Measurements of the Cosmic Microwave Background Radiation

- first (involuntary) measurements by Penzias and Wilson in 1965 \Rightarrow Nobel Prize in 1978
- COBE (Cosmic Background Explorer) is launched in 1989, takes data until 1991
 - FIRAS (Far Infrared Absolute Spectrophotometer) measures the frequency distribution in 1990
 - \Rightarrow the CMB is a thermal blackbody radiation with T $\sim 2.725\,\text{K}$
 - DMR (Differential Microwave Radiometer)
 discovers the primary temperature anisotropy in 1992
 - \Rightarrow Nobel Prize in 2006
- BOOMERanG and MAXIMA measure the acoustic oscillations in the angular power spectrum of the CMB anisotropy in 1999

How is the Cosmic Microwave Background Radiation measured ?

- it is mainly a microwave radiation \Rightarrow radio antenna
 - ⇒ directional measurements possible
- for higher accuracy in temperature differences
 - \Rightarrow differential measurement
 - * comparing the radiation coming from two different directions
- WMAP (Wilkinson Microwave Anisotropy Probe) measured
 - in 5 radio bands (23, 33, 41, 61, and 94 GHz with \sim 22% bandwidth)
 - 393,216 sky pixels with a solid angle of (0.77, 0.44, 0.26, 0.12, and 0.05) degree
 * each sky pixel is measured 1000 to 5000 times per year



WMAP





Thomas Gajdosik – Concepts of Modern Cosmology 23.04.2012

WMAP gives the temperature $T(\theta, \phi)$ of the CMB radiation

• the average temperature is

$$\langle \mathsf{T} \rangle = \frac{1}{4\pi} \int \mathsf{T}(\theta, \phi) \sin \theta \, d\theta \, d\phi = 2.725 \,\mathsf{K}$$

- the temperature fluctuation

$$\frac{\delta \mathsf{T}}{\mathsf{T}}(\theta,\phi) = \frac{\mathsf{T}(\theta,\phi) - \langle \mathsf{T} \rangle}{\langle \mathsf{T} \rangle} = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell m} Y_{\ell}^{m}(\theta,\phi)$$

can be described by spherical harmonics

- the spherical harmonics are orthonormal basis functions $Y_{\ell}^{m}(\theta,\phi) = Ne^{im\phi}P_{\ell}^{m}(\cos\theta) \quad \text{with} \quad \int Y_{\ell}^{*m}Y_{\ell'}^{m'}\sin\theta \,d\theta \,d\phi = \delta_{\ell\ell'}\delta^{mm'}$

that satisfy an addition theorem

$$\sum_{m=-\ell}^{\ell} Y_{\ell}^{*m}(\hat{n}_{1}) Y_{\ell}^{m}(\hat{n}_{2}) = \frac{2\ell+1}{4\pi} P_{\ell}(\cos\theta_{12})$$

with Legendre polynomial P_ℓ and the angle $\cos \theta_{12} = \hat{n}_1 \cdot \hat{n}_2$

multipoles of the CMB radiation

• the multipoles are given by

$$a_{\ell m} = \int Y_{\ell}^{m*}(\theta,\phi) \frac{\delta \mathsf{T}}{\mathsf{T}}(\theta,\phi) \sin \theta \, d\theta \, d\phi$$

• the two point correlation is

$$C(\theta_{12}) = \langle \frac{\delta T}{T}(\hat{n}_1) \frac{\delta T}{T}(\hat{n}_2) \rangle$$

= $\sum_{\ell_1, \ell_2, m_1, m_2} a_{\ell_1 m_1} a_{\ell_2 m_2} \int Y_{\ell_1}^{m_1}(\hat{n}_1) Y_{\ell_2}^{m_2}(\hat{n}_2) \sin \theta \, d\theta \, d\phi$

– use Clebsch-Gordan coefficients to express the product of two $Y \mathrm{s}$

$$Y_{\ell_1}^{m_1}(\hat{n}_1)Y_{\ell_2}^{m_2}(\hat{n}_2) = |\ell_1 m_1\rangle \otimes |\ell_2 m_2\rangle = |\ell_1 m_1 \ell_2 m_2\rangle$$

as a sum over single Ys

 $|(\ell_1\ell_2)\ell_3m_3\rangle = \sum_{m_1,m_2} |\ell_1m_1\ell_2m_2\rangle \langle \ell_1m_1\ell_2m_2|(\ell_1\ell_2)\ell_3m_3\rangle$

- since we integrate over the angles $\Rightarrow \ell_3 = m_3 = 0$
 - \Rightarrow m_1 and m_2 have to sum up to zero and $\ell_1 = \ell_2$

 \Rightarrow only the "diagonal" terms contribute:

$$C(\theta_{12}) = \langle (\frac{\delta T}{T})^2 \rangle$$

multipoles of the CMB radiation

• using the addition theorem (and $C_\ell = \sum_m |a_{\ell m}|^2$) we get

$$C(\theta_{12}) = \sum_{\ell,m} a_{\ell m} a_{\ell m}^* \int Y_{\ell}^{m*}(\hat{n}_1) Y_{\ell}^m(\hat{n}_2) \sin \theta \, d\theta \, d\phi$$
$$= \sum_{\ell} C_{\ell} \frac{2\ell+1}{4\pi} P_{\ell}(\cos \theta_{12})$$

- we are looking for the autocorrelation of density fluctuations
 - the angle between a direction and the same direction is zero $\Rightarrow \cos \theta_{12} = 1$ and $P_{\ell}(1) = 1$
- when dealing with a large sum, one can estimate it with an integral
 - in this case it is convenient to display the logarithm of ℓ

$$C = \sum_{\ell=0}^{\infty} C_{\ell} \frac{2\ell+1}{4\pi} P_{\ell}(1) \sim \int C_{\ell} \frac{\ell(2\ell+1)}{4\pi} d(\ln \ell)$$

– the interesting quantity is the integrand, more exactly C_ℓ

How can we predict the multipoles of the CMB radiation ?

- first we have to realize how the CMB is produced
 - the radiation left over from the hot big bang
- how exactly?
 - studying the distribution of photons
 - * coming from the pair annihilations and scatterings
 - * of the available particles during the expansion

⇒ coupled Einstein-Boltzmann equations



the Boltzman transport equation

$$\frac{d}{dt}f_i(\vec{r},\vec{p},t) = \left(\frac{\partial}{\partial t} + \vec{v}\cdot\nabla_{\vec{r}} + \vec{F}\cdot\nabla_{\vec{p}}\right)f_i(\vec{r},\vec{p},t) = \left(\frac{\partial f}{\partial t}\right)_{\text{coll}}$$

- describes the change in the phase space density of particle i:
 - the flow \vec{v} of particles changes their number in a region of space
 - the force \vec{F} acting on the particles changes their momentum
 - the collisions (and decays) can change number and momentum
- how can we understand this equation in a covariant way?
 - we have 3 + 1 and not only 3 dimensions . . .
- \Rightarrow field equations (equations of motion) constrain p^{μ} :
 - in flat space:

$$m^2 = p^2 \quad \Rightarrow \quad p^0 = E = \sqrt{m^2 + \vec{p}^2}$$

in curved space

- field equations (equations of motion)
 - couple to the Einstein equations
- taking the Robertson-Walker metric as a background:

$$g_{\mu\nu} \approx \left(\begin{array}{ccc} 1 & & & \\ & -a^2 & \\ & & -a^2 & \\ & & & -a^2 \end{array}\right)$$

- with scalar metric perturbations

$$g_{00} = 1 - 2\Phi$$
 and $g_{jk} = -\delta_{jk}a^2(1 - 2\Psi)$

- $\ast~\Phi$ corresponds to the Newtonian potential
- $\ast~\Psi$ is the curvature perturbation
- one gets the constraint on the momentum

$$m^{2} = g_{\mu\nu}p^{\mu}p^{\nu} = E^{2}(1 - 2\Phi) - \vec{p}^{2}a^{2}(1 - 2\Psi)$$

 $\ast\,$ that contains already a dependence on the curvature . . .

curvature is still given by Einsteins equations

- but now linearlized
 - the scale factor a is determined without perturbations
 - Φ and Ψ are determined by the first order in the perturbations
- the stress energy tensor is given by the particles
 - weighted by their densities:

$$T^{\mu\nu} = \sum_{i=\text{all particles}} n_i T_i^{\mu\nu}$$

- the density is given by the phase space density

$$n_i = g_i \int \frac{d^3p}{(2\pi)^3} f_i(\vec{r}, \vec{p}, t)$$

- * with g_i the number of degrees of freedom per particle
- * which is also the zero-order "moment" of the phase space density
- the velocity $\vec{v_i}$ is then the first-order moment

$$\vec{v}_i = \frac{g_i}{n_i} \int \frac{d^3p}{(2\pi)^3} \frac{\vec{p}_i}{E_i} f_i(\vec{r}, \vec{p}, t) = \left\langle \frac{\vec{p}_i}{E_i} \right\rangle$$

the integrated collision term C[f] is given by

$$C[f] = \int_{p_1} \left(\frac{\partial f}{\partial t}\right)_{\text{coll}} = \int_{p_1} \int_{p_2} \int_{p_3} \int_{p_4} (2\pi)^4 \delta^4 (p_1^{\mu} + p_2^{\mu} - p_3^{\mu} - p_4^{\mu}) |\mathcal{M}|^2 \\ \times \{f_3 f_4 [1 \pm f_1] [1 \pm f_2] - f_1 f_2 [1 \pm f_3] [1 \pm f_4] \}$$

• with $\int_{p_i} := \int \frac{d^3 p_i}{(2\pi)^3 2E_i}$

- ${\mathcal M}$ describes the matrix element for the process $1+2\rightleftharpoons 3+4$
 - * to be calculated in Quantum Field Theory (next semester)
- \pm describes Bose enhancement / Pauli blocking (+/–) for bosons / fermions
- for high temperatures these factors become less important
 - \Rightarrow the distributions $f = [e^{\frac{E-\mu}{kT}} \mp 1]^{-1}$ approach the Boltzmann distribution $e^{-\frac{E-\mu}{kT}}$
 - \ast with the chemical potential μ , which is related to the density

$$\frac{n_i}{g_i} = \int \frac{d^3 p}{(2\pi)^3} f_i(\vec{r}, \vec{p}, t) = e^{\mu/kT} \int \frac{d^3 p}{(2\pi)^3} e^{-E/kT} \approx \begin{cases} \left(\frac{m_i kT}{2\pi}\right)^{3/2} e^{-m_i/kT} & m_i \gg kT \\ \frac{(kT)^3}{\pi^2} & m_i \ll kT \end{cases}$$

- in equilibrium $C[f] = 0 = (e^{(\mu_1 + \mu_2)/kT} e^{(\mu_3 + \mu_4)/kT}) \int |\mathcal{M}|^2$
 - \Rightarrow the chemical potentials have to be equal: $\mu_1 + \mu_2 = \mu_3 + \mu_4$

introducing the equilibrium density $n_i^{(0)} = g_i \int \frac{d^3p}{(2\pi)^3} e^{-E/kT}$

• one can rewrite
$$e^{\mu/kT} = n_i/n_i^{(0)}$$

• and defined the thermally averaged cross section

$$\langle v\sigma \rangle := \frac{(2\pi)^4}{n_1^{(0)}n_2^{(0)}} \int_{p_1} \int_{p_2} \int_{p_3} \int_{p_4} \delta^4 (p_1^{\mu} + p_2^{\mu} - p_3^{\mu} - p_4^{\mu}) |\mathcal{M}|^2 e^{-(E_1 + E_2)/kT}$$

• then the Boltzmann equation for the number density becomes

$$\int_{p_1} \frac{d}{dt} f_1(\vec{r}, \vec{p}, t) = \frac{d(a^3 n_1)}{a^3 dt} = n_1^{(0)} n_2^{(0)} \langle v\sigma \rangle \left\{ \frac{n_3}{n_3^{(0)}} \frac{n_4}{n_4^{(0)}} - \frac{n_1}{n_1^{(0)}} \frac{n_2}{n_2^{(0)}} \right\}$$

• now
$$\frac{d(a^3n_1)}{a^3dt} \sim Hn_1$$
 if $H \ll n_2 \langle v\sigma \rangle$
 \Rightarrow the bracket has to become zero: $\frac{n_3}{n_3^{(0)}} \frac{n_4}{n_4^{(0)}} = \frac{n_1}{n_1^{(0)}} \frac{n_2}{n_2^{(0)}}$
 $*$ chemical equilibrium ... for heavy relics of the early universe
 $*$ nuclear statistical equilibrium ... for Big Bang nucleosynthesis
 $*$ Saha equation ... for recombination and ionization balance

- 7. General Relativity fluctuation of densities
- applying this ansatz to
 - dark matter particles and SM particles
 - \Rightarrow dark matter abundance
 - $\ast\,$ needs non-equilibrium solution for freeze-out
 - protons, neutrons and nuclei
 - \Rightarrow Big Bang nucleosynthesis
 - \ast needs non-equilibrium solution for neutron capture and decay
 - electrons, positrons, photons, and neutrinos
 - ⇒ CMB temperature versus neutrino temperature
 - electrons, nuclei, and photons
 - \Rightarrow recombination, CMB photon spectrum
 - * still have to calculate the density fluctuations
 - ⇒ we have to solve the linearized Einstein-Boltzmann equations

linearized Einstein-Boltzmann equations

- introducing the metric perturbations Φ and Ψ
- introducing linearized density fluctuations for all particles:
 - for the photons $\Theta(\vec{x}, \hat{p}, t)$, indepentent of $|\vec{p}|$:

$$f_{\gamma}(\vec{x}, \vec{p}, t) = \left[\exp\left\{ \frac{|\vec{p}|}{T(t)[1 + \Theta(\vec{x}, \hat{p}, t)]} \right\} - 1 \right]^{-1}$$

- for the other particles as a density contrast $\delta_i(\vec{x},t)$ and a velocity $\vec{v}_i(\vec{x},t)$

$$n_i(\vec{x},t) = n_i^{(0)}(\vec{x},t) \left[1 + \delta_i(\vec{x},t)\right] \quad \text{and} \quad \vec{v}_i(\vec{x},t) = \left\langle \frac{\vec{p}_i}{E_i} \right\rangle$$

 $\ast\,$ both, $\delta_i(\vec{x},t)$ and $\vec{v}_i(\vec{x},t)$, are considered first order

- * the zero-order velocity is in equilibrium $\Rightarrow \vec{v}_i^{(0)} = 0$
- minimal amount of relevant particles:
 - * photon * baryon (includes e^- !) * neutrino * dark matter
- \Rightarrow 10 coupled partial differential equations
 - Fourier transform $\Theta(\vec{x}) = \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot\vec{x}} \Theta(\vec{k})$ decouples the Fourier modes of Θ_{γ}
 - \Rightarrow CMB power spectrum in the multipoles $C_{\ell} = \frac{1}{(-i)^{\ell}} \int_{-1}^{1} \frac{d(\hat{p}.\hat{k})}{2} P_{\ell}(\hat{p}.\hat{k}) \Theta(\vec{k})$
 - $ec{v_i}$ is already the "dipole" of n_i

7. General Relativity — Big Bang nucleosynthesis (following Dodelson)

cosmic plasma at a temperature of 1 MeV (universe a few minutes old)

- in equilibrium: electrons, positrons, photons ... relativistic
- decoupled neutrinos ... still relativistic
 - for $e\nu \rightleftharpoons e\nu$ the coupling $\langle v\sigma \rangle < H$
- coupled baryons: protons and neutrons ... non-relativistic
 - antibaryons have annihilated with the baryons
 - the remaining baryons can come from Baryogenesis (introduced by Sakharov)
 - * the baryon asymmetry is estimated as $(n_b-n_{\overline{b}})/spprox 10^{-10}$
 - * compatible with the ratio of baryons to photons today: $n_b/n_\gamma pprox 5.5 \cdot 10^{-10}$
 - these baryons can form nuclei ... step by step
 - ? coupled equations for all the elements until iron
- simplifications:
 - with $\sim 10^{10}$ photons per baryon and a temperature $kT \sim 0.1$ MeV
 - * a binding energy of deuterium (²H) of $E_b = 2.2$ MeV
 - * there are $10^{10} \times e^{-2.2 \text{MeV}/0.1 \text{MeV}} \sim 2.2$ photons with $E_{\gamma} > E_b$ per nucleon
 - $\Rightarrow\,$ nearly all nuclei are disintegrated by high energy photons
 - Li, Be, and B are less bound than He
 - \Rightarrow ^{*n*}H and ^{*n*}He are relevant, ^{*n*}Li only marginal

- 7. General Relativity Big Bang nucleosynthesis (following Dodelson) cosmic plasma at a temperature of 0.1 MeV (universe a few minutes old)
 - for the reaction $p + n \rightleftharpoons D + \gamma$ we have the equilibrium

$$\frac{n_D}{n_p n_n} = \frac{n_D^{(0)}}{n_p^{(0)} n_n^{(0)}} = \frac{g_D \int \frac{d^3 p}{(2\pi)^3} e^{-E_D/kT}}{g_p \int \frac{d^3 p}{(2\pi)^3} e^{-E_p/kT} g_n \int \frac{d^3 p}{(2\pi)^3} e^{-E_n/kT}}$$

$$\approx \frac{3 \left[\frac{m_D kT}{(2\pi)}\right]^{3/2} e^{-\frac{m_D}{kT}}}{2 \left[\frac{m_p kT}{(2\pi)}\right]^{3/2} e^{-\frac{m_P}{kT}} 2 \left[\frac{m_n kT}{(2\pi)}\right]^{3/2} e^{-\frac{m_n}{kT}}} = \frac{3}{4} \left[\frac{2\pi m_D}{m_p m_n kT}\right]^{3/2} e^{\frac{m_p + m_n - m_D}{kT}}$$

- $m_p \simeq m_n \simeq m_D/2 \simeq 1$ GeV, but $m_p + m_n - m_D = E_b \sim 2.2$ MeV - at $kT \sim 1$ MeV the densities for p and n are similar to n_b

$$\frac{n_D}{n_b} = \frac{n_D}{n_p n_n} n_n \approx \frac{3}{4} \left[\frac{2\pi m_D}{m_p m_n kT} \right]^{3/2} e^{\frac{E_b}{kT}} \frac{n_b}{n_\gamma} n_\gamma \approx \frac{3}{4} \left[\frac{4\pi}{m_p kT} \right]^{3/2} e^{\frac{E_b}{kT}} 5.5 \cdot 10^{-10} \frac{(kT)^3}{\pi^2}$$
$$\approx 1.86 \cdot 10^{-9} \left[\frac{kT}{m_p} \right]^{3/2} e^{\frac{E_b}{kT}}$$

– which becomes smaller than 1 for kT > 63.6 keV

$$\Rightarrow n_D$$
 is at higher kT exponentially suppressed: $\frac{n_D}{n_b}\Big|_{1 \text{ MeV}} \approx 5.3 \cdot 10^{-13}$

7. General Relativity — Big Bang nucleosynthesis (following Dodelson)

cosmic plasma at a temperature of 0.1 MeV (universe a few minutes old)

- the neutron-proton ratio comes mainly from $p + e \rightleftharpoons n + \nu$
 - for equilibrium we have $n_p^{(0)}/n_n^{(0)} = e^{(m_n m_p)/kT} = e^{\mathcal{Q}/kT}$
 - the electrons are still in thermal equilibrium: $n_e = n_e^{(0)}$
 - the neutrinos decouple with this reaction completely
 - starting with the Boltzman equation

$$\frac{d(a^3 n_n)}{a^3 dt} = n_n^{(0)} n_\nu^{(0)} \langle v\sigma \rangle \left\{ \frac{n_p}{n_p^{(0)}} - \frac{n_n}{n_n^{(0)}} \right\} = n_\nu^{(0)} \langle v\sigma \rangle \left\{ n_p e^{-\mathcal{Q}/kT} - n_n \right\}$$

– $\lambda_{np}=n_{\nu}^{(0)}\langle v\sigma\rangle$ describes the rate for neutron-proton conversion

- the neutron fraction $X_n = \frac{n_n}{n_p + n_n}$ "freezes" below $kT \sim 0.5$ MeV
- nucleosynthesis starts at kT = 70 keV with $X_n \approx 0.11$
 - \Rightarrow He mass ratio $\sim 4 \cdot X_n/2 \approx 22\%$
 - * all neutrons are bound in He, since $E_b(^4\text{He}) \sim 28 \text{ MeV} \gg E_b(^2\text{D}) \sim 2.2 \text{ MeV}$

7. General Relativity — Big Bang nucleosynthesis (following Dodelson)

cosmic plasma at a temperature < 0.1 MeV (universe a few minutes old)

- only at kT = 70 keV nucleosynthesis really starts
 - nearly all D is processed further to ${}^{4}\text{He}$
 - the left over D depends on the baryon density n_b
 - \Rightarrow measurement of the ratio D/H $\sim 3 \cdot 10^{-5}$ determines $\Omega_b \sim 0.04$
 - the produced 7 Li gives also tight bounds

7. General Relativity — Recombination (following Dodelson)

cosmic plasma at a temperature < 14 eV

- Recombination goes by the process $p + e \rightleftharpoons H + \gamma$
 - for equilibrium we have $\frac{n_e n_p}{n_H} = \frac{n_e^{(0)} n_p^{(0)}}{n_H^{(0)}}$
 - with the free electron fraction $X_e = \frac{n_e}{n_e + n_H} = \frac{n_p}{n_p + n_H}$ we get

$$\frac{X_e^2}{1 - X_e} = \frac{1}{n_e + n_H} \left(\frac{m_e kT}{2\pi}\right)^{3/2} e^{(m_H - m_p - m_e)/kT}$$

- for $kT \sim \epsilon_0 = m_p + m_e - m_H$ all Hydrogen is ionized

- recombination has to end in an excited state
 - * a photon from ground state recombination has $E_{\gamma} \geq \epsilon_0$
 - \Rightarrow instant reionization
- solving the equation for the electron fraction
 - \Rightarrow determines the decoupling temperature (or redshift \sim 1000)
- \Rightarrow CMB pattern: C_{ℓ} -distribution, CMB polarization

7. General Relativity — dark matter

dark matter balance

• for simplicity we take a single particle \boldsymbol{X}

- with a (very) weak coupling $X + X \rightleftharpoons Y + Z$

• the Standard Model particles Y and Z are in thermal equilibrium $\Rightarrow n_{Y,Z} = n_{Y,Z}^{(0)}$ and the number density equation becomes

$$\frac{d(a^3 n_X)}{a^3 dt} = \langle v\sigma \rangle \left\{ (n_X^{(0)})^2 - (n_X)^2 \right\}$$

- eventually we want to express the density in terms of the temperature kT* the temperature scales inverse to the scale factor: $T \sim a^{-1}$ * in the radiation dominated time $H(a) = H(a_1)(a_1/a)^2$ * with $x = m_X a$ we have $\frac{dx}{dt} = m\dot{a} = mHa = xH_m(x_m/x)^2 = H_m/x$ * using $Y := a^3n_X$ and $\frac{d}{dt} = \frac{dx}{dt}\frac{d}{dx} = \frac{H_m}{x}\frac{d}{dx}$ we get $\frac{H_m}{x}\frac{dY}{dx} = \frac{m_X^3}{x^3}\langle v\sigma \rangle \left(Y_{\text{EQ}}^2 - Y^2\right)$ or $\frac{dY}{dx} = -\frac{\lambda}{x^2}\left(Y^2 - Y_{\text{EQ}}^2\right)$ - Riccati equation with $\lambda = m_X^3\langle v\sigma \rangle/H_m$ 7. General Relativity — dark matter

dark matter balance

- for estimating $\lambda = m_X^3 \langle v\sigma \rangle / H_m$ we need
 - the mass m_X
 - the cross section $\sigma_{X+X \to Y+Z}$
 - the Hubble parameter H_m at the mass scale m_X
- \bullet for X from Supersymmetry (SUSY) we know the cross section σ
 - we get limits on the mass m_X
 - v and H_m are given by the Einstein-Boltzmann equations
 - \Rightarrow for the lightest supersymmetric particle (LSP) \neq gravitino

$$\Omega_X \sim 0.3 \left(\frac{x_f}{10}\right) \left(\frac{g_*(m_X)}{100}\right)^{1/2} \frac{10^{-39} \text{cm}^2}{\langle v\sigma \rangle}$$

- the gravitino couples as $\frac{E_X^2}{M_P^2} \sim \frac{(10^3 \,\text{GeV})^2}{(10^{19} \,\text{GeV})^2} \sim 10^{-32} \ll \alpha_{\text{em}}$

- $\Rightarrow\,$ it is only at the Plank epoch in thermal equilibrium
 - * all estimates for Ω_X have to be reassessed