Homework: Symmetries **— due 2018/02/27, 17:00**

David Griffiths, Chapter 4, pp. 137-138, n. 4.1, n. 4.2, n. 4.6, and n. 4.7
and David Griffiths, Chapter 3, pp. 100-102, n. 3.8:

4.1. Prove that I, R_+, R_-, R_a, R_b, and R_c are *all* the symmetries of the equilateral triangle. 2 POINTS

4.2. Construct a "multiplication table" for the triangle group.
Is this an Abelian group? How can you tell, just by looking at the multiplication table? 2 POINTS

4.6. Consider a vector $\vec{a}$ in two dimensions. Suppose its components with respect to Cartesian axes x, y, are (a_x, a_y). What are its components (a'_x, a'_y) in a system x', y' which is rotated, counterclockwise, by an angle θ, with respect to x, y? Express your answer in the for of a 2×2 matrix $R(\theta)$:

$$\begin{pmatrix} a'_x \\ a'_y \end{pmatrix} = R(\theta) \begin{pmatrix} a_x \\ a_y \end{pmatrix}$$

Show that R is an orthogonal matrix. What is its determinant? The set of *all* such rotations constitutes a group; what is the name of this group? By multiplying the matrices show that $R(\theta_1)R(\theta_2) = R(\theta_1 + \theta_2)$; is this an Abelian group? 2 POINTS

4.7. Consider the matrix $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$. Is it in the group $O(2)$? How about $SO(2)$? What is its effect on the vector $\vec{a}$ of Problem 4.6? Does it describe a possible rotation of the plane? 2 POINTS

3.8. A second–rank tensor is called *symmetric* if it is unchanged when you switch the indices ($s^{\nu\mu} = s^{\mu\nu}$); it is called *antisymmetric* if it changes sign ($a^{\nu\mu} = -a^{\mu\nu}$).

(a) How many independent elements are there in a symmetric tensor? (Since $s^{12} = s^{21}$, these would count as only *one* independent element.) 0.2 POINTS

(b) How many independent elements are there in an antisymmetric tensor? 0.2 POINTS

(c) If $s^{\mu\nu}$ is symmetric, show that $s_{\mu\nu}$ is also symmetric.
If $a^{\mu\nu}$ is antisymmetric, show that $a_{\mu\nu}$ is antisymmetric. 0.2 POINTS

(d) If $s^{\mu\nu}$ is symmetric and $a^{\mu\nu}$ is antisymmetric, show that $s^{\mu\nu}a_{\mu\nu} = 0$. 0.2 POINTS

(e) Show that any second–rank tensor ($t^{\mu\nu}$) can be written as the sum of an antisymmetric part ($a^{\mu\nu}$) and a symmetric part ($s^{\mu\nu}$): ($t^{\mu\nu} = a^{\mu\nu} + s^{\mu\nu}$). Construct ($a^{\mu\nu}$) and ($s^{\mu\nu}$) explicitly, given ($t^{\mu\nu}$). 0.2 POINTS

Remark: It helps to read the corresponding chapters in Griffiths ...

... the numbers of the exercises are from the second edition, 1987; the library has a newer edition with different numbers ...

if this is not enough for 3.8, it might help to read "Special Relativity for Particle Physics":
`http://web.vu.lt/ff/t.gajdosik/files/2014/01/sr4wop.pdf`

Homework: Lorenztransformations — due 2018/03/13, 17:00

David Griffiths, Chapter 3, pp. 100-102, n. 3.3, n. 3.4, n. 3.6, n. 3.7, and 3.10:

3.3. Transformation between the frames S' and S, which are moving with the speed v relative to each other:

(a) How do *volumes* transform? Specifically, if a container has volume V' in its own rest frame, S', what is its volume V as measured by an observer in S, with respect to which is is moving at speed v. 0.4 POINTS

(b) How do *densities* transform? — If a container holds ρ' molecules per unit volume in its own rest frame, S', how many molecules per unit volume does it carry in S? 0.4 POINTS

3.4. Cosmic ray muons are produced high in the atmosphere (at 8000 m, say) and travel toward the earth at very nearly the speed of light, (0.998 c, say). The speed of light is roughly $3. \times 10^8$ m/s.

(a) Given the lifetime of the muon (2.2×10^{-6} s), how far would the average muon go before disintegrating, according to prerelativistic physics? Would the muon make it to ground level? 0.4 POINTS

(b) Now answer the same question using *relativistic* physics. (Because of time dilation, the muons last longer, so they travel farther.) 0.4 POINTS

(c) Now analyze the same process from the perspective of the *muon*. (In *its* reference frame it only lasts 2.2×10^{-6} s; how then does it make it to the ground?) 0.4 POINTS

(d) Pions are also produced in the upper atmosphere. [In fact, the sequence is: proton (from outer space) hits proton (in atmosphere) $\rightarrow p + p +$ pions. The pions then decay into muons: $\pi^{\pm} \rightarrow \mu^{\pm} + \nu_\mu(\bar{\nu}_\mu)$.] But the lifetime of the pion is much shorter, a hundredth of the muon. Should the pions reach ground level? (Assume that the pions have also a speed of 0.998 c.) 0.4 POINTS

3.6. Find the matrix $\mathbf{M}$ that inverts the Lorentz transformation Λ: $x^\mu = M^\mu_\nu x^{\nu\prime}$. Show that $\mathbf{M}$ is the matrix inverse of Λ: $\Lambda\mathbf{M} = 1$. 0.5 POINTS

3.7. Show that $\mathbf{I} = x_\mu x^\mu$ is invariant under the Lorentz transformation Λ. 0.4 POINTS

3.10. Consider a collision in which particle A (mass m_A and velocity $\vec{v}_A$) hits particle B (mass m_B and velocity $\vec{v}_B$), producing particles C (m_C, $\vec{v}_C$) and D (m_D, $\vec{v}_D$). Assume energy momentum conservation in system S (i.e. $p^\mu_A + p^\mu_B = p^\mu_C + p^\mu_D$). Using the Lorentz transformation $\Lambda(\eta)$, show that energy and momentum are also conserved in system S'. 0.7 POINTS

Homework: Particle kinematics I — due 2018/04/03, 17:00
David Hoggs, Chapter 6, p. 34, Problems 6.7, 6.8, 6.9, and 6.10,
and David Griffiths, Chapter 3, pp. 100-102, n. 3.22 :

6.7. A particle of mass M, at rest, decays into two smaller particles of masses m_1 and m_2. What are their energies and momenta? 1 POINTS

6.8. Solve problem 6.7 again for the case $m_2 = 0$. Solve the equations for p and E_1 and then take the limit $m_1 \to 0$. 1 POINTS

6.9. If a massive particle decays into photons, explain using 4-momenta why it cannot decay into a single photon, but must decay into two or more. Does your explanation still hold if the particle is moving at high speed when it decays? 2 POINTS

6.10. A particle of rest mass M, travelling at speed v in the x-direction, decays into two photons, moving in the positive and negative x-direction relative to the original particle. What are their energies? What are the photon energies and directions if the photons are emitted in the positive and negative y-direction relative to the original particle (i.e., perpendicular to the direction of motion, in the particles rest frame). 2+2 POINTS

3.22. In a two–body scattering event, $(A + B \to C + D)$, it is convenient to introduce the *Mandelstam variables*

$$s = (p_A + p_B)^2 \qquad t = (p_A - p_C)^2 \qquad u = (p_A - p_D)^2$$

(a) Show that $s + t + u = m_A^2 + m_B^2 + m_C^2 + m_D^2$. 0.7 POINTS

The *theoretical* virtue of the Mandelstam variables is that they are Lorentz invariants, with the same value in any inertial system. *Experimentally*, though, the more accessible parameters are energies and scattering angles.

(b) Find the CM energy of A, in terms of s, t, u, and the masses. 0.7 POINTS

(c) Find the Lab (B at rest) energy of A. 0.7 POINTS

(d) Find the total CM energy ($E_{\mathrm{TOT}} = E_A + E_B = E_C + E_D$). 0.7 POINTS

Homework: Particle kinematics II — due 2018/04/17, 17:00

David Griffiths, Chapter 3, pp. 100-102, n. 3.14, n. 3.16, n. 3.18 and n. 3.24:

3.14. Particle A (energy E) hits particle B (at rest), producing particles C_1, C_2, $\ldots C_n$. Calculate the threshold (i.e. the minimum E) for this reaction, in terms of the various particle masses. 1 POINTS

3.16. Particle A, at rest, decays into particles B and C ($A \to B + C$).

(a) Find the energy of the outgoing particles in terms of the various masses. 0.7 POINTS

(b) Find the magnitude of the outgoing momenta. 0.7 POINTS

(c) $|\vec{p}_B|$ goes to zero when $m_A = m_B + m_C$, and runs imaginary when $m_A < m_B + m_C$. Explain. 0.5 POINTS

3.18. (a) A pion at rest decays into a muon and a neutrino ($\pi^- \to \mu^- + \bar{\nu}_\mu$). On the average, how far will the muon travel (in vacuum) before disintegrating? 1 POINTS

(b) The length in the muon track in Figure 1.7 is about 0.6 mm (the photograph has been enlarged). How do you explain this? 0.5 POINTS

3.24. (Compton scattering.) A photon of wavelength λ collides elastically with a charged particle of mass m. If the photon scatters at angle θ, find its outgoing wavelength, λ'; use $E_\gamma = h/\lambda$. 1.8 POINTS