Supersymmetry (SUSY) — Overview

General features of Supersymmetry

- connects bosons and fermions
- provides an extension to the Poincaré algebra
- unbroken global Supersymmetry (in quantum field theory)
  - bosons and fermions of the same superfield have the same mass
  - the energy of the vacuum is always positive
  - loop corrections involving superfields vanish

    $\Rightarrow$ there is no renormalisation of couplings or masses
- broken global Supersymmetry (in quantum field theory)
  - bosonic and fermionic loop corrections nearly cancel each other

    $\Rightarrow$ solves the hierarchy problem of the Standardmodel
  - better unification of couplings: Grand unified Theories (GUTs)
- local Supersymmetry
  - includes Gravity $\Rightarrow$ Supergravity (SUGRA)
- needed for consistent 10 dimensional Stringtheory $\Rightarrow$ Superstrings

Supersymmetry (SUSY) — Overview

Supersymmetry as an extension of the Poincaré algebra

- the Poincaré algebra describes space-time transformations (of particles)
- bosons and fermions transform separately
  - angular momentum operator changes spin by integer units
  - different statistics of bosons and fermions

⇒ so all "normal" symmetries are bosonic

- according to the Coleman-Mandula theorem (a no-go theorem)
  - all symmetry generators have to commute
    with the generators of the Poincaré algebra
    ⇒ internal symmetries cannot change space-time properties of particles
  - no other symmetries are allowed for a meaningful QFT
- Supersymmetry evades this restriction by using anticommutators
  - Supersymmetry generators are fermionic
  - they extend the Poincaré algebra to the Super-Poincaré algebra

The Super-Poincaré algebra

- the Poincaré algebra with generators M_{mn} and P_m
  (I will use greek letters for spinor indices and latin letters for space time indices)

$$[M_{mn}, M_{k\ell}] = i(g_{mk}M_{n\ell} - g_{nk}M_{m\ell} - g_{m\ell}M_{nk} + g_{n\ell}M_{mk}) \quad ,$$

$$[P_m, P_n] = 0 \qquad \text{and} \qquad [M_{mn}, P_k] = i(g_{mk}P_n - g_{nk}P_m)$$

- is extended by the Supersymmetry generators Q_α and $\bar{Q}_{\dot{\alpha}}$ to

$$[M_{mn}, Q_\alpha] = \tfrac{1}{2}(\sigma_{mn})_\alpha{}^\beta Q_\beta \quad , \qquad [M_{mn}, \bar{Q}_{\dot{\alpha}}] = -\tfrac{1}{2}(\bar{\sigma}_{mn})^{\dot{\beta}}{}_{\dot{\alpha}}\bar{Q}_{\dot{\beta}} \quad ,$$

$\Rightarrow$ Q_α and $\bar{Q}_{\dot{\alpha}}$ transform as spinors under Lorentz transformations

$$\{Q_\alpha, \bar{Q}_{\dot{\alpha}}\} = 2\sigma^m_{\alpha\dot{\alpha}}P_m \quad , \qquad \text{and} \qquad [Q_\alpha, P_m] = [\bar{Q}_{\dot{\alpha}}, P_m] = 0$$

$\Rightarrow$ the algebra closes

- mass dimensions of the generators:
  - P_m (like the energy) has 1: $[P_m] = 1$
  - M_{mn} (like the rotation) has 0: $[M_{mn}] = 0$
  - Q, $\bar{Q}$ have half the dimension of P^μ $\Rightarrow$ $[Q] = [\bar{Q}] = \frac{1}{2}$

Superspace

- the normal coordinates x^μ can be extended to $x^M = (x^m, \theta^\mu, \bar{\theta}_{\dot{\mu}})$
  - θ^α, $\bar{\theta}_{\dot{\alpha}}$ are Grassman valued coordinates with indices $\alpha, \dot{\alpha} = 1, 2$
  - ^{M} represents the set of indices $(^m, ^\mu, _{\dot{\mu}})$

- The SUSY generators can be represented by differential operators:

$$Q_\alpha = \frac{\partial}{\partial\theta^\alpha} + i\sigma^m_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\frac{\partial}{\partial x^m} \qquad \text{and} \qquad \bar{Q}_{\dot{\alpha}} = -\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} - i\theta^\alpha\sigma^m_{\alpha\dot{\alpha}}\frac{\partial}{\partial x^m}$$

  - similar to $P_m = -i\frac{\partial}{\partial x^m}$

- it is convenient to introduce differential operators

$$D_\alpha = \frac{\partial}{\partial\theta^\alpha} - i\sigma^m_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\frac{\partial}{\partial x^m} \qquad \text{and} \qquad \bar{D}_{\dot{\alpha}} = -\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} + i\theta^\alpha\sigma^m_{\alpha\dot{\alpha}}\frac{\partial}{\partial x^m}$$

  - they anticommute with Q_α and $\bar{Q}_{\dot{\alpha}}$ and have a similar algebra:

$$\{D_\alpha, Q_\beta\} = \{D_\alpha, \bar{Q}_{\dot{\beta}}\} = \{\bar{D}_{\dot{\alpha}}, Q_\beta\} = \{\bar{D}_{\dot{\alpha}}, \bar{Q}_{\dot{\beta}}\} = 0$$

$$\{D_\alpha, D_\beta\} = \{\bar{D}_{\dot{\alpha}}, \bar{D}_{\dot{\beta}}\} = 0 \qquad \text{and} \qquad \{D_\alpha, \bar{D}_{\dot{\beta}}\} = 2i\sigma^m_{\alpha\dot{\beta}}\partial_m$$

Superfields in Superspace

- a Superfield S is a function of $x^M = (x^m, \theta^\mu, \bar{\theta}_{\dot{\mu}})$
- it can be expanded in component fields
  - the expansion terminates since $(\theta^\alpha)^2 = (\bar{\theta}_{\dot{\alpha}})^2 = 0$
  - the highest term is the coefficient of $\theta\theta := \theta^\alpha\theta_\alpha$ or $\bar{\theta}\bar{\theta} := \bar{\theta}_{\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}$

$$S = s + \theta\phi + \bar{\theta}\bar{\chi} + \theta\theta\, M + \bar{\theta}\bar{\theta}\, N + \theta\sigma^m\bar{\theta}v_m + \theta\theta\,\bar{\theta}\bar{\lambda} + \bar{\theta}\bar{\theta}\,\theta\psi + \theta\theta\,\bar{\theta}\bar{\theta}\, D$$

- The SUSY transformation with spinorial parameter η is

$$\delta_\eta = \eta Q + \bar{\eta}\bar{Q} = \eta^\alpha Q_\alpha + \bar{\eta}_{\dot{\alpha}}\bar{Q}^{\dot{\alpha}} = \eta^\alpha Q_\alpha - \bar{\eta}^{\dot{\alpha}}\bar{Q}_{\dot{\alpha}}$$

  - it gives the transformation property of the component fields:

$$\begin{aligned} \delta_\eta S &= \delta_\eta s + \theta\delta_\eta\chi + \bar{\theta}\delta_\eta\bar{\psi} + \theta\theta\,\delta_\eta M + \bar{\theta}\bar{\theta}\,\delta_\eta N + \theta\sigma^\mu\bar{\theta}\delta_\eta v_\mu \\ &\quad + \theta\theta\,\bar{\theta}\delta_\eta\bar{\lambda} + \bar{\theta}\bar{\theta}\,\theta\delta_\eta\lambda + \theta\theta\,\bar{\theta}\bar{\theta}\,\delta_\eta D \\ &= (\eta Q + \bar{\eta}\bar{Q})S = (\eta\tfrac{\partial}{\partial\theta} + i(\eta\sigma^m\bar{\theta})\partial_m - \bar{\eta}\tfrac{\partial}{\partial\bar{\theta}} - i(\theta\sigma^m\bar{\eta})\partial_m)S \end{aligned}$$

  ... by comparing the coefficients of the θs

Supersymmetry (SUSY) — Theory

Superfields containing bosons and fermions

- Superfields describe multiplets of component fields
  - all fields in S , i.e. $(s, \chi, \bar{\chi}, M, N, v_\mu, \bar{\lambda}, \lambda, D)$,

    have to have all the same quantum numbers — except spin

- but there are too many (unnecessary) degrees of freedom (dof)
  - in a similar way like in gauge fields ...

$\Rightarrow$ using constraints:

- the differential operators D anticommute with the Qs

  $\Rightarrow$ constraints using Ds are unaffected by SUSY transformations
  - this is their convenience

Constraints on Superfields

- chiral Superfields are defined by $\bar{D}_{\dot{\alpha}}\Phi = D_\alpha \Phi^\dagger = 0$
  - this constraint can be solved explicitely by

$$\Phi(x,\theta,\bar{\theta}) = \Phi(y,\theta,0) \qquad \text{with} \quad y^m = x^m - i\theta\sigma^m\bar{\theta}$$

$\Rightarrow$ it reduces the field to three components: (no $\bar{\theta}$ can appear)

$$\Phi(y,\theta) = \phi(y) + \theta\psi(y) + \tfrac{1}{2}\theta\theta\, F(y)$$

  - the SUSY transformation δ_η acting on this chiral superfield is

$$\begin{aligned}
\delta_\eta &= (\eta^\alpha \tfrac{\partial}{\partial\theta^\alpha} + \bar{\eta}_{\dot{\alpha}} \tfrac{\partial}{\partial\bar{\theta}_{\dot{\alpha}}} + i(\eta^\alpha\sigma^m_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}} - \theta^\alpha\sigma^m_{\alpha\dot{\alpha}}\bar{\eta}^{\dot{\alpha}})\partial_m) \\
&= (\eta^\alpha[\tfrac{\partial}{\partial\theta^\alpha} + \tfrac{\partial y^n}{\partial\theta^\alpha}\partial^y_n] - \bar{\eta}_{\dot{\alpha}} \tfrac{\partial}{\partial\bar{\theta}_{\dot{\alpha}}} + \bar{\eta}^{\dot{\alpha}}\tfrac{\partial y^n}{\partial\bar{\theta}^{\dot{\alpha}}}\partial^y_n + i(\eta^\alpha\sigma^m_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}} - \theta^\alpha\sigma^m_{\alpha\dot{\alpha}}\bar{\eta}^{\dot{\alpha}})[\partial_m y^n]\partial^y_n) \\
&= (\eta^\alpha \tfrac{\partial}{\partial\theta^\alpha} - i\eta^\alpha\sigma^n_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial^y_n - \bar{\eta}_{\dot{\alpha}} \tfrac{\partial}{\partial\bar{\theta}_{\dot{\alpha}}} + \bar{\eta}^{\dot{\alpha}} i\theta^\alpha\sigma^n_{\alpha\dot{\alpha}}\partial^y_n + i(\eta^\alpha\sigma^m_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}} - \theta^\alpha\sigma^m_{\alpha\dot{\alpha}}\bar{\eta}^{\dot{\alpha}})[\delta^n_m]\partial^y_n) \\
&= (\eta^\alpha \tfrac{\partial}{\partial\theta^\alpha} - \bar{\eta}_{\dot{\alpha}} \tfrac{\partial}{\partial\bar{\theta}_{\dot{\alpha}}} - 2i\theta^\alpha\sigma^n_{\alpha\dot{\alpha}}\bar{\eta}^{\dot{\alpha}}\partial^y_n)
\end{aligned}$$

  - so we get

$$\begin{aligned}
\delta_\eta\Phi(y,\theta) &= (\eta^\alpha \tfrac{\partial}{\partial\theta^\alpha} - \bar{\eta}_{\dot{\alpha}} \tfrac{\partial}{\partial\bar{\theta}_{\dot{\alpha}}} - 2i\theta^\alpha\sigma^n_{\alpha\dot{\alpha}}\bar{\eta}^{\dot{\alpha}}\partial^y_n)[\phi(y) + \theta\psi(y) + \tfrac{1}{2}\theta\theta\, F(y)] \\
&= \eta^\alpha \tfrac{\partial}{\partial\theta^\alpha}[\phi(y) + \theta\psi(y) + \tfrac{1}{2}\theta\theta\, F(y)] - 2i\theta^\alpha\sigma^n_{\alpha\dot{\alpha}}\bar{\eta}^{\dot{\alpha}}\partial^y_n[\phi(y) + \theta\psi(y)] \\
&= \eta\psi(y) + \eta\theta\, F(y) - 2i\theta\sigma^n\bar{\eta}\partial^y_n\phi(y) - 2i\theta^\alpha\sigma^n_{\alpha\dot{\alpha}}\bar{\eta}^{\dot{\alpha}}\theta^\beta\partial^y_n\psi_\beta(y)
\end{aligned}$$

Constraints on Superfields

- comparing the coefficients of the θs

$$\delta_\eta \Phi(y,\theta) = \delta_\eta \phi(y) + \theta^\alpha \delta_\eta \psi_\alpha(y) + \frac{1}{2}\theta\theta\, \delta_\eta F(y)$$

  – we get

$$\begin{aligned} \delta_\eta \phi(y) &= \eta\psi(y) \\ \theta^\alpha \delta_\eta \psi_\alpha(y) &= \theta^\alpha [\eta_\alpha F(y) - 2i\sigma^n_{\alpha\dot\alpha}\bar\eta^{\dot\alpha}\partial^y_n \phi(y)] \\ \frac{1}{2}\theta\theta\, \delta_\eta F(y) &= -2i\theta^\beta\theta^\alpha\sigma^n_{\alpha\dot\alpha}\bar\eta^{\dot\alpha}\partial^y_n\psi_\beta(y) = i\theta\theta\,\partial^y_n\psi^\alpha(y)\sigma^n_{\alpha\dot\alpha}\bar\eta^{\dot\alpha} \end{aligned}$$

- mass dimensions
  – since $[Q] = \frac{1}{2}$ we have to have $[\theta] = [\bar\theta] = [\eta] = [\bar\eta] = -\frac{1}{2}$
  – taking $[\phi] = 1$ (normal scalar field) and $[\psi] = \frac{3}{2}$ (normal spinor field)

  $\Rightarrow$ $[F] = 2$ $\Rightarrow$ auxiliary field $\Rightarrow$ F cannot have a kinetic term

  * . . . no propagation . . . no physical d.o.f.
  * the complex scalar ϕ has 2 d.o.f.
  * the Weyl spinor ψ has 2 d.o.f.

Constraints on Superfields

- Vector Superfields are defined by $V = V^\dagger$
  - a transformation $V \to V' = V - \Phi - \Phi^\dagger$ still has $V' = V'^\dagger$
  - with

$$V = s + \theta[\chi + i(\theta\sigma^m\bar\theta)\partial_m\chi] + \bar\theta[\bar\chi - i(\theta\sigma^m\bar\theta)\partial_m\bar\chi] + \tfrac{1}{2}\theta\theta\, M + \tfrac{1}{2}\bar\theta\bar\theta\, M^* + \theta\sigma^m\bar\theta v_m + \tfrac{1}{2}\theta\theta\,\bar\theta\bar\lambda + \tfrac{1}{2}\bar\theta\bar\theta\,\theta\lambda + \tfrac{1}{4}\theta\theta\,\bar\theta\bar\theta\, D$$

and

$$\begin{aligned}\Phi(x,\theta,\bar\theta) &= \Phi(y^m = x^m - i\theta\sigma^m\bar\theta, \theta, 0) = f(y) + \theta\psi(y) + \tfrac{1}{2}\theta\theta\, F(y)\\ &= f(x) - i\theta\sigma^m\bar\theta\partial_m f(x) + \tfrac{1}{2}(-i\theta\sigma^m\bar\theta\partial_m)(-i\theta\sigma^n\bar\theta\partial_n)f(x)\\ &\quad + \theta^\alpha[\psi_\alpha(x) - i\theta\sigma^m\bar\theta\partial_m\psi_\alpha(x)] + \tfrac{1}{2}\theta\theta\, F(x)\\ &= f - i\theta\sigma^m\bar\theta\partial_m f + \theta\psi - \tfrac{1}{2}\theta\theta\, i[\partial_m\psi]\sigma^m\bar\theta + \tfrac{1}{2}\theta\theta\,[F - \tfrac{1}{2}\partial^2 f]\end{aligned}$$

we get

$$\begin{aligned}V' = V - \Phi - \Phi^\dagger &= s + \theta\chi + \bar\theta\bar\chi + \tfrac{1}{2}\theta\theta\, M + \tfrac{1}{2}\bar\theta\bar\theta\, M^*\\ &\quad + (\theta\sigma^m\bar\theta)v_m + \tfrac{1}{2}\theta\theta\,\bar\theta\bar\lambda - \tfrac{i}{2}\theta\theta\,[\partial_m\chi]\sigma^m\bar\theta + \tfrac{1}{2}\bar\theta\bar\theta\,\theta\lambda - \tfrac{i}{2}\bar\theta\bar\theta\,\theta\sigma^m[\partial_m\bar\chi] + \tfrac{1}{4}\theta\theta\,\bar\theta\bar\theta\, D\\ &\quad - f + i\theta\sigma^m\bar\theta\partial_m f - \theta\psi + \tfrac{i}{2}\theta\theta\,[\partial_m\psi]\sigma^m\bar\theta - \tfrac{1}{2}\theta\theta\,[F - \tfrac{1}{2}\partial^2 f]\\ &\quad - f^* - i\theta\sigma^m\bar\theta\partial_m f^* - \bar\theta\bar\psi - \tfrac{i}{2}\bar\theta\bar\theta\,\theta\sigma^m[\partial_m\bar\psi] - \tfrac{1}{2}\bar\theta\bar\theta\,[F^* - \tfrac{1}{2}\partial^2 f^*]\\ &= [s - 2\mathcal{R}e[f]] + (\theta\sigma^m\bar\theta)\,[v_m - 2\partial_m\mathcal{I}m[f]] + \theta[\chi - \psi] + \bar\theta[\bar\chi - \bar\psi]\\ &\quad + \tfrac{1}{2}\theta\theta\,\bar\theta\bar\lambda - \tfrac{i}{2}\theta\theta\,(\partial_m[\chi - \psi])\sigma^m\bar\theta + \tfrac{1}{2}\bar\theta\bar\theta\,\theta\lambda + \tfrac{i}{2}\bar\theta\bar\theta\,\theta\sigma^m(\partial_m[\bar\chi - \bar\psi])\\ &\quad + \tfrac{1}{2}\theta\theta\,[M - F + \tfrac{1}{2}\partial^2 f] + \tfrac{1}{2}\bar\theta\bar\theta\,[M^* - F^* + \tfrac{1}{2}\partial^2 f^*] + \tfrac{1}{4}\theta\theta\,\bar\theta\bar\theta\, D\end{aligned}$$

Constraints on Superfields

- taking $\Phi + \Phi^\dagger$ as a gauge transformation
  - with the gauge parameter $2\mathcal{I}m[f]$

  $\Rightarrow$ so the vectorfield changes: $v_m \to v'_m = v_m - 2\partial_m \mathcal{I}m[f]$
  - and setting $\mathcal{R}e[f] = \frac{1}{2}s$, $\psi = \chi$, and $F = M + \frac{1}{2}\partial^2 f$
  - we get the Wess Zumino gauge (WZ gauge):

$$\begin{aligned} V' &= [s - 2\mathcal{R}e[f]] + (\theta\sigma^m\bar{\theta})\,[v_m - 2\partial_m\mathcal{I}m[f]] + \theta[\chi - \psi] + \bar{\theta}[\bar{\chi} - \bar{\psi}] \\ &\quad + \tfrac{1}{2}\theta\theta\,\bar{\theta}\bar{\lambda} - \tfrac{i}{2}\theta\theta\,(\partial_m[\chi - \psi])\sigma^m\bar{\theta} + \tfrac{1}{2}\bar{\theta}\bar{\theta}\,\theta\lambda + \tfrac{i}{2}\bar{\theta}\bar{\theta}\,\theta\sigma^m(\partial_m[\bar{\chi} - \bar{\psi}]) \\ &\quad + \tfrac{1}{2}\theta\theta\,[M - F + \tfrac{1}{2}\partial^2 f] + \tfrac{1}{2}\bar{\theta}\bar{\theta}\,[M^* - F^* + \tfrac{1}{2}\partial^2 f^*] + \tfrac{1}{4}\theta\theta\,\bar{\theta}\bar{\theta}\,D \\ &= (\theta\sigma^m\bar{\theta})v'_m + \theta\theta\,\bar{\theta}\bar{\lambda} + \bar{\theta}\bar{\theta}\,\theta\lambda + \tfrac{1}{4}\theta\theta\,\bar{\theta}\bar{\theta}\,D \end{aligned}$$

  - only (v_m, λ, D) are gauge invariant $\Rightarrow$ vector superfield
  - powers of the vector superfield in WZ gauge are extremely simple:

$$V^2 = -\tfrac{1}{2}\theta\theta\,\bar{\theta}\bar{\theta}v^\mu v_\mu \qquad \text{and} \qquad V^3 = 0$$

  $\Rightarrow$ allows the exponential formulation of the gauge transformation: e^{gV}

supersymmetric flat space

- as in GR it will be helpful to use forms
  - they are easily generalized to superspace
- as in GR we can choose any basis, not only $dx^M = (dx^m, d\theta^\mu, d\bar{\theta}_{\dot{\mu}})$
  - then the basis vectors are given by $\partial_m = \frac{\partial}{\partial x^m} \quad \partial_\mu = \frac{\partial}{\partial \theta^\mu} \quad \bar{\partial}^{\dot{\mu}} = -\frac{\partial}{\partial \bar{\theta}_{\dot{\mu}}}$
  - a more convenient basis is given by the supersymmetric covariant derivatives

$$D_a = \frac{\partial}{\partial x^a} \qquad D_\alpha = \frac{\partial}{\partial \theta^\alpha} - i\sigma^\mu_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\frac{\partial}{\partial x^\mu} \qquad \bar{D}^{\dot{\alpha}} = -\frac{\partial}{\partial \bar{\theta}_{\dot{\alpha}}} + i\theta_\alpha\sigma^\mu_{\alpha\dot{\beta}}\epsilon^{\dot{\beta}\dot{\alpha}}\frac{\partial}{\partial x^\mu}$$

$\Rightarrow$ with $e^A = (e^a, e^\alpha, e_{\dot{\alpha}})$ we have $d = dx^M \partial_M = e^A D_A$

- $*$ this relation also defines the vielbein: $e^A = dx^M e_M{}^A$ and $D_A = e_A{}^M \partial_M$

– for the natural coordinates we still have $d\,dz^P = d^2 z^P = 0$, since

$$\begin{aligned} d\,d \;&=\; [dx^m\partial_m + d\theta^\mu\tfrac{\partial}{\partial\theta^\mu} + d\bar{\theta}_{\dot{\mu}}(-\tfrac{\partial}{\partial\bar{\theta}_{\dot{\mu}}})][dx^n\partial_n + d\theta^\nu\tfrac{\partial}{\partial\theta^\nu} + d\bar{\theta}_{\dot{\nu}}(-\tfrac{\partial}{\partial\bar{\theta}_{\dot{\nu}}})] \\ &=\; dx^m dx^n\partial_m\partial_n + dx^m d\theta^\nu\partial_m\tfrac{\partial}{\partial\theta^\nu} - dx^m d\bar{\theta}_{\dot{\nu}}\partial_m\tfrac{\partial}{\partial\bar{\theta}_{\dot{\nu}}} - d\theta^\mu dx^n\tfrac{\partial}{\partial\theta^\mu}\partial_n + d\theta^\mu d\theta^\nu\tfrac{\partial}{\partial\theta^\mu}\tfrac{\partial}{\partial\theta^\nu} \\ &\quad -d\theta^\mu d\bar{\theta}_{\dot{\nu}}\tfrac{\partial}{\partial\theta^\mu}\tfrac{\partial}{\partial\bar{\theta}_{\dot{\nu}}} + d\bar{\theta}_{\dot{\mu}}dx^n\tfrac{\partial}{\partial\bar{\theta}_{\dot{\mu}}}\partial_n - d\bar{\theta}_{\dot{\mu}}d\theta^\nu\tfrac{\partial}{\partial\bar{\theta}_{\dot{\mu}}}\tfrac{\partial}{\partial\theta^\nu} + d\bar{\theta}_{\dot{\mu}}d\bar{\theta}_{\dot{\nu}}\tfrac{\partial}{\partial\bar{\theta}_{\dot{\mu}}}\tfrac{\partial}{\partial\bar{\theta}_{\dot{\nu}}} \;=\; 0 \end{aligned}$$

- $*$ as the one forms $d\theta^\mu$ and $d\bar{\theta}_{\dot{\mu}}$ now commute : $\quad d\theta^\mu d\bar{\theta}_{\dot{\mu}} = d\bar{\theta}_{\dot{\mu}} d\theta^\mu$

supersymmetric flat space

- the "orthonormal" basis $e^A = (e^a, e^\alpha, e_{\dot\alpha})$ respects Supersymmetry:

$$\delta_\eta D_A = (\eta Q + \bar\eta \bar Q) D_A = D_A \delta_\eta \quad \text{and} \quad D_A e^B = \delta_A^B \quad \Rightarrow \quad \delta_\eta e^B = e^B \delta_\eta$$

  - but the outer derivative of the basis does not vanish

$$de^A = dx^N \partial_N dx^M {e_M}^A = dx^M dx^N \partial_N {e_M}^A \neq 0$$

    - like in curved space

- one defines supersymmetric flat space
  - to be compatible with the SUSY transformations

  ⇒ it is given by $\{e^A\}$ (and not by dx^M)

- but even in flat space, the supersymmetric torsion does not vanish:

$$T_{\alpha\dot\beta}{}^c = T_{\dot\beta\alpha}{}^c = 2i\sigma^c_{\alpha\dot\beta}$$

  - SUSY "twists" flat space ... like a Möbius strip
    - we have $(\theta)^2 \sim x$, so $\theta \sim \sqrt{x}$ ... which is never a linear function

local Supersymmetry

- SUSY always includes fermions $\Rightarrow$ describing curvature by
  - the local vielbein $E_M{}^A$ and the spin connection $\omega_M{}^A{}_B$
    - A and B are called Lorentz indices (transforming with Lorentz transformations)
    - M and N are Einstein indices (transforming with general coordinate transformations)
- the Lorentz group can be seen as the local symmetry group
  - with local Lorentz transformations (LLTs) $\Lambda_B{}^A = (\Lambda_b{}^a, \Lambda_\beta{}^\alpha, \Lambda^{\dot\beta}{}_{\dot\alpha})$
  - the space-time and spinorial parts of the LLTs are linked:

$$\sigma^a_{\alpha\dot\alpha}\sigma^b_{\beta\dot\beta}\Lambda_{ab} = -2\epsilon_{\alpha\beta}\Lambda_{\dot\alpha\dot\beta} + 2\epsilon_{\dot\alpha\dot\beta}\Lambda_{\alpha\beta}$$

- a general coordinate transformation in superspace $x'^M = x^M - \xi^M$
  - changes the position $x^M \to x'^M$
  - and the direction of the vierbein: $E^A \to E'^A = E^B\Lambda_B{}^A$
  - the change of coordinates of the vector $V = V^A D_A$ becomes

$$\delta_\xi V^A = -\xi^B E_B{}^M \partial_M V^A + V^B\Lambda_B{}^A = -\xi^B\nabla_B V^A + V^B\xi^C\omega_{CB}{}^A + V^B\Lambda_B{}^A$$

  - taking $\Lambda_B{}^A = -\xi^C\omega_{CB}{}^A$ (i.e. a gauge fixing of the LLTs with the translations) gives
    - supergauge transformations or gauged supersymmetry transformations

Supersymmetry (SUSY) — Theory

local Supersymmetry = Supergravity (SUGRA)

- all of the introduced quantites are superfields
  - they can be expanded in the superspace coordinates
  - the metric is replaced by the vielbein as the dynamic quantity
  - constraints on the torsion: $T_{\alpha\dot{\beta}}{}^{c} = T_{\dot{\beta}\alpha}{}^{c} = 2i\sigma^{c}_{\alpha\dot{\beta}}$

    $\Rightarrow$ express the connection in terms of the vielbein
  - the vielbein contains graviton and gravitino

- gives a scenario for SUSY breaking
  - with mechanisms to motivate the soft breaking terms

    $\Rightarrow$ MSSM (Minimal Supersymmetric Standard Model)

- but this SUSY breaking happens at high energies
  - relevant for Cosmology
    - but only for the first nanoseconds

SUGRA does not make GR a renormalisable QFT

The Minimal Supersymmetric Standard Model (MSSM)

- takes the SM particles (fields) and supersymmetrizes them
  - in a "minimal" way
  - particles have to appear in supersymmetric multiplets

⇒ it gives a superpartner to each particle in the Standardmodel:
  - fermions get the scalar sfermions
  - vector bosons (gauge bosons) get the fermionic gauginos
  - the two doublets of Higgs bosons get the fermionic higgsinos

the Lagrangian consists of

- supersymmetric parts:
  - vector superfields in the gauge group $SU(3)_{\text{color}} \times SU(2)_L \times U(1)_Y$
  - chiral superfields give the matter fields and the higgses
  - the interaction between chiral and vector superfields
- and soft breaking terms
  - they parametrize the effect of Supersymmetry breaking

  ⇒ particles in the multiplets no longer have the same mass

MSSM and Cosmology

- the MSSM has an unbroken discrete symmetry: conserved R-parity
  - it is a multiplicative quantum number
    - SM particles have $(+1)$, SUSY partners have (-1)
  - supersymmetric particles can only be produced in pairs
  - a SUSY particle can only decay into another SUSY particle
  - $\Rightarrow$ the lightest supersymmetric particle (LSP) is stable
    - this is usually the neutralino $\tilde{\chi}_1^0$ with $m_{\tilde{\chi}_1^0} > 50\text{GeV}$

$\Rightarrow$ the MSSM provides a Dark Matter candidate

- since SUSY interactions are just SM gauge interactions
  - $\Rightarrow$ (nearly) all properties of the DM candidate are known!
- if SUSY particles are found by LHC
  - $\Rightarrow$ some properties of dark matter can be investigated