## How do we measure particles

- We can measure tracks of individual charged particles with
  - pixel detectors
  - wire chambers

  ⇒ we measure the 3-momentum $\vec{p}$

- We can measure the energy of particles in
  - calorimeters

  ⇒ we measure the energy $E$

- We can distinguish different kinds of particles by
  - sets of Cherenkov counters
  - comparing calorimeter- and track-data

  ⇒ reconstructing the 4-momentum: $E^2 = \vec{p}^2 + m^2$

## How do we measure intermediate particles

- masses are measured for instance with "invariant-mass distributions"
- example: $e^+e^- \to Z \to \mu^+\mu^-$
  - the 4-momenta of the muons are reconstructed
  - these 4-momenta are summed up and squared: $M^2$
  - events with 2 muons are distributed according to $M^2$:
    * this gives a Breit-Wigner distribution (on top of a background)
  - measured mass = position of the peak in the Breit-Wigner curve

- couplings are measured with the strength of the interaction
- example: $H \to \mu^+\mu^-$ versus $H \to b\bar{b}$
  - identifying the decaying particle in all events
  - relative coupling strength $= \frac{\#\text{events}(H\to\mu^+\mu^-)}{\#\text{events}(H\to b\bar{b})}$

## How do we calculate

- we generate all Feynman diagrams $\{f\}$ for the process
- we calculate the amplitude for each Feynman diagram $\mathcal{M}_f$
- we add up all amplitudes $\mathcal{M} = \sum_f \mathcal{M}_f$
- we square the modulus $|\mathcal{M}|^2$
- we integrate over the Lorentz invariant phase space $d$Lips

$$d\sigma = \frac{symmetry}{flux} |\mathcal{M}|^2 d\text{Lips}(p_1^\mu \dots p_n^\mu)$$

  - momentum-conservation $P^\mu = P_{in}^\mu = \sum_i p_i^\mu$
  - and each particle $i$ in the final state contributes, so

$$d\text{Lips}(p_1^\mu \dots p_n^\mu) = (2\pi)^4 \delta^4(P - \sum_i p_i) \prod_i \frac{d^3\vec{p}_i}{(2\pi)^3\, 2E_{\vec{p}_i}}$$

- in perturbation theory, we expand in orders of the coupling constant $g$:

$$\mathcal{M} = \sum_k g^k \mathcal{M}^{(k)} \qquad \text{where} \qquad \mathcal{M}^{(k)} = \sum_f \mathcal{M}_f^{(k)}$$

## How do we calculate

- When perturbation theory is applicable: $|g^n \mathcal{M}^{(n)}| > |g^{n+1} \mathcal{M}^{(n+1)}|$
  - the leading order (LO) is the smallest $n$ with $|\mathcal{M}^{(n)}| > 0$
  - the next-to leading order (NLO) is then $n+1$ or $n+2$.
- the cross section is also a sum over powers of the coupling constant $g$:

$d\sigma = \sum_{k=2n} g^k d\sigma^{(k)}$ where $d\sigma^{(2n)} = |g^n \mathcal{M}^{(n)}|^2 d\text{Lips}$ and

$$d\sigma^{(2n+1)} = \left[(g^n \mathcal{M}^{(n)})^\dagger (g^{n+1} \mathcal{M}^{(n+1)}) + (g^n \mathcal{M}^{(n)})(g^{n+1} \mathcal{M}^{(n+1)})^\dagger\right] d\text{Lips}$$

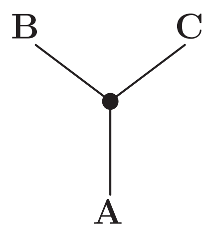
- we identified the "physical" coupling $g$ from the measurement of $\sigma$
  - LO: $\sigma^{\text{LO}} = \int d\sigma^{(2n)}$
  - NLO: $\sigma^{\text{NLO}} = \int (d\sigma^{(2n)} + d\sigma^{(2n+1)})$
- if $\sigma^{\text{LO}} \neq \sigma^{\text{NLO}}$ ...what is now the right $\sigma$ ? $\Rightarrow$ Renormalisation

## Example of Renormalisation: ABC-theory

- we have 3 real scalar fields $A$, $B$, and $C$ and 1 coupling $g$:

$$\mathcal{L}_0 = -\tfrac{1}{2}\Phi_0(\partial^2 + m_{0,\Phi}^2)\Phi_0 + g_0 A_0 B_0 C_0 \qquad \text{where} \quad \Phi = A, B, C$$

  - masses are the poles in the propagators $i \times [p^2 - m_{0,\Phi}^2 + i\epsilon]^{-1}$
  - the coupling connects all three different fields

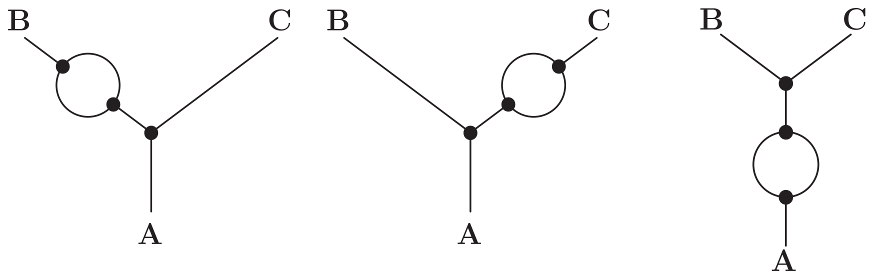



- having the physical masses as $m_A > m_B + m_C$,
  - $A$ will decay into $B$ and $C$:
  - at LO there is only one diagram, which gives $g^1\mathcal{M}^{(1)} = g$
  - $d\text{Lips} = (2\pi)^4\delta^4(p_A - p_B - p_C)\frac{d^3\vec{p}_B}{(2\pi)^3\, 2E_{\vec{p}_B}}\frac{d^3\vec{p}_C}{(2\pi)^3\, 2E_{\vec{p}_C}} = \frac{d\Omega}{(4\pi)^2}\kappa_{ABC}$

$$\sigma^{\text{LO}}(A \to BC) = \frac{1}{2m_A}\int |g|^2 \frac{d\Omega}{(4\pi)^2}\kappa_{ABC} = \frac{g^2}{4\pi}\frac{\kappa_{ABC}}{2m_A}$$

- $g$ or $\alpha = \frac{g^2}{4\pi}$ can be determined from the decay!

## Example of Renormalisation: ABC-theory



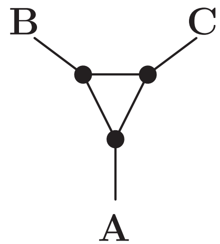

we calculate now $\sigma^{\text{NLO}}(A \to BC)$

- there is no diagram of order $g^2$
- there are 4 diagrams of order $g^3$
  - 3 of them have a "bubble" on an external line

    $\Rightarrow$ we will have to renormalise also the propagators!

    * as it turns out, they will not contribute (on-shell scheme)

  - the $4^{\text{th}}$ diagram is just a triangle
    * which is usually not zero
    * but depends on incoming and outgoing momenta




  - so $g^3 \mathcal{M}^{(3)} = g^3 t_{ABC}$

$$\sigma^{\text{NLO}}_{(A\to BC)} = \frac{1}{2m_A} \int \left[ |g|^2 + g \times (g^3 t_{ABC})^\dagger + g^\dagger \times (g^3 t_{ABC}) \right] \frac{d\Omega}{(4\pi)^2} \kappa_{ABC}$$

$$= \frac{g^2}{4\pi} \frac{\kappa_{ABC}}{2m_A} \left( 1 + 2g^2 T_{ABC} \right) \quad \text{with} \quad T_{ABC} = \int \frac{d\Omega}{4\pi} \mathcal{R}e[t_{ABC}]$$

## Example of Renormalisation: ABC-theory

- but we calculated with $g_0$, not with the measured $g$ !
- the same happens to the other parameters of the theory
- writing

  $$g_0 = g + \delta g \ , \quad m^2_{0,\Phi} = m^2_\Phi + \delta m^2_\Phi \ , \text{ and } \quad \Phi_0 = \sqrt{Z_\Phi}\Phi = \Phi + \tfrac{1}{2}\delta Z_\Phi \Phi$$

  we get a new Lagrangian

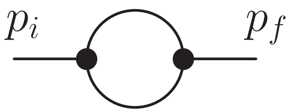
  $$\mathcal{L}_0 = -\tfrac{1}{2} Z_\Phi \Phi(\partial^2 + m^2_\Phi + \delta m^2_\Phi)\Phi + (g + \delta g)\sqrt{Z_A Z_B Z_C}\, ABC = \mathcal{L} + \delta\mathcal{L}$$

  with

  $$\begin{aligned} \mathcal{L} &= -\tfrac{1}{2}\Phi(\partial^2 + m^2_\Phi)\Phi + gABC \\ \delta\mathcal{L} &= -\tfrac{1}{2}\Phi[\delta Z_\Phi(\partial^2 + m^2_\Phi) + \delta m^2_\Phi]\Phi + [\tfrac{g}{2}(\delta Z_A + \delta Z_B + \delta Z_C) + \delta g]\, ABC \end{aligned}$$

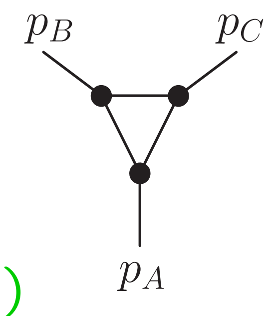
  where only terms linear in the expansion of the parameters are kept.
- We assume only one-loop accuracy for now.

## Example of Renormalisation: ABC-theory

- Introducing abbreviations for the loops
- The self energies $i\Pi(p^2) =$ 

$= i\frac{g^2}{(4\pi)^2} B_0(p^2; m_0^2, m_1^2)$
- Using the Feynman rules we get to one loop

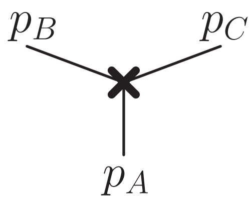
$$
\begin{aligned}
& i\Pi(p_i^2)(2\pi)^4\delta^4(p_i - p_f) \\
&= \int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\frac{(ig)(2\pi)^4\delta^4(p_i - k_1 - k_2)i}{k_1^2 - m_1^2}\frac{(ig)(2\pi)^4\delta^4(k_1 + k_2 - p_f)i}{k_2^2 - m_0^2} \\
&= i(2\pi)^4\delta^4(p_i - p_f)\frac{g^2}{(4\pi)^2}\frac{1}{i\pi^2}\int d^4q\,[q^2 - m_0^2]^{-1}[(q + p_i)^2 - m_1^2]^{-1} \\
&=: i(2\pi)^4\delta^4(p_i - p_f)\frac{g^2}{(4\pi)^2}B_0(p_i^2; m_0^2, m_1^2)
\end{aligned}
$$

- In a similar way the vertex correction $iT(p_A^2, p_B^2, p_C^2) =$ 


  is written with the function $\frac{g^3}{(4\pi)^2}C_0(p_C^2, p_A^2, p_B^2; m_A^2, m_B^2, m_C^2)$

## Example of Renormalisation: ABC-theory

- Now we have 7 counterterms that we should detemine: $\delta m_\Phi^2$, $\delta Z_\Phi$, $\delta g$
- they also give rise to new diagrams:
  - six counterterms are used by $i\delta Z_\Phi(p^2 - m_\Phi^2) - i\delta m_\Phi^2 =$ 

  - the 7$^{\text{th}}$ counterterm by $iC_g = \frac{ig}{2}(\delta Z_A + \delta Z_B + \delta Z_C) + i\delta g =$ 

- We have to choose Renormalisation conditions for them.

  We can take (without proof):

  1. the pole of the full propagator should be at $m_\Phi^2$

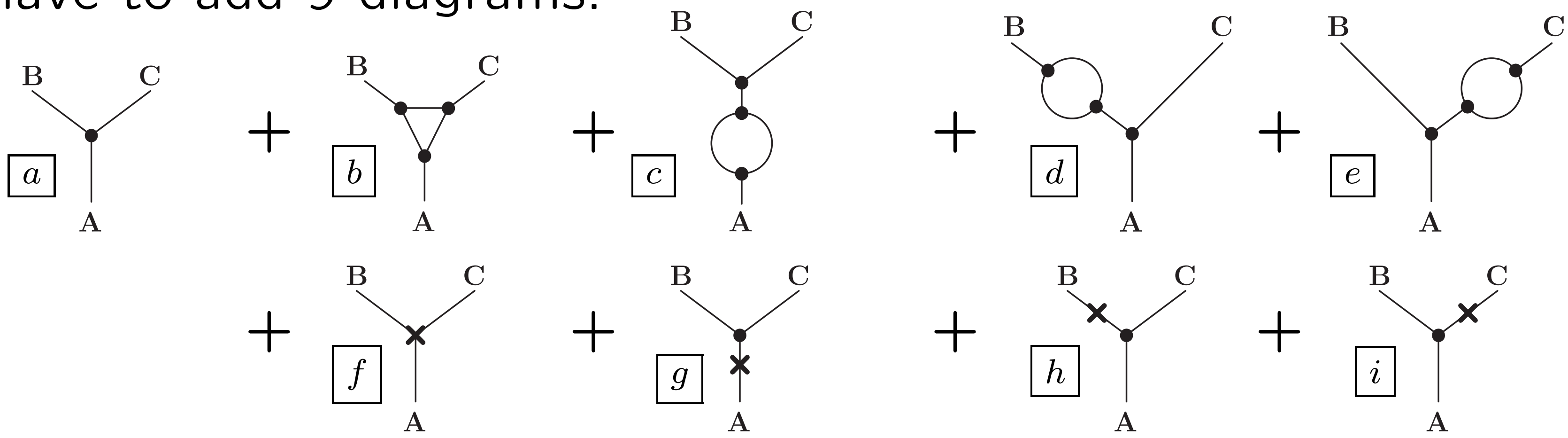
     $\Rightarrow \quad \delta m_\Phi^2 = \Pi(m_\Phi^2)$
  2. the residuum of the full propagator should be 1

     $\Rightarrow \quad \delta Z_\Phi = -\frac{d}{dx}\Pi(x)\big|_{x=m_\Phi^2} = -\Pi'(m_\Phi^2)$
  3. the full decay width of $A$ should be like in the tree level.

     $\Rightarrow \quad \delta g = -T(m_A^2, m_B^2, m_C^2) - \frac{g}{2}(\delta Z_A + \delta Z_B + \delta Z_C)$

## Example of Renormalisation: ABC-theory

- Calculating the NLO matrix element for the decay of $A \to B + C$, we have to add 9 diagrams:




- adding $\boxed{c} + \boxed{g}$ and Taylor expanding around $p_A^2 = m_A^2$ gives

$$-g\frac{\Pi(m_A^2) + \Pi'(m_A^2)(p_A^2 - m_A^2) + \mathcal{O}\left((p_A^2 - m_A^2)^2\right)}{p_A^2 - m_A^2} - g\frac{\delta Z_A(p_A^2 - m_A^2) - \delta m_A^2}{p_A^2 - m_A^2}$$

$$= \mathcal{O}\left((p_A^2 - m_A^2)\right)$$

## Example of Renormalisation: ABC-theory

- $\boxed{a}$ gives now $g^1\mathcal{M}^{(1)} = g$
- $\boxed{c} + \boxed{g}$, $\boxed{d} + \boxed{h}$, and $\boxed{e} + \boxed{i}$ are all $\mathcal{O}\left((p_A^2 - m_A^2)\right)$
- so $g^3\mathcal{M}^{(3)} = \lim_{p_\Phi^2 \to m_\Phi^2}(\boxed{b} + \cdots + \boxed{i}) = \lim_{p_\Phi^2 \to m_\Phi^2}(\boxed{b} + \boxed{f})$

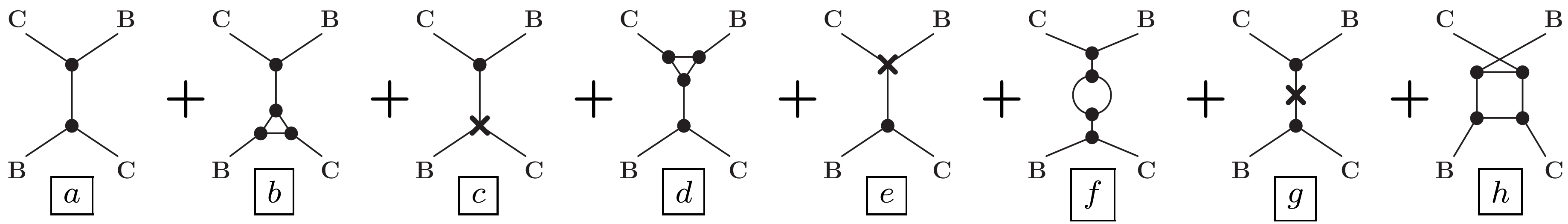
$$= T(m_A^2, m_B^2, m_C^2) + \frac{g}{2}(\delta Z_A + \delta Z_B + \delta Z_C) + \delta g = 0$$

- So now $\sigma^{\text{LO}} = \sigma^{\text{NLO}}$ for this decay !
- $g$ can be determined unambiguously.
- but where do come new predictions?
  - $A \to 3B + C$ or $A \to B + 3C$ or . . .
  - scattering $B + C \to B + C$: $d\sigma(BC \to BC)$

## Example of Renormalisation: ABC-theory, $d\sigma(BC \xrightarrow{s} BC)$

- with the same argument as before: no corrections on external legs
- contributing diagrams to the s-channel,

  initial momenta $k$, final momenta $p$:




- $k_B^2 = p_B^2 = m_B^2$, $k_C^2 = p_C^2 = m_C^2$, and $(k_B + k_C)^2 = (p_B + p_C)^2 = s$.
- $\boxed{a} = (ig)^2 \frac{i}{s-m_A^2}$ so $g^2\mathcal{M}^{(2)} = -\frac{g^2}{s-m_A^2}$
- $$\boxed{b}+\boxed{c}+\boxed{d}+\boxed{e} = \frac{i^3 g T(s,k_B^2,k_C^2)}{s-m_A^2} + \frac{i^3 g C_g}{s-m_A^2} + \frac{i^3 g T(s,p_B^2,p_C^2)}{s-m_A^2} + \frac{i^3 g C_g}{s-m_A^2}$$
  $$= -2ig\frac{T(s,m_B^2,m_C^2) - T(m_A^2,m_B^2,m_C^2)}{s-m_A^2} = \mathcal{O}\left((s-m_A^2)^0\right)$$

## Example of Renormalisation: ABC-theory, $d\sigma(BC \xrightarrow{s} BC)$

- $\boxed{f} + \boxed{g} \;=\; \frac{(ig)^2 i^2 i\Pi(s)}{(s-m_A^2)^2} + \frac{(ig)^2 i^2 i\delta Z_A(s-m_A^2) - \delta m_A^2}{(s-m_A^2)^2}$

  $= \; ig^2 \frac{\Pi(s) - \delta m_A^2 + \delta Z_A(s-m_A^2)}{(s-m_A^2)^2} = \mathcal{O}\left((p_A^2 - m_A^2)^0\right)$

- $\boxed{h} = g^4 D_{\text{box}}(p_B^2, p_C^2, k_C^2, k_B^2; (-p_B - p_C)^2, (-p_C + k_C)^2; m_A^2, m_C^2, m_A^2, m_B^2)$

- $g^4 \mathcal{M}^{(4)} = \frac{g^2[\Pi(s) - \Pi(m_A^2)]}{(s-m_A^2)^2} - \frac{2g[T(s) - T(m_A^2)] + g^2\Pi'(m_A^2)}{s-m_A^2} + D_{\text{box}} =: \frac{g^2}{s-m_A^2}\Delta\mathcal{M}$

- so $\quad d\sigma^{\text{LO}}_{(BC \xrightarrow{s} BC)} = \frac{1}{2\kappa_{sBC}} \left| \frac{-g^2}{s-m_A^2} \right|^2 \frac{d\Omega}{(4\pi)^2} \frac{\kappa_{sBC}}{2s} = \left(\frac{g^2}{4\pi}\right)^2 \frac{d\Omega}{4s(s-m_A^2)^2}$

- and $\; d\sigma^{\text{NLO}}_{(BC \xrightarrow{s} BC)} \;=\; \frac{1}{2\kappa_{sBC}} \left[ \frac{g^4}{(s-m_A^2)^2} + \frac{-g^2}{s-m_A^2} \cdot 2\mathcal{R}e[g^4 \mathcal{M}^{(4)}] \right] \frac{d\Omega}{(4\pi)^2} \frac{\kappa_{sBC}}{2s}$

  $= \left(\frac{g^2}{4\pi}\right)^2 \frac{d\Omega}{4s(s-m_A^2)^2} (1 - 2\mathcal{R}e[\Delta\mathcal{M}]) \neq d\sigma^{\text{LO}}_{(BC \xrightarrow{s} BC)}$

## Regularisation

- is the prescription, how to deal with undefined integrals
- examples:
  - only integrating up to $p^2 < \Lambda$
  - subtracting the integrand with a regulator mass (Pauli-Villars)
  - making the dimension continuous (dimensional regularisation)
- the physical result cannot depend on the regularisation
- the counterterms can "absorb" the regularisation constants

  $\Rightarrow$ renormalisable theory

## Regularisation in ABC

- only the selfenergies had ill defined integrals
- $\delta Z_\Phi$ and $\delta m_\Phi^2$ contain regularisation constants
- but they drop out in all physical processes