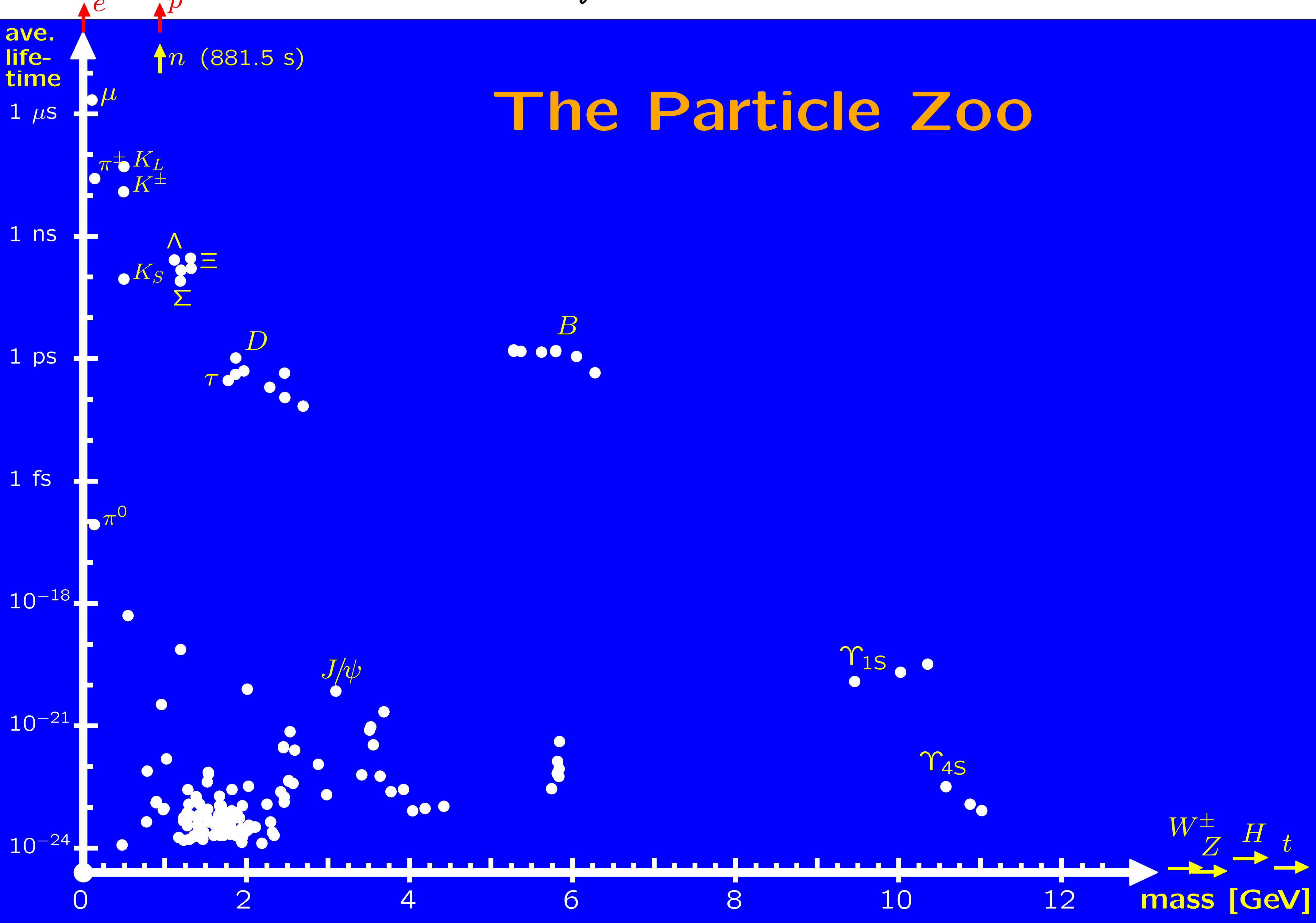
Symmetries

The Particle Zoo



Looking for some order in this ”chaos” . . .

1. properties of particles:

- order by **mass** (approximately, rather to be seen historically):

leptons	(Greek: ”light”)	electrons, muons, neutrinos, . . .
mesons	(”medium-weight”)	pions, kaons, . . .
baryons	(”heavy”)	protons, neutrons, lambda, . . .

- order by **charge**:

neutral	neutrons, neutrinos, photons, . . .
± 1 elementary charge	proton, electron, muon, . . .
± 2 elementary charge	Δ^{++}, Σ_c^{++}, . . .

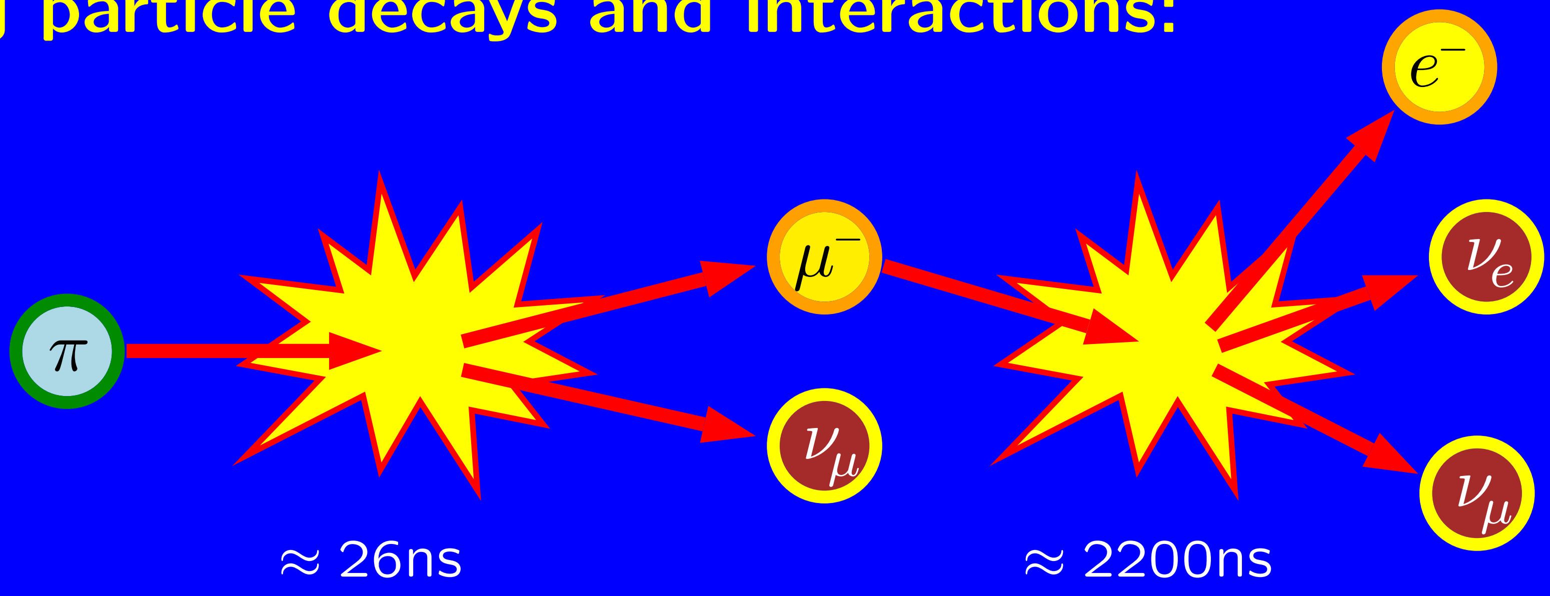
- order by **spin**:

fermions	(spin $\frac{1}{2}$, $1\frac{1}{2}$, . . .)	electrons, protons, neutrinos, . . .
bosons	(spin 0, 1, . . .)	photons, pions, . . .

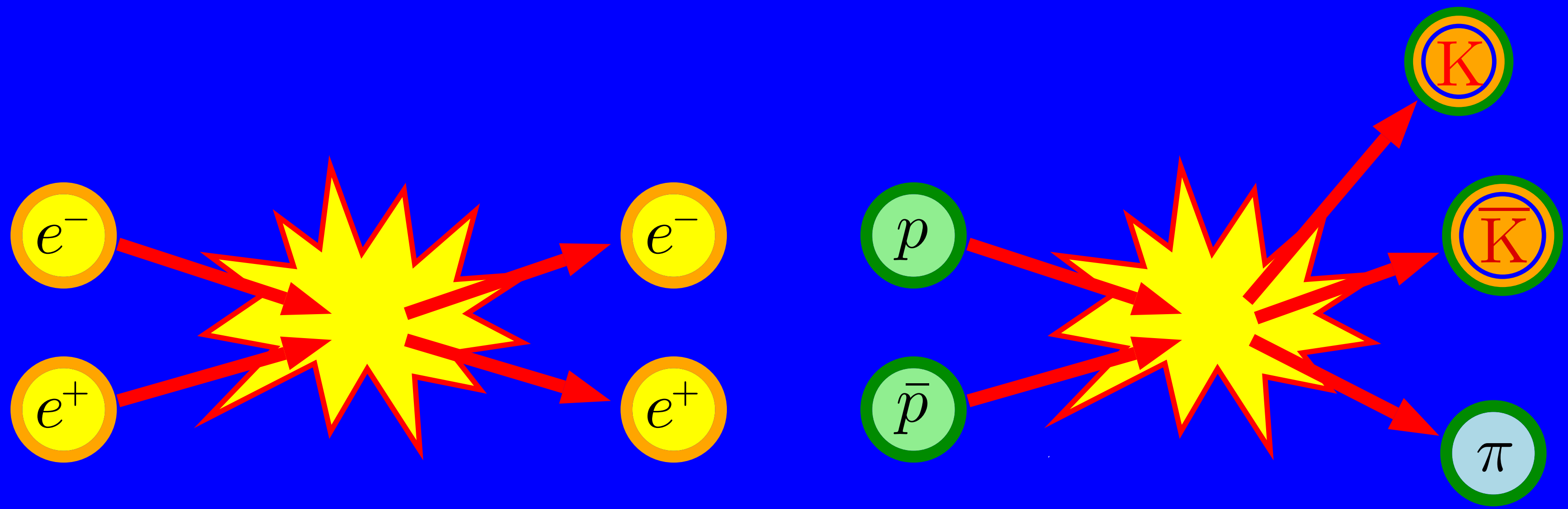
- order by **”strangeness”**, **parity**, . . .

Observing particle decays and interactions:

Decay



Scattering



Looking for some order in this "chaos" ...

2. conservation laws for particles:

- conservation of **energy**:

  $n \to p + \ldots$ but not $\pi^0 \to \pi^{\pm} + \ldots$

- conservation of **charge**:

  $n \to p + e^- + \ldots$ but not $n \to p + e^+ + \ldots$

- conservation of **lepton number**:

  $n \to p + e^- + \bar{\nu}_e$ but not $n \to p + e^- + \nu_e$

- conservation of **baryon number**:

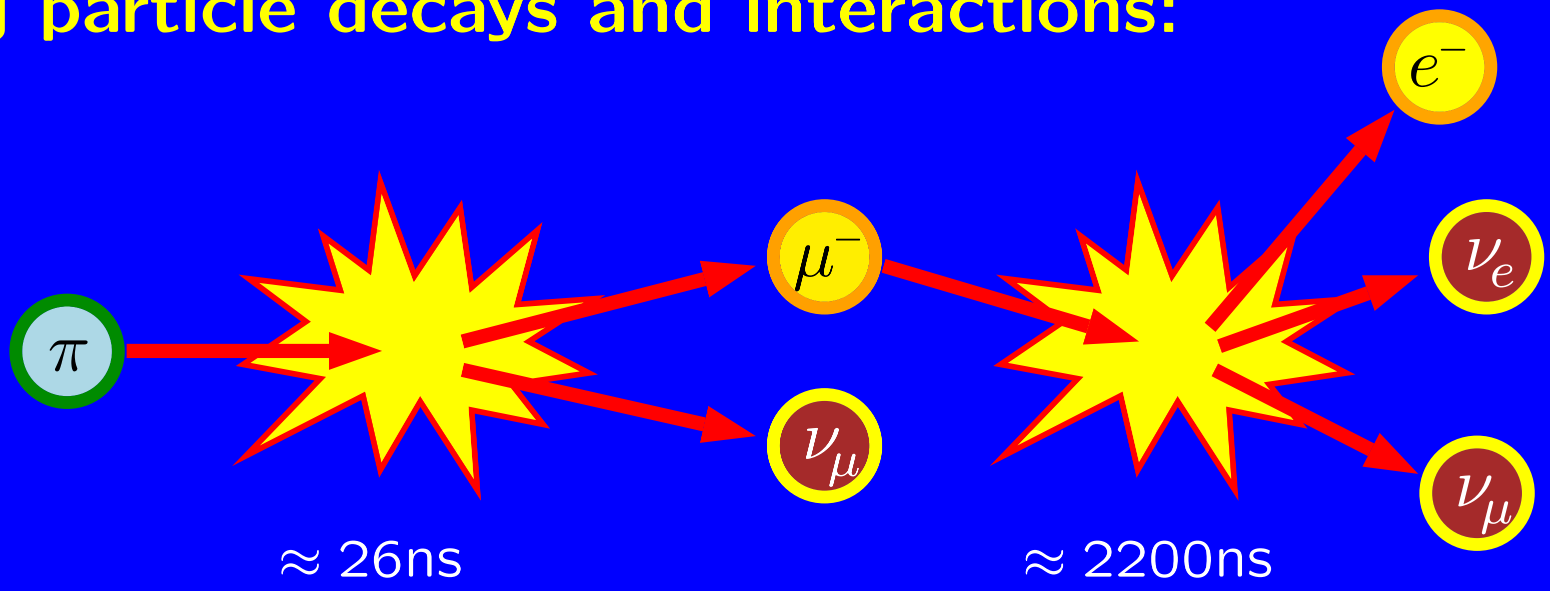
  $n \to p + \ldots$ but not $n \to \pi^+ + \pi^- + \ldots$

- conservation of **strangeness** (only in "fast" processes):

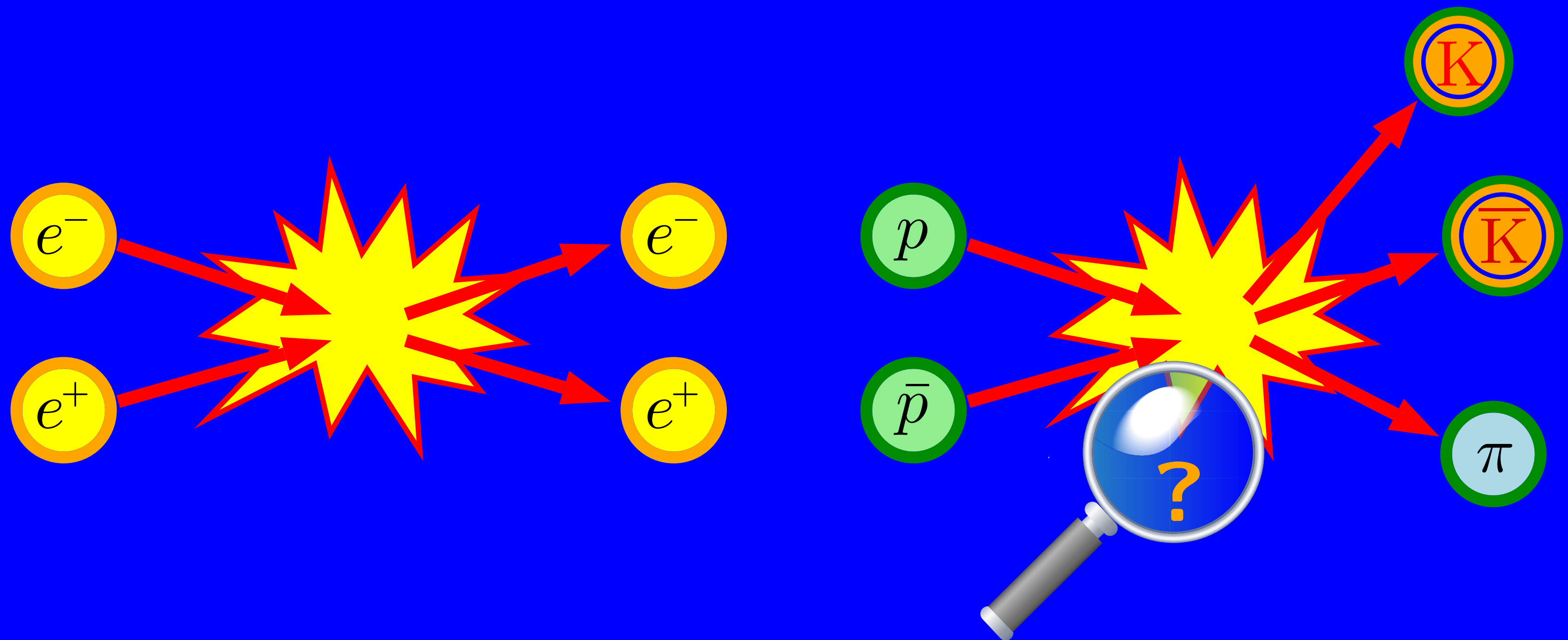
  fast $K^* \to K + \pi$ but only "slow" $K \to \pi + \pi$

Observing particle decays and interactions:

Decay

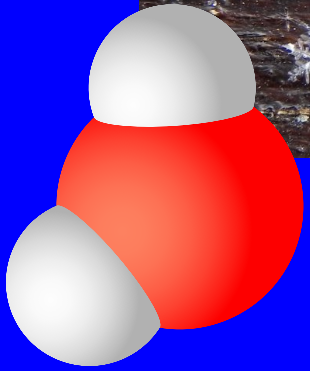


Scattering

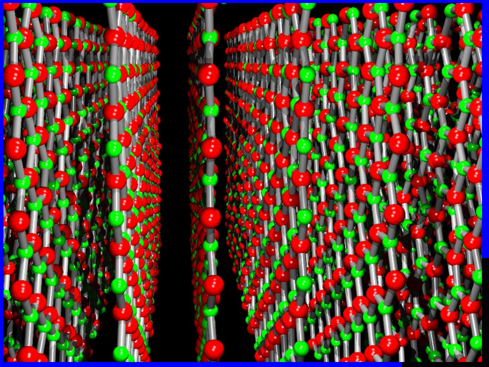


Symmetries & where do we find them?

→ **everywhere in nature:**



- **snowflakes exhibit a 6-fold symmetry**



- **crystals build lattices**

→ **symmetries of the microcosm are also visible in the macrocosm**

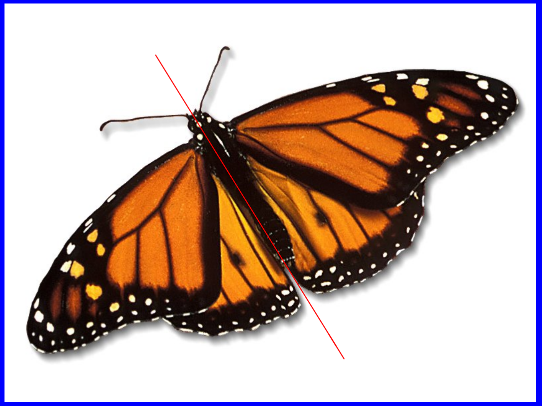
How do symmetries look like in theory?

★ symmetries are described by symmetry transformations:

Example 1: Butterfly

symmetry transformation S_0:

mirror all points at a line



formally: W = ''original picture'' $\Rightarrow$ W' = ''mirrored picture''

apply symmetry in operator notation: $S_0 W = W'$

a symmetry is given if and only if $S_0 W = W$!

How do symmetries look like in theory?

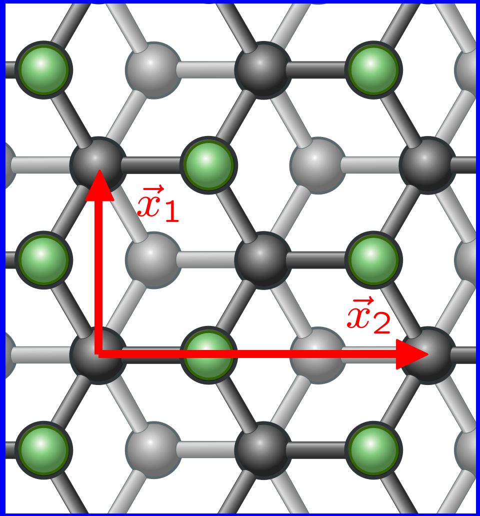
★ symmetries are described by symmetry transformations:

Example 2: crystal lattice

symmetry transformations S_i:

move all points

by the same vectors ($\vec{x}_1$ or $\vec{x}_2$)



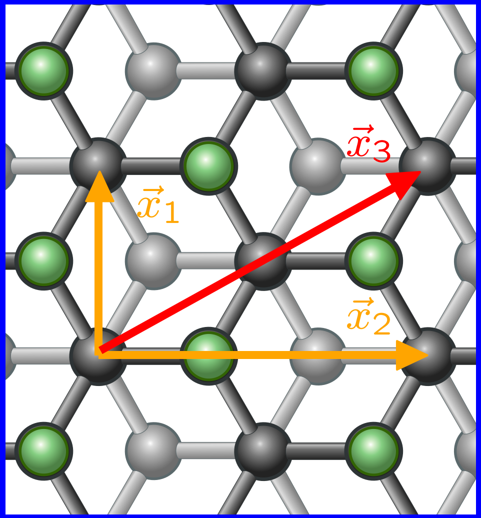
formally: $W =$ "original picture" $\Rightarrow$ $W' =$ "moved picture"

apply symmetry in operator notation: $S_i W = W'$

a symmetry is given <u>if and only if</u> $S_i W = W$!

How do symmetries look like in theory?

★ symmetry transformations form group structures



Example: translations

symmetry transformations S_1 and S_2:

move all points

by the same vector ($\vec{x}_1$ or $\vec{x}_2$)

movement of all points by the vector

$$\vec{x}_3 = \vec{x}_1 + \vec{x}_2$$

is also a symmetry transformation !

$$S_3 = S_1 \circ S_2$$

Groups, mathematically:

a **group** $(G, \circ)$ is a **set** $G = \{a, b, c, \ldots\}$ with a **binary operation** $\circ$ that fulfills (the axioms)

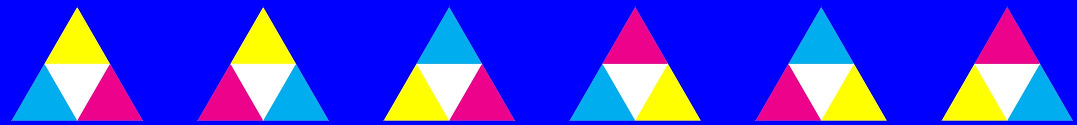
- **closure:** $c = a \circ b \in G \quad \Leftrightarrow \quad a, b \in G$
- **associativity:** $(a \circ b) \circ c = a \circ (b \circ c)$
- **identity:** $\exists\, e$ with $a \circ e = e \circ a = a \quad \forall a \in G$
- **inverse:** $\forall a \in G \quad \exists\, b = a^{-1}$ (the inverse)
  with $a \circ b = b \circ a = e$

an **Abelian group** fulfills an additional relation

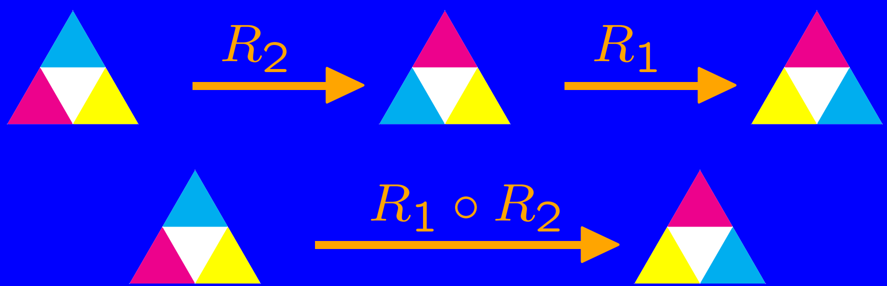
- **commutativity:** $a \circ b = b \circ a \quad \forall a, b \in G$

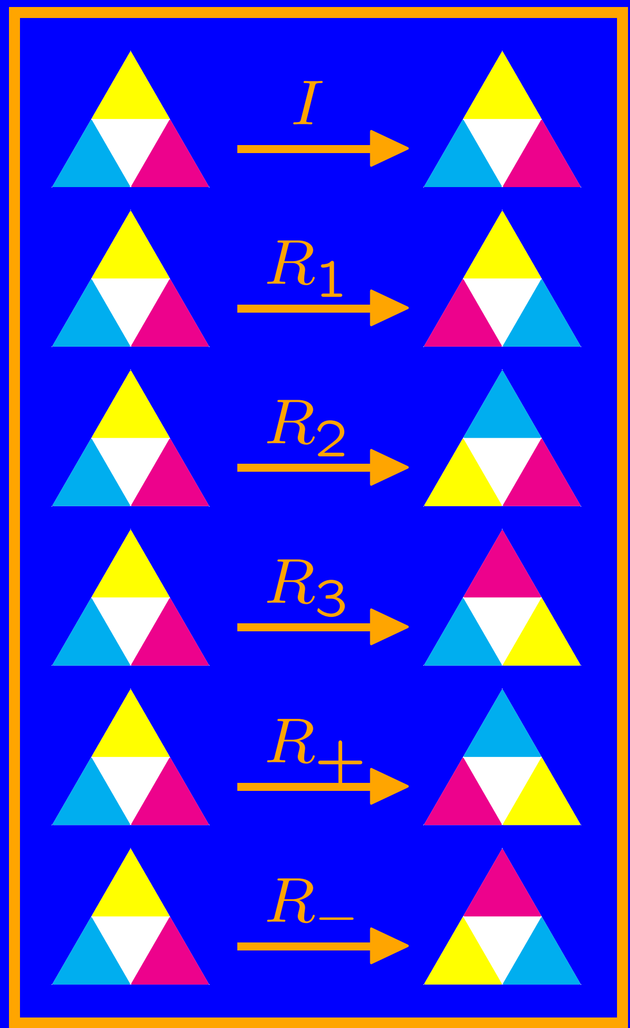
Groups, an example:

using these six triangles,
we can construct a group:



- the triangles themselves will not be elements
  - as we have no clue, how to connect them
- their **relations** will be elements of a group!
  - we know, how we can transform one triangle into the other
  - then the set is $\{I, R_1, R_2, R_3, R_+, R_-\}$
- then these transformations can be connected:
  - do first one, then the other:
    $R_1 \circ R_2 =$ doing first R_2, then R_1





Which symmetries do we encounter in particle physics?

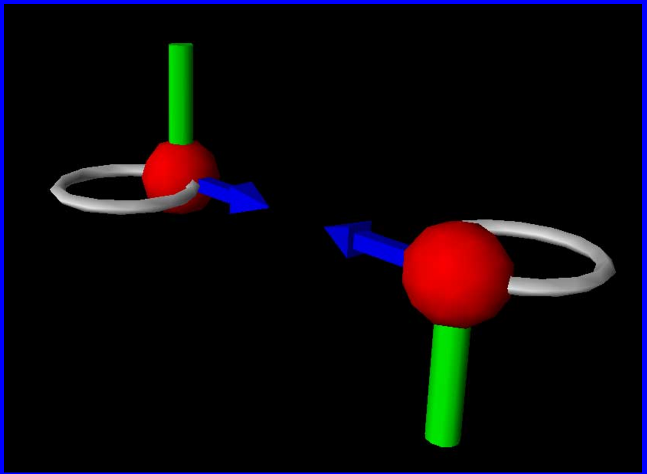
★ **discrete** symmetry transformations:

parity transformation $\mathbf{P}$

- to mirror at a plane (a mirror) is easy to understand, but depends on the (arbitrary) position and orientation of the plane.
- a more general definition: **mirror at the origin**

  (space inversion, parity transformation):

  $$\mathbf{P}W(t, x, y, z) := W(t, -x, -y, -z)$$



- the parity transformation corresponds to a rotation followed by a mirroring at a plane

Which symmetries do we encounter in particle physics?

★ **discrete** symmetry transformations:

time reversal T (reversal of the ”arrow of time”)

- corresponds to a movie played backwards
- in case of a movie (= everday physics), this is spotted at once (i.e. there is no symmetry)
- however, the laws of mechanics are time-symmetric (example: billiard)

- definition:

$$\mathbf{T}W(t,x,y,z) := W(-t,x,y,z)$$

Which symmetries do we encounter in particle physics?

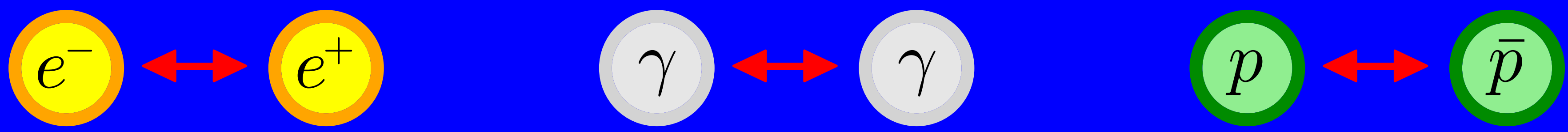
★ **discrete** symmetry transformations:

charge conjugation C (exchanging matter and anti-matter)

- for every known particle, there is also a anti-partner
- anti particles are identical to their partners with respect to some properties (e.g. mass), and opposite w.r.t. others (e.g. charge)
- charge conjugation exchanges all particles with their (anti)partners (and vice versa)

- definition:

$$\mathrm{C}W(t,x,y,z) := \bar{W}(t,x,y,z) = W^{\dagger}(t,x,y,z)$$



Which symmetries do we encounter in particle physics?

★ **continuous** symmetry transformations:

– they can be performed in **arbitrary small steps**

- **time shift**: physics(today) ⟶ physics(tomorrow)
  – more accurately: shift by a time-step Δt

$$e^{\Delta t \frac{\partial}{\partial t}} W(t, x, y, z) = W(t + \Delta t, x, y, z)$$

- **space shift**: physics(here) ⟶ physics(there)
  – more accurately: shift in space by a vector $\Delta\vec{r} = (\Delta x, \Delta y, \Delta z)$

$$e^{\Delta\vec{r}\cdot\vec{\nabla}} W(t, x, y, z) = W(t, x + \Delta x, y + \Delta y, z + \Delta z)$$

Which symmetries do we encounter in particle physics?

★ **continuous** symmetry transformations:

– they can be performed in **arbitrary small steps**

- **orientation**: physics(north) ➡ physics(west)
  – more accurately: rotation around an arbitrary axis in space

$$DW(t,x,y,z) = W(t,x',y',z')$$

Which symmetries do we encounter in particle physics?

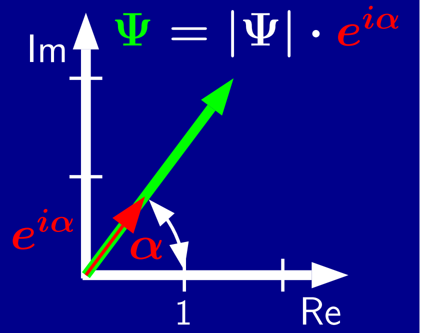
★ **continuous** symmetry transformations:

– they can be performed in **arbitrary small steps**

- $U(1)$ **transformation**:
  - does not affect the outer coordinates (t, x, y, z), but inner properties of particles
  - $U(1)$ is a transformation, which rotates the phase of a particle field (denoted as Ψ) by an angle α:

$$U(1)\Psi(t,x,y,z) = e^{i\alpha}\Psi(t,x,y,z)$$

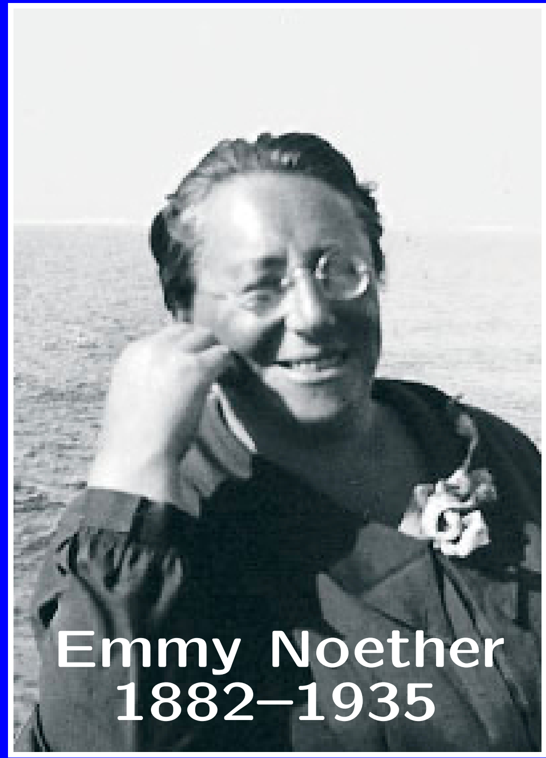
<u>**insertion**</u>: **particles** are represented by fields in quantum field theory. At each point in space and time, the field Ψ can have a certain complex phase.



The fundamental importance of symmetries

★ Noether's theorem:

to each symmetry of a field theory corresponds a certain conserved quantity → conservation law



that means: if a field theory remains unchanged under a certain symmetry transformation S, then there is a mathematical procedure to calculate a property of the field which does not change with time, whatever complicated processes are involved.

The fundamental importance of symmetries

★ applications of Noether's theorem:

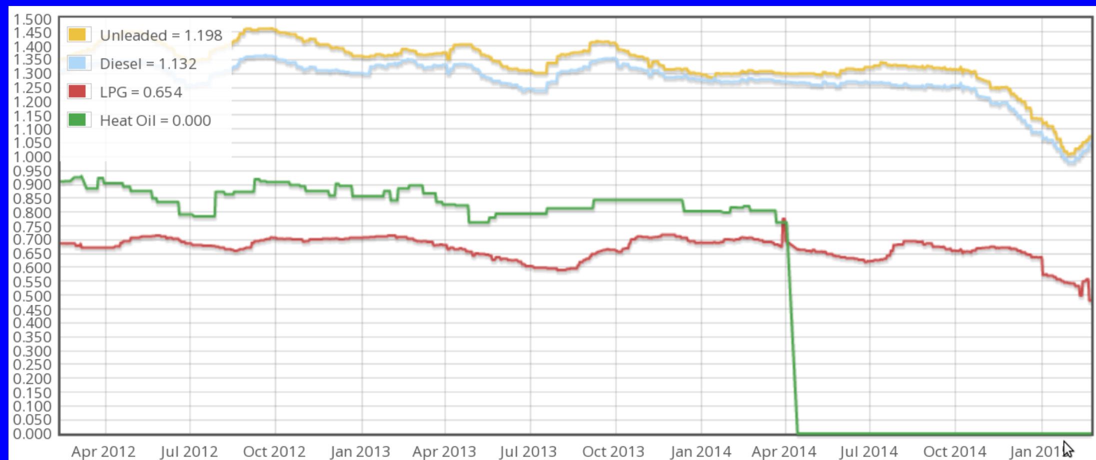
"also tomorrow the sun will rise"

→ conservation of energy

- the laws of physics do not change with time
- more accurate: the corresponding field theory is invariant under time shifts:

$$e^{\Delta t \frac{\partial}{\partial t}} W(t,x,y,z) = W(t+\Delta t,x,y,z) \doteq W(t,x,y,z)$$

From Noether's theorem follows the conservation of a well-known property: energy!



The fundamental importance of symmetries

★ applications of Noether's theorem:

''also tomorrow the sun will rise''

$\longrightarrow$ conservation of energy

- the laws of physics do not change with time
- more accurate: the corresponding field theory is invariant under time shifts:

$$e^{\Delta t \frac{\partial}{\partial t}} W(t,x,y,z) = W(t+\Delta t,x,y,z) \doteq \boldsymbol{W(t,x,y,z)}$$

From Noether's theorem follows the conservation of a well-known property: **energy**!

$E = mc^2$

. . . and that's it!

just to be clear:

★ we are talking about properties of the underlying theory, not a certain physics scenario:

Example: chess:

- there is virtually an infinite number of ways a game of chess can develop
- a game tomorrow can be completely different from a game today

but:

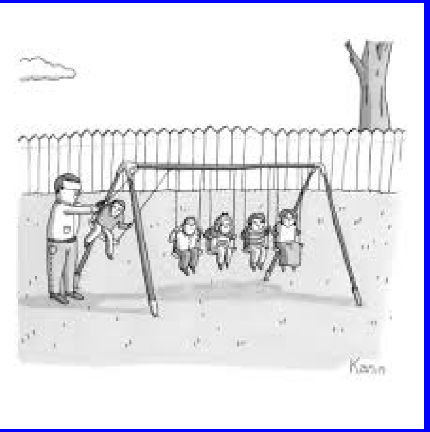
- the rules of chess remain the same, they are invariant under time shifts!

The fundamental importance of symmetries

★ applications of Noether's theorem:

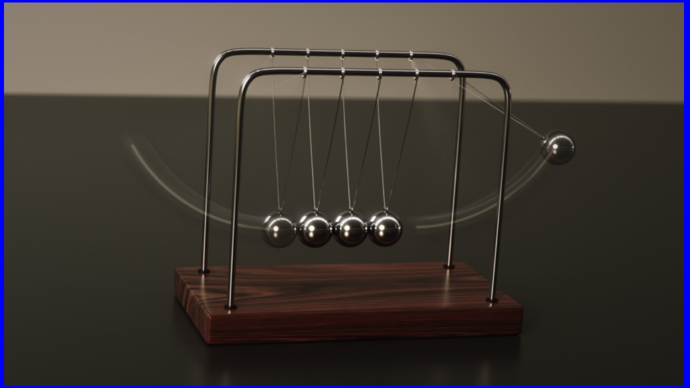
''it's the same everywhere''

→ **conservation of momentum**



- the laws of physics do not depend on where you are
- more accurate: the corresponding field theory is invariant under space shifts:

$$e^{\Delta\vec{r}\cdot\vec{\nabla}} W(t,\vec{r}) = W(t,\vec{r}+\Delta\vec{r}) \doteq \boldsymbol{W(t,\vec{r})}$$



From Noether's theorem follows the conservation of a well-known property: momentum!

The fundamental importance of symmetries

★ applications of Noether's theorem:

''going round and round''

→ conservation of angular momentum

- the laws of physics do not depend on which way you look
- more accurate: the corresponding field theory is invariant under rotations:

$$DW(t,\vec{r}) = W(t,\vec{r}') \doteq \boldsymbol{W(t,\vec{r})}$$

From Noether's theorem follows the conservation of a well-known property: angular momentum!

The fundamental importance of symmetries

★ applications of Noether's theorem:

even more abstract symmetries get a meaning:

★ conservation of charge

- as it turns out, the field theory of electro-dynamics is invariant under a global* U(1) transformation:

$$U(1)\Psi(t,x,y,z) = e^{i\alpha}\Psi(t,x,y,z)$$
$$\Rightarrow W'(t,x,y,z) \doteq W(t,x,y,z)$$

* global means: affecting all space-points (t,x,y,z) in the same way

From Noether's theorem follows the conservation of charge!

Overview

symmetries and conservation laws

symmetry	conservation law
time shift	energy
space shift	momentum
rotation	angular momentum
$U(1)$ phase	charge