# 3. Outlook — Strings

## History of Stringtheory

- 1960s: describing strong dynamics with string models
  - Yoichiro Nambu; and later Lenny Susskind and Holger Nielsen

- 1970s: string theory as quantum gravity
  - bosonic string theory: John H. Schwarz and Joel Scherk

- 1984: Green-Schwarz anomaly cancellation mechanism
  - Michael Green and John H. Schwarz

  ⇒ the first Superstring revolution

- 1990: D-branes by Polchinsky

- 1995: M-theory by Edward Witten

  ⇒ the second Superstring revolution

- 1997: AdS/CFT correspondence conjecture by Juan Maldacena

- since then: stringtheory becomes a mathematical tool

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## Ideas and Consequences of Stringtheory (Ed. Witten)

- replacing the point particle with a string
- quantizing the string $\Rightarrow$ mass spectrum
  - lowest energy state of the bosonic string has $m^2 < 0$ $\Rightarrow$ tachyonic
  - second lighest state is a massless spin-2 particle $\Rightarrow$ graviton
- including SUSY $\Rightarrow$ the tachyonic state vanishes
- Feynman diagrams in QFT have 2 parts: propagators and vertices
  - they are in principle different
  - vertices happen when particles meet/decay $\Rightarrow$ space-time
- strings have no vertices, no exact location of an interaction
  $\Rightarrow$ space-time itself becomes fuzzy
- general predictions of stringtheory:
  - Gravity $+$ Gauge Symmetry $+$ Supersymmetry

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## Bosonic String, Nambu-Goto action, Polyakov action

- in a fixed Pseudo-Riemannian space-time of dimension $D$ with
  - coordinates $X = (X^\mu)$, $\mu = 0, \ldots, D-1$
  - metric $G_{\mu\nu}(X) = (1, -1, \ldots, -1) = (1)(-1)^{D-1}$
- the motion of a string forms a two-dimensional world-sheet $\Sigma$ with
  - coordinates $\sigma^\alpha$, $\alpha = 0, 1$
  - induced metric $G_{\alpha\beta} = \frac{\partial X^\mu}{\partial \sigma^\alpha}\frac{\partial X^\nu}{\partial \sigma^\beta} G_{\mu\nu}$
  - intrinsic metric $h_{\alpha\beta}$
- the Nambu-Goto action is the area of the string world-sheet

$$S_{\text{NG}} = \frac{1}{2\pi\alpha'} \int_\Sigma d^2\sigma \, \left|\text{det}\left(G_{\alpha\beta}\right)\right|^{1/2}$$

- the Polyakov action is the same, just using the intrinsic metric

$$S_{\text{P}} = \frac{1}{4\pi\alpha'} \int_\Sigma d^2\sigma \, \sqrt{h} h^{\alpha\beta} \partial_\alpha X^\mu \partial_\beta X^\nu G_{\mu\nu}$$

where $h = \text{det}(h_{\alpha\beta})$ and $\alpha'$ is the energy per length or tension.

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## Polyakov action

- the equations of motion of $h_{\alpha\beta}$ the Polyakov action are algebraic
  - the intrinsic metric $h_{\alpha\beta}$ is non-dynamical
  - the energy-momentum tensor of the $2D$ field theory is

$$T_{\alpha\beta} := \frac{1}{4\pi\alpha'\sqrt{h}}\frac{\delta S_{\mathrm{P}}}{\delta h^{\alpha\beta}} = \partial_\alpha X^\mu \partial_\beta X_\mu - h_{\alpha\beta}\partial_\gamma X^\mu \partial^\gamma X_\mu$$

  - so the equations of motion are the $2D$ Einstein equations

- the Polyakov action has three local symmetries:

$$\sigma^\alpha \to \tilde{\sigma}^\alpha(\sigma^0, \sigma^1) \qquad \text{and} \qquad h_{\alpha\beta}(\sigma) \to e^{\Lambda(\sigma)} h_{\alpha\beta}(\sigma)$$

  - can be used to gauge-fix the metric $h_{\alpha\beta} \doteq \eta_{\alpha\beta} = \mathrm{diag}(1, -1)$
  - introduce light cone coordinates $\sigma^\pm = \sigma^0 \pm \sigma^1$
  - but still has conformal invariance:

$$\sigma^+ \to \tilde{\sigma}^+(\sigma^+) \qquad \text{and} \qquad \sigma^- \to \tilde{\sigma}^-(\sigma^-)$$

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## Polyakov action

- in flat space $G_{\mu\nu} = \eta_{\mu\nu}$ one gets the $2D$ wave equation:

$$\partial^2 X^\mu = \partial_- \partial_+ X^\mu = 0$$

  with the general solutions

$$X^\mu(\sigma) = X^\mu_L(\sigma^+) + X^\mu_R(\sigma^-)$$

- imposing the boundary conditions gives open or closed strings
  - closed strings are obtained by $X^\mu(\sigma^0, \sigma^1 + \pi) = X^\mu(\sigma^0, \sigma^1)$ or

$$X^\mu(\sigma) = x^\mu + 2\alpha' p^\mu \sigma^0 + i\sqrt{2\alpha'} \sum_{n \neq 0} \frac{\alpha^\mu_n}{n} e^{-2in\sigma^+} + \frac{\tilde{\alpha}^\mu_n}{n} e^{-2in\sigma^-}$$

  - for open strings we can choose
    - Neumann boundary conditions $\partial_1 X^\mu|_{\sigma^1 = 0,\pi} = 0$ or
    - Dirichlet boundary conditions $X^\mu|_{\sigma^1 = 0,\pi} = x^\mu_{1,2}$
  - for Dirichlet boundary conditions the string has to end on a surface

    $\Rightarrow$ D-branes

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## Quantised bosonic string

- it turns out that QFT for the string is an unnecessary complication
- using light cone coordinates, one immediately gets $T_{+-} = T_{-+} = 0$
- Fourier transforming $T_{\pm\pm}$ gives the Virasoro generators

$$L_m := \int_0^{2\pi} d\sigma^+ T_{++} e^{im\sigma^+} \qquad \tilde{L}_m := \int_0^{2\pi} d\sigma^- T_{--} e^{im\sigma^-}$$

  which fulfill classically the Witt algebra of Poisson brackets

$$\{L_m, L_n\}_{\text{P.B.}} = i(m-n)L_{m+n}$$

- in terms of the $\alpha^\mu$: $L_m = -\frac{1}{2}\sum_{n=-\infty}^{\infty} \eta_{\mu\nu}\alpha^\mu_{m-n}\alpha^\nu_n$
- quantisation extends the Witt algebra to the Virasoro algebra

$$[L_m, L_n] = (m-n)L_{m+n} + \frac{c}{12}(m^3 - m)\delta_{m+n,0}$$

  - the central charge $c = \eta^{\mu\nu}\eta_{\mu\nu} = D$
  - the algebra only closes for $D = 26$ ⇒ critical bosonic string

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## Quantised bosonic string with gauge group

- introducing charges to an open string or an oriented string
  $=$ Chen-Paton factors; introduces gauge groups to the strings

## Superstring

- introduction of fermionic states
  - as target space fermions: Green-Schwarz superstring
  - by introducing world-sheet Supersymmetry: (RNS)
    - Ramond (R) with periodic boundary conditions
    - Neveu-Schwarz (NS) with antiperiodic boundary conditions
  - since there are left chiral and right chiral fermions
    - four sets of boundary conditions: R-R, NS-R, R-NS, NS-NS
  - gives a new critical dimension $D = 10$
- the GSO projection (Gliozzi, Scherk and Olive) removes the tachyon
  - and allows two different choices of chirality projections

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## Types of Superstring theories

- strings can be: open or closed, oriented or non-oriented, differently GSO projected, with different gauge groups

- taking the bosonic string and curling up the 16 additional dimensions:

  ⇒ a $10D$ string with a gauge group

  – combining with another superstring ⇒ heterotic string

| Type | open/closed | oriented | chiral | SUSY | gauge group |
|---|---|---|---|---|---|
| I | both | no | yes | $N = 1$ | $SO(32)$ |
| IIA | closed | yes | no | $N = 2A$ | — |
| IIB | closed | yes | yes | $N = 2B$ | — |
| heterotic | closed | yes | yes | $N = 1$ | $E_8 \times E_8$ |
| heterotic | closed | yes | yes | $N = 1$ | $SO(32)$ |

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## Compactification

- how to connect the $10D$ of Superstrings to our $4D$ world?
  - in 1921 Kaluza extended GR to $5D$
  - in 1926 Klein proposed, that the $4^{th}$ spatial dimension is curled up
  - ⇒ compactification of extra dimensions is called Kaluza-Klein (KK)

- $10D$ Superstrings use Calabi-Yau manifolds for compactification
  - a string can live on the curled up dimension
    - ∗ the momentum in that direction is quantised as $\frac{n}{R}$
    - ∗ the quantised momentum is seen like an addition to the mass
    - ⇒ KK-tower of states: $m^2 = m_0^2 + \frac{n}{R}$
  - a closed string can also wind around a curled up dimension
    - ∗ the quantised momentum gets an addition $n \cdot R$
    - ⇒ winding number states: $m^2 = m_0^2 + n \cdot R$
  - ⇒ duality of large and small radius of the compactification

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## Dualities

- T-duality relates compactification radii:

| $\mathrm{I} \Leftrightarrow \mathrm{I}'$ | $\mathrm{IIA} \Leftrightarrow \mathrm{IIB}$ | heterotic $SO(32) \Leftrightarrow$ heterotic $E_8 \times E_8$ |
|---|---|---|

- S-duality relates couplings
  - strong coupling in one theory to weak coupling in another:

| $\mathrm{I} \Leftrightarrow$ heterotic $SO(32)$ | $\mathrm{IIB} \Leftrightarrow \mathrm{IIB}$ |
|---|---|

- 1995 Ed. Witten found that $11D$ Supergravity is dual to Strings:

| | | |
|---|---|---|
| IIA | $\Leftrightarrow$ | $11D$ Sugra compactified on a circle |
| heterotic $E_8 \times E_8$ | $\Leftrightarrow$ | $11D$ Sugra compactified on an interval |

$\Rightarrow$ all five types of string theories are related

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## $M$-theory

- has the 5 Superstring theories and $11D$ Sugra as perturbative limits
- all 6 are related by dualities: "there has to be something in between"
  $\Rightarrow$ $M$-theory

## "Branes"-theory and black holes

- since Stringtheory today looks mostly at branes as dynamical objects
  $\Rightarrow$ "Branes"-theory rather than Stringtheory

**but:** branes as fundamental objects are not "free", even in a flat background

- black holes can be described by dimensional reduction of $p$-branes:
  - the Bekenstein-Hawking entropy $S_{\text{BH}} = \frac{A}{4}$ could be calculated by the microscopic degrees of freedom of the strings $S = \log N$

## Links

- `http://www.sukidog.com/jpierre/strings/index.html`
- `http://theory.tifr.res.in/%7Emukhi/Physics/string2.html`
- `http://online.kitp.ucsb.edu/online/plecture/witten/`