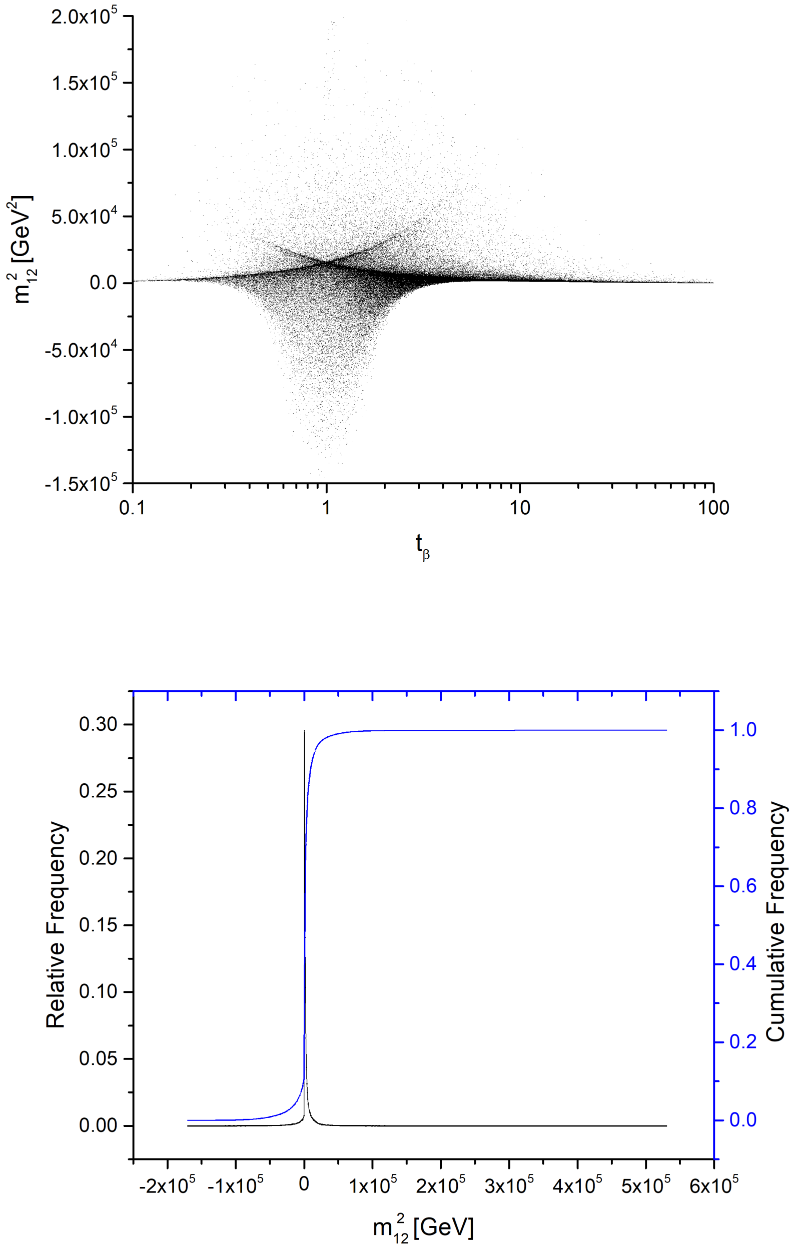

Figure 10: Result of generating m_{12}^2 parameter after applying the UPS check. Scatter plot of m_{12}^2 as a parameter t_β (top). Frequency distribution of m_{12}^2 (bottom).

At this point, after applying all of the mentioned tweaks we performed a 500 hour run. We managed to get 224 619 valid points in the mass eigenstates basis. This results in around 450 points per hour. That is 64 times more effective computing compared with the default 2HDMC. Size was reduced from 360 bytes to 244 bytes. That is more than 30 per cent reduction.

Further we analyse the Peskin-Takeuchi parameters by comparing them with values from the Gfitter group. In our analysis we inspect these values directly without correlation coefficients between the S, T and U parameters. As expected, the most severe constraints came from the T parameter, which is a good sign. The valid region of the model after applying the Gfitter group results is presented in fig. 11. After applying constraints on Peskin-Takeuchi parameters, 224 619 points were reduced to 23 822.

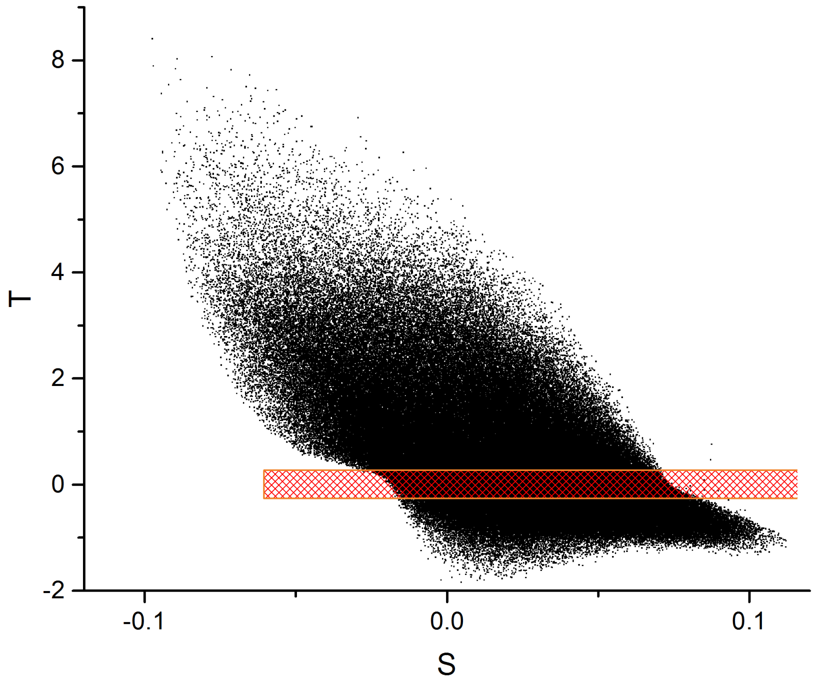

Figure 11: S-T plane. The shaded area is the allowed contribution from the new physics based on Gfitter values.

Finally, the decay rates are checked. For this check the program Higgs Bounds (HB) [61] is used. We check the fermiophobic model where the Yukawa couplings of the second Higgs doublet are zero. This way we get the minimal constraint on the Higgs bosons decay rates. After applying the HB check, the number of parameters is reduced from 23 822 to 2 977. Afterwards we check if such model is possible in the Type III 2HDM by randomly generating 10 000 Yukawa couplings. It turned out that all 2 977 model points are possible.

5.2 Results of Our Analysis of the CP-Conserving 2HDM Potential

In this section we discuss the possible values of the Higgs bosons masses in the CP-conserving 2HDM potential. We think that in the upcoming years LHC will be able to cover the mass interval of the Higgs sector up to 1200 GeV. Although the results are based on the CP-conserving 2HDM potential, nevertheless they can be achieved in the general model. We provide the frequency distribution of possible mass values at different 2HDM check steps in fig.(12, 13, 14). The maximum values are presented in tab.5.

Table 5: The maximum mass values, in GeV, of the additional 2HDM Higgs bosons at different check steps.

	UPS	UPS+STU	UPS+STU+HB
m_H	1178.31	1178.31	1178.31
m_A	1126.57	1098.43	1098.43
$m_{H^\pm}$	1162.94	1162.94	1162.94

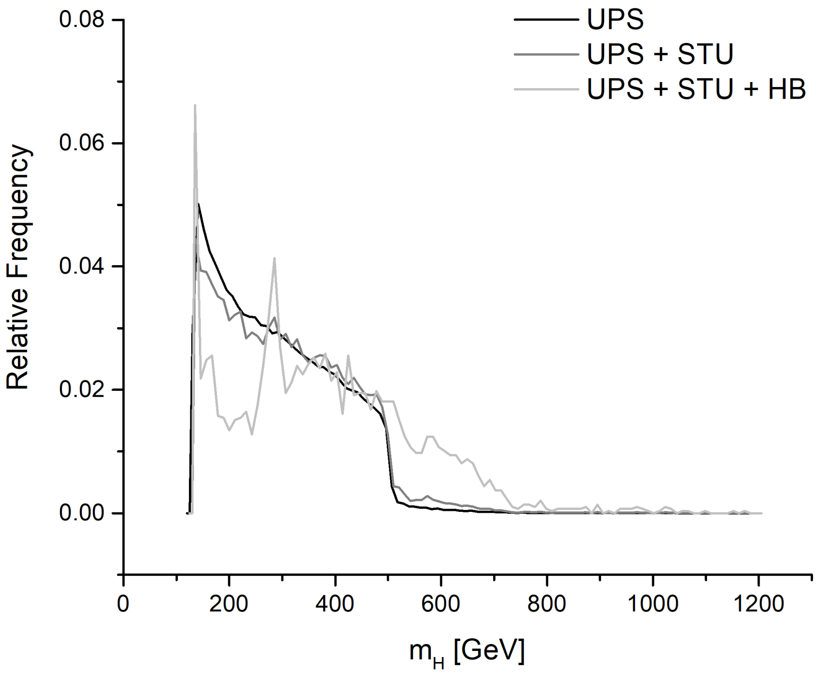

Figure 12: Frequency distribution of the mass of the CP-even Higgs boson H at various 2HDM potential check steps.

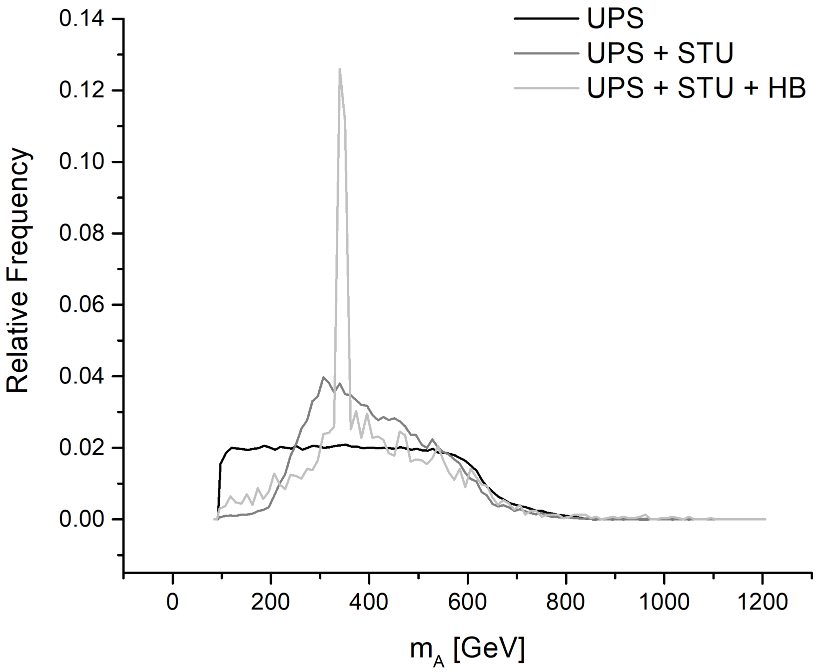

Figure 13: Frequency distribution of the mass of the CP-odd Higgs boson A at various 2HDM potential check steps.

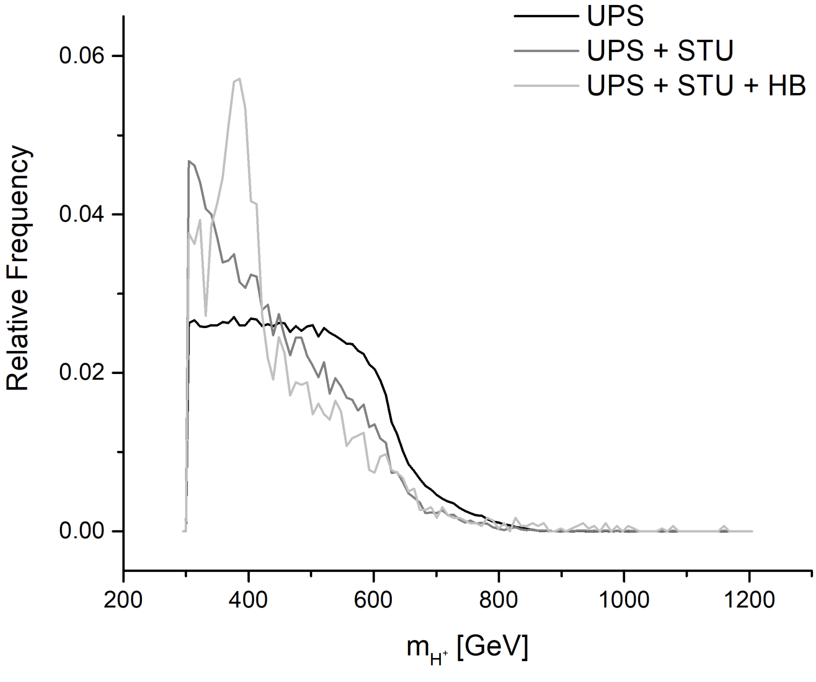

Figure 14: Frequency distribution of the mass of the charged Higgs boson $H^{\pm}$ at various 2HDM potential check steps.

From the plot fig.12 it can be seen that there is a steep at the value of $m_H = 500$ GeV. We analysed this region and it turns out that this is due to the quartic Higgs perturbation condition. In the plot fig.13 the peak after all three checks corresponds to the value $m_A = 2m_t$. This is an undistinguishable decay process by HB.

Also we provide more detailed results after applying all of the three discussed 2HDM constraints. For convenience, frequency distribution of all three Higgs bosons is presented on one plot. The result can be seen in fig.15.

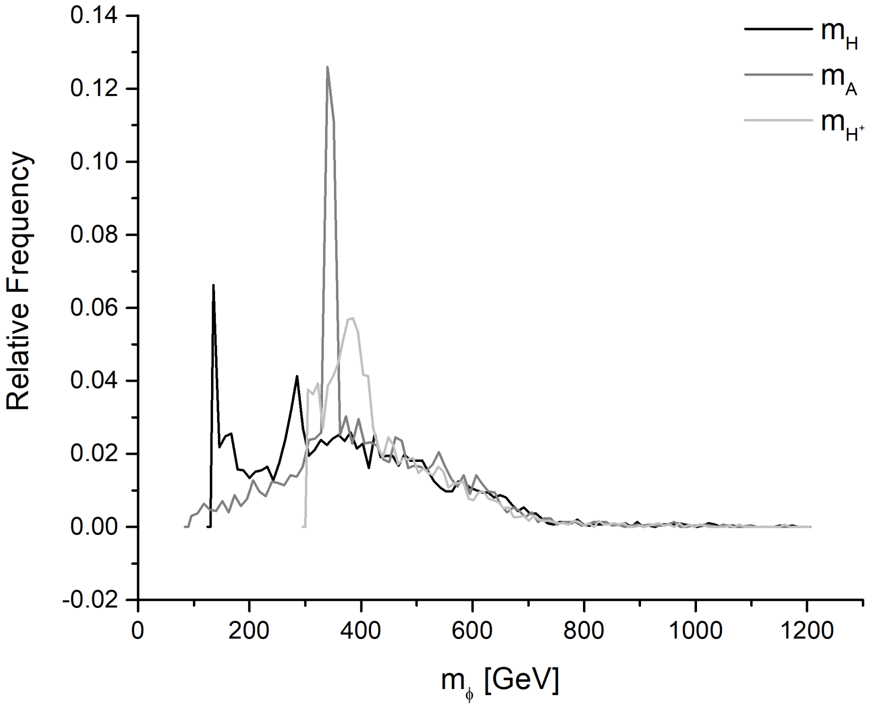

Figure 15: Frequency distribution of the 2HDM additional Higgs bosons after applying all three check steps.

A useful value for the determination of couplings between the 2HDM is the physical mixing angle $s_{\beta-\alpha}$. Possible values of $s_{\beta-\alpha}$ are presented in fig.16. It can be seen that the most favourable values are around $s_{\beta-\alpha} = \pm 1$. This indicates that the 2HDM sector is SM-like.

We also present relation between α and β in fig.17. An interesting conclusion can be made about the value of the CP-even Higgs mixing α. The most probable value of α corresponds to the non-degenerate case. This explains the peak at minimum values of the plot fig.12.

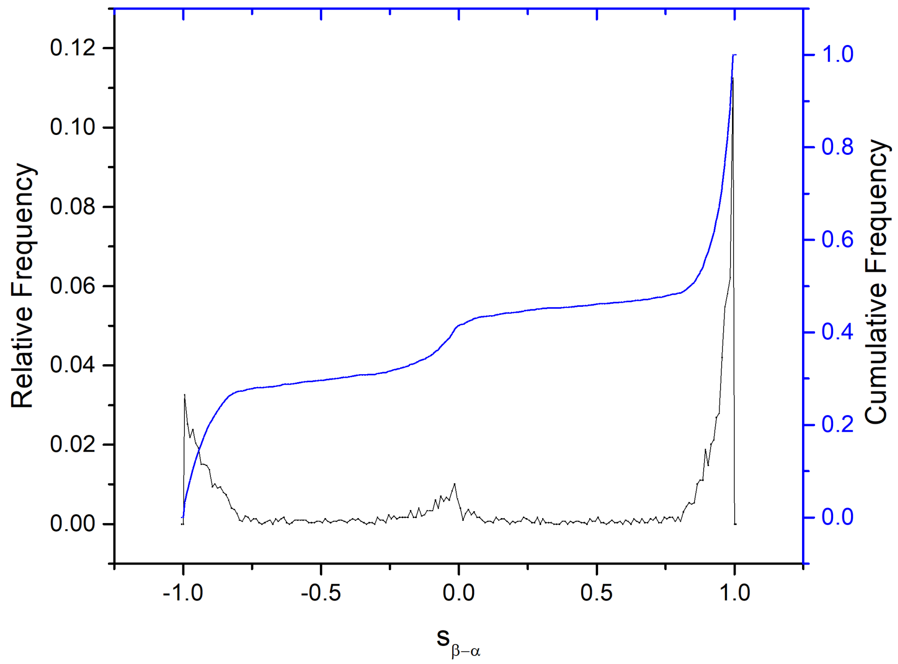

Figure 16: Frequency distribution of the 2HDM mixing $s_{\beta-\alpha}$ after applying all three constraints.

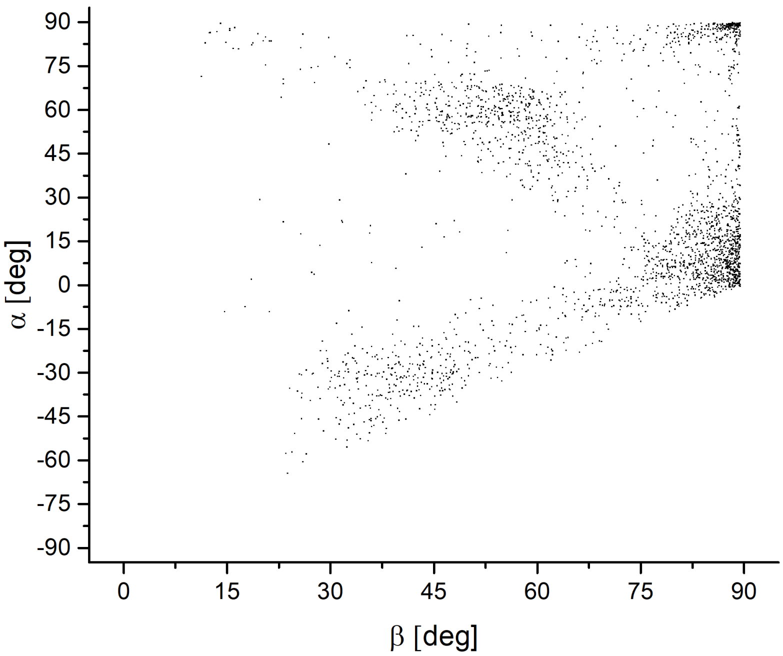

Figure 17: Scatter plot of two 2HDM angles α and β after applying all three check steps.

It turns out that in our model the most possible value of t_β is around one. This corresponds to the of the maximal mixing between Higgs doublets. This can be seen in fig.18.

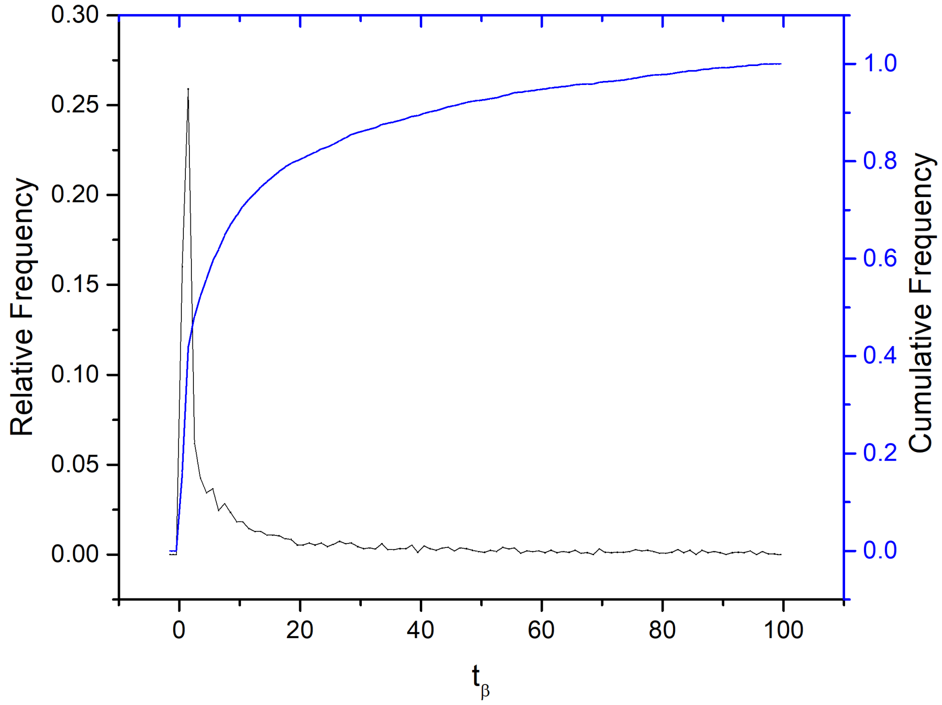

Figure 18: Frequency distribution of the ratio of VEVs relation after applying all three check steps.

We face the fact that in most articles generic bases and Higgs bases are used for the analysis. Therefore we show our results in terms of couplings λ and Z. The generic basis is presented in fig.19 and Higgs bases in fig.20. We found that possible values of coupling coefficients lay in the following intervals:

$$
\begin{array}{rcccl}
& & & & \\
0.01 & < & \lambda_1 & < & 12.13, \\
0.01 & < & \lambda_2 & < & 10.53, \\
-5.54 & < & \lambda_3 & < & 19.27, \\
-13.66 & < & \lambda_4 & < & 7.5, \\
-8.67 & < & \lambda_5 & < & 6.5, \\
& & & &
\end{array}
\qquad
\begin{array}{rcccl}
0.26 & < & Z_1 & < & 4.14, \\
0.01 & < & Z_2 & < & 4.17, \\
-0.01 & < & Z_3 & < & 19.27, \\
-9.16 & < & Z_4 & < & 6.34, \\
-8.67 & < & Z_5 & < & 6.64, \\
-3.9 & < & Z_6 & < & 3.93, \\
-4.42 & < & Z_7 & < & 4.04
\end{array}
\tag{5.4}
$$

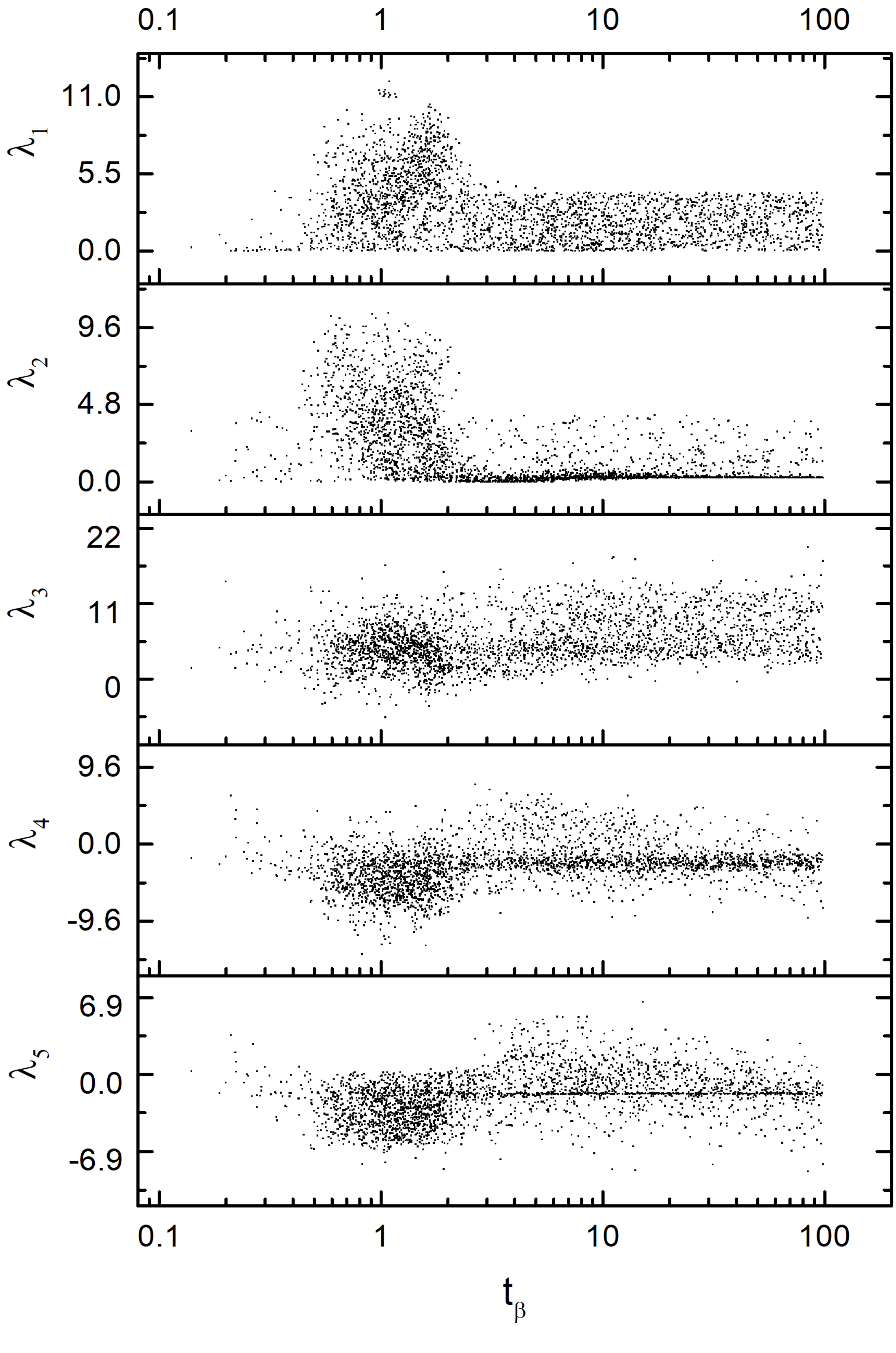

Figure 19: Distribution of coupling coefficients λ as a function of t_β in the generic basis after applying all three check steps.

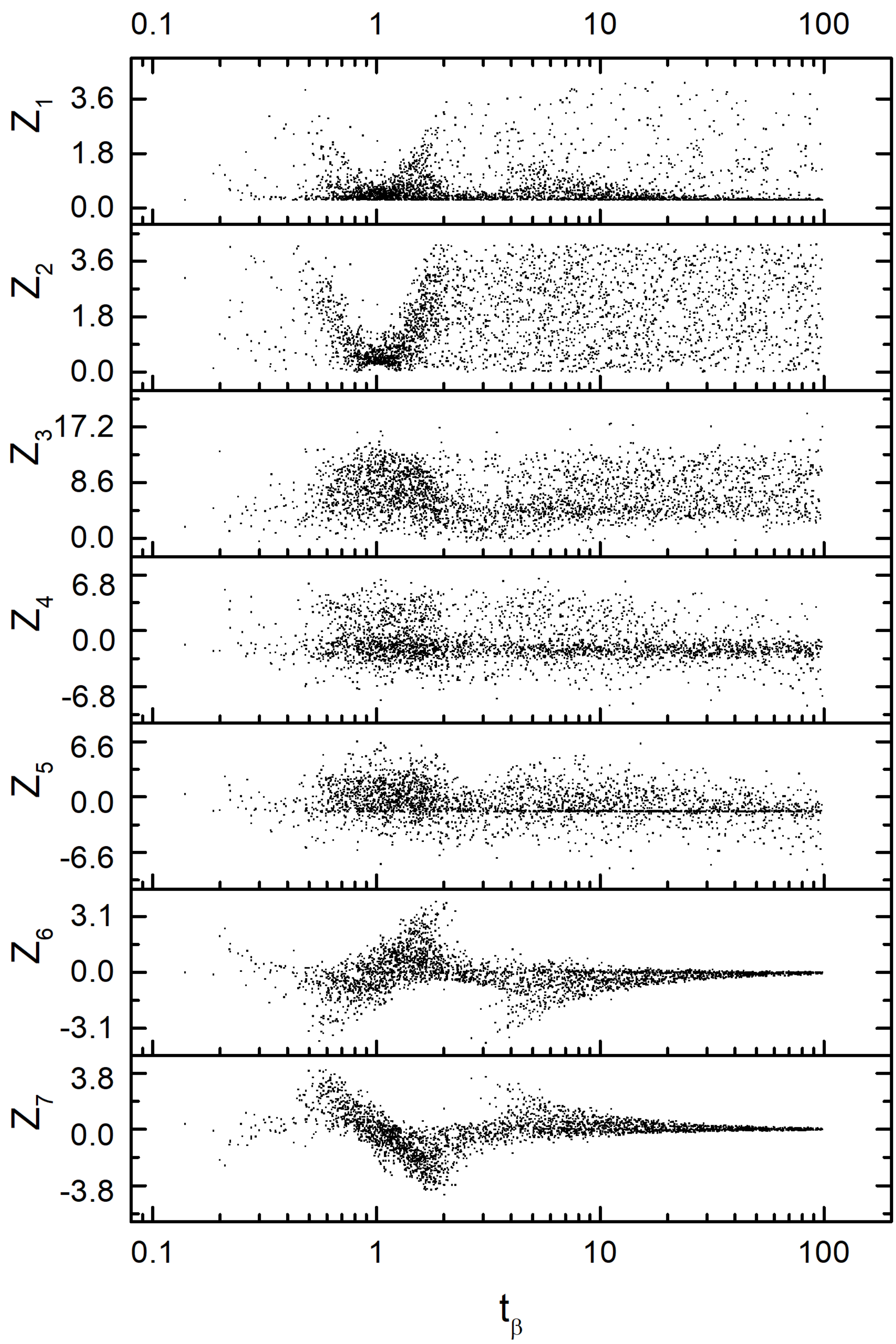

Figure 20: Distribution of coupling coefficients Z as a function of t_β in Higgs bases after applying all three check steps.

6 Results and Conclusions

In the process of the research we obtained several results:

- We worked out possible transformations of the 2HDM CP-conserving potential between the generic basis, Higgs bases and the mass eigenstates basis
- We derived and applied theoretical constraints of the 2HDM CP-conserving potential:
  - Stability of the potential
  - S matrix tree-level unitarity
  - Quartic Higgs bosons perturbativity
  - Peskin-Takeuchi parameters
- Using the 2HDMC and our own code we managed to get stable 2HDM CP-conserving potential

By performing multiple transformations of the 2HDM CP-conserving potential we managed to get to the original values. Therefore the derived transformations during the research are correct. Numerical results of both 2HDMC and our code coincide. This implies that theoretical constraints were derived correctly. After applying all of the constraints on the 2HDM CP-conserving potential we got the following maximum values of the Higgs bosons masses:

- $m_H = 1178.31$ GeV
- $m_A = 1098.43$ GeV
- $m_{H^\pm} = 1162.94$ GeV

All of these conditions are in the interval $0.9 \leq t_\beta \leq 1.2$. This might point out that the generic basis is more preferable in the CP-conserving 2HDM.

We managed to improve computing results of our Monte-Carlo sampling by at least 60 times. After applying all possible 2HDM check steps we got 2 977 valid CP-conserving parameter points in the 2HDM parameter-space.

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