Particle Cosmology

Lectures: 2 hours each week: room 510, Thursday 12^{00} to 14^{00}
and 2 hours for discussions: room 207, Wednesday 13^{00} to 15^{00}

02/07 Overview, Introduction; description of the course; assignements of seminars

02/07 Special Relativity: [1, 2]

02/08 Special Relativity: Astronomical relevance [1]

02/15 GR 1: General covariance [3, 6, 7]

02/22 GR 2: Riemannian geometry & Einstein equations [3, 6, 7]

03/01 GR 3: Vacuum solutions: Schwarzschild, Reissner-Nordström, Kerr-Newman [3, 6, 7]

03/08 GR 4: symmetric solutions: de Sitter, anti de Sitter, FLRW; time evolution [3, 6, 7]

03/15 GR 5: Big Bang [3, 6, 7, 8, 9]

03/22 GR 6: Inflation [3, 6, 7, 8, 9]

04/05 GR 7: CMB [6, 7, 8, 9]

04/12 Special Relativity: Algebra of the Poincaré group [15]

04/19 The Standard Model: Particle content [2, 10, 11, 12, 13, 14, 16, 17]

04/26 Supersymmetry (SUSY) & Dark Matter from SUSY [11, 8, 15, 18]

05/03 Particle detection, DM Searches

05/10 Questions, review of homework

Attendance optional; will count towards the grade

Homework suggested; will count towards the grade; less credit for late homework;

Grading: 100 points = 100%,
available points:

28 attendance

20 seminar presentation

22 homework

40 final exam: written and oral; 50% required to pass the course.

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webpage: `http://web.vu.lt/ff/t.gajdosik/ve/`

Books are available

trivial mathematical background: `http://web.vu.lt/ff/t.gajdosik/files/2014/01/sr4wop.pdf`

Group Theory: Peskin & Schroeder [12], Chapter 15.4, pp. 495 – 502
and much deeper: `http://www.teorfys.uu.se/people/minahan/Courses/Mathmeth/notes_v3.pdf`

Reading assignments

The understanding of Special Relativity is needed for most parts of modern physics, although it might be hidden, like in electro-magnetism. But it is *essential* for particle physics and obviously also for cosmology, which today relies on the understanding of General Relativity. Therefore I *strongly* recommend the reading of the very short and very good introduction into Special Relativity by David Hogg [1]. In the lecture I want to stress additional features, which are not covered by David Hogg, but I will rely on the basic understanding, as it is taught by David Hogg.

Seminar presentation

The idea of the presentation is to involve the students into the discussion about the connection between cosmology and particle physics and related areas.

The student chooses a subject for the presentation and clarifies with me, if the subject is suitable or not. If it is suitable the student will get a time during the discussion hours to present the subject to the fellow students. The presentation can be given in English or in Lithuanian. The presentation should be rather short, i.e. about 5 minutes, and it has to be presented using the computer.

- The presentation has to be prepared in a computer readable format:
  - **.pdf** is recommended, as the presentation will look the same, independent of the computer.
  - it might be shown with the own computer, but I want the electronic form, too.
- The presentation should be given orally. It is recommended, that the student does not just read a text, but explains the subjects freely in her/his own words.
- The student should be able to answer questions from her/his fellow students. That does not mean, that she/he has to have all the answers.

The presentation helps also practicing the necessary presentation of the masters thesis at the end of the students masters studies.

Homework

Without calculating some problems any lecture in theorectical physics remains a fairy tale. In that sense the homework is required to profit from this lecture. The solving of problems helps to understand, whether the student has understood the material or not. At the exam it is too late to recognise, that one has not learned the required material.

The students are invited to come before the homework is due to discuss the problems and ask. I will gladly help them to understand the problem and guide them to the solution. The best way to arrange for a meeting is to write an email to arrange a time, as I can not guarantee that I will have always immediately time for the questions or that I will be always in my room (509).

I plan to give less points for homework that is brought later than its due date. It will nevertheless help to do the homework, even if it is late, as the exam will have questions and problems to solve similar to the homework, too.

Exam

The exam will be a written test, that I want to discuss afterwards with the student.

References

[1] Lecture notes by David Hogg:
`http://cosmo.nyu.edu/hogg/sr/sr.pdf`

[2] David Griffiths, *Introduction to Elementary Particles*,
John Wiley & Sons, Inc.; ISBN 0-471-60386-4 (1987)

[3] S. M. Carroll, *Spacetime and geometry: An introduction to general relativity*,
San Francisco, USA: Addison-Wesley (2004) 513 p
free preprint: *Lecture Notes on General Relativity*, 1st section
gr-qc/9712019

[4] B. F. Schutz,
A First Course In General Relativity,
Cambridge, Uk: Univ. Pr. (1985) 376p

[5] C. W. Misner, K. S. Thorne and J. A. Wheeler,
Gravitation,
San Francisco 1973, 1279p

[6] T. P. Cheng, *Relativity, Gravitation, And Cosmology: A Basic Introduction*,
Oxford, UK: Univ. Pr. (2010) 435 p

[7] P. J. E. Peebles, *Principles of physical cosmology*,
Princeton, USA: Univ. Pr. (1993) 718 p

[8] D. H. Perkins, *Particle astrophysics*,
Oxford, UK: Univ. Pr. (2003) 256 p

[9] S. Dodelson, *Modern cosmology*,
Amsterdam, Netherlands: Academic Pr. (2003) 440 p

[10] The particle adventure:
`http://www.particleadventure.org/`

[11] A. Zee, *Quantum Field Theory in a Nutshell*,
Princeton University Press; ISBN 0-691-01019-6 (2003)

[12] Michael E. Peskin and Daniel V. Schroeder, *An Introduction to Quantum Field Theory*,
Reading, USA: Addison-Wesley; ISBN 0-201-50397-2 (1995)

[13] David Tong, *Quantum Field Theory*,
University of Cambridge Part III Mathematical Tripos
`http://www.damtp.cam.ac.uk/user/tong/qft/qft.pdf`

[14] I. J. R. Aitchison and A. J. G. Hey,
Gauge theories in particle physics: A practical introduction.
Vol. 1: From relativistic quantum mechanics to QED,
Bristol, UK: IOP (2003) 406 p
Vol. 2: Non-Abelian gauge theories: QCD and the electroweak theory,
Bristol, UK: IOP (2004) 454 p

[15] I. J. R. Aitchison, *Supersymmetry in particle physics: An elementary introduction*,
Cambridge, UK: Univ. Pr. (2007) 222 p

[16] Stefan Pokorsky, *Gauge Field Theories*,
Cambridge University Press; ISBN 0-521-47816-2 (2000)

[17] Steven Weinberg, *The Quantum Theory of Fields, I and II*,
Cambridge University Press; ISBN 0-521-58555-4 (1995)

[18] Steven Weinberg, *The Quantum Theory of Fields, III*,
Cambridge University Press; ISBN 0-521-66000-9 (2000)

[19] Warren Siegel, *Fields*
`http://insti.physics.sunysb.edu/~siegel/plan.html` (2002)

Homework: Vector, Tensors — **due 2018/02/22, 12:00**
David Griffiths, Chapter 4, pp. 137-138, n. 4.6, and n. 4.7,
and David Griffiths, Chapter 3, pp. 100-102, n. 3.8 :

4.6. Consider a vector $\vec{a}$ in two dimensions. Suppose its components with respect to Cartesian axes x, y, are (a_x, a_y). What are its components (a'_x, a'_y) in a system x', y' which is rotated, counterclockwise, by an angle θ, with respect to x, y? Express your answer in the for of a 2×2 matrix $R(\theta)$:

$$\begin{pmatrix} a'_x \\ a'_y \end{pmatrix} = R(\theta) \begin{pmatrix} a_x \\ a_y \end{pmatrix}$$

Show that R is an orthogonal matrix. What is its determinant? The set of *all* such rotations constitutes a group; what is the name of this group? By multiplying the matrices show that $R(\theta_1)R(\theta_2) = R(\theta_1 + \theta_2)$; is this an Abelian group?

0.3+0.1+0.1+0.1+0.1+0.1 POINTS

4.7. Consider the matrix $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$. Is it in the group $O(2)$? How about $SO(2)$? What is its effect on the vector $\vec{a}$ of Problem 4.6? Does it describe a possible rotation of the plane? 0.1+0.1+0.2+0.2 POINTS

3.8. A second–rank tensor is called *symmetric* if it is unchanged when you switch the indices ($s^{\nu\mu} = s^{\mu\nu}$); it is called *antisymmetric* if it changes sign ($a^{\nu\mu} = -a^{\mu\nu}$).

(a) How many independent elements are there in a symmetric tensor? (Since $s^{12} = s^{21}$, these would count as only *one* independent element.) 0.1 POINTS

(b) How many independent elements are there in an antisymmetric tensor?
0.1 POINTS

(c) If $s^{\mu\nu}$ is symmetric, show that $s_{\mu\nu}$ is also symmetric. If $a^{\mu\nu}$ is antisymmetric, show that $a_{\mu\nu}$ is antisymmetric. 0.1 POINTS

(d) If $s^{\mu\nu}$ is symmetric and $a^{\mu\nu}$ is antisymmetric, show that $s^{\mu\nu}a_{\mu\nu} = 0$. 0.1 POINTS

(e) Show that any second–rank tensor $(t^{\mu\nu})$ can be written as the sum of an antisymmetric part $(a^{\mu\nu})$ and a symmetric part $(s^{\mu\nu})$: $(t^{\mu\nu} = a^{\mu\nu} + s^{\mu\nu})$. Construct $(a^{\mu\nu})$ and $(s^{\mu\nu})$ explicitly, given $(t^{\mu\nu})$. 0.1 POINTS

Homework: Lorenztransformations **— due 2018/03/08, 12:00**

David Hoggs, Chapter 3, p. 14, Problems 3.3 and 3.4, and Chapter 4, p.22, Problems 4.7 and 4.8:

3.3. A rocket ship of proper length ℓ_0 travels at constant speed v in the $\hat{x}$-direction relative to a frame S. The nose of the ship passes the point $x = 0$ (in S) at time $t = 0$, and at this event a light signal is sent from the nose of the ship to the rear.

(a) Draw a space-time diagram showing the worldlines of the nose and rear of the ship and the photon in S. 0.3 POINTS

(b) When does the signal get to the rear of the ship in S? 0.3 POINTS

(c) When does the rear of the ship pass $x = 0$ in S? 0.3 POINTS

3.4. At noon a rocket ship passes the Earth at speed $\beta = 0.8$. Observers on the ship and on Earth agree that it is noon. Answer the following questions and draw complete spacetime diagrams in both the Earth and rocket ship frames, showing all events and worldlines:

(a) At 12:30 p.m., as read by a rocket ship clock, the ship passes an interplanetary navigational station that is fixed relative to the Earth and whose clocks read Earth time. What time is it at the station? 0.3 POINTS

(b) How far from Earth, in Earth coordinates, is the station? 0.3 POINTS

(c) At 12:30 p.m. rocket time, the ship reports by radio back to Earth. When does Earth receive this signal (in Earth time)? 0.3 POINTS

(d) Earth replies immediately. When does the rocket receive the response (in rocket time)? 0.3 POINTS

(e) The spacetime diagrams 0.2+0.2 POINTS

4.7. In an interplanetary race, slow team X is travelling in their old rocket at speed 0.9c relative to the finish line. They are passed by faster team Y, observing Y to pass X at 0.9c. But team Y observes fastest team Z to pass Ys own rocket at 0.9c. What are the speeds of teams X, Y and Z relative to the finish line? 0.4 POINTS

4.8. An unstable particle at rest in the Lab frame splits into two identical pieces, which fly apart in opposite directions at Lorentz factor $\gamma = 100$ relative to the Lab frame. What is one particles Lorentz factor relative to the other? What is its speed relative to the other? 0.3 POINTS

Homework: Particle kinematics I **— due 2018/03/22, 12:00**

David Hoggs, Chapter 6, p. 34, Problems 6.7, 6.8, 6.9, and 6.10:

6.7. A particle of mass M, at rest, decays into two smaller particles of masses m_1 and m_2. What are their energies and momenta? 0.2 POINTS

6.8. Solve problem 6.7 again for the case $m_2 = 0$. Solve the equations for p and E_1 and then take the limit $m_1 \to 0$. 0.3 POINTS

6.9. If a massive particle decays into photons, explain using 4-momenta why it cannot decay into a single photon, but must decay into two or more. Does your explanation still hold if the particle is moving at high speed when it decays? 1 POINTS

6.10. A particle of rest mass M, travelling at speed v in the x-direction, decays into two photons, moving in the positive and negative x-direction relative to the original particle. What are their energies? What are the photon energies and directions if the photons are emitted in the positive and negative y-direction relative to the original particle (i.e., perpendicular to the direction of motion, in the particles rest frame).

0.5+0.5 POINTS

Homework: FLRW universes **— due 2018/04/19, 12:00**

The Friedmann equations

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8}{3}\pi G\rho - \frac{k}{a^2} \qquad \frac{\ddot{a}}{a} = -\frac{4}{3}\pi G(\rho + 3\mathbf{p}) = -\frac{4}{3}\pi G(1+3w)\rho$$

can be solved explicitely for specific matter content. $\dot{a} := \frac{\partial a}{\partial t}$ and $\ddot{a} := \frac{\partial^2 a}{\partial t^2}$

Show that the parametric solutions are really solutions and determine the parameter b for each solution:

A.1 For $w = 0$ we have $\rho = m * a^{-3}$ and

for $k = -1$: $a = b(\cosh\phi - 1)$, $t = b(\sinh\phi - \phi)$

for $k = 0$: $a = \left(\frac{9b}{2}\right)^{1/3} t^{2/3}$

for $k = +1$: $a = b(1 - \cos\phi)$, $t = b(\phi - \sin\phi)$

0.7+0.7+0.7 points

A.2 For $w = \frac{1}{3}$ we have $\rho = E * a^{-4}$ and

for $k = -1$: $a = [(2b+t)t]^{1/2}$

for $k = 0$: $a = (4b)^{1/4} t^{1/2}$

for $k = +1$: $a = [(2b-t)t]^{1/2}$

0.7+0.7+0.7 points

A.3 For $w = -1$ we have $\rho = \frac{\Lambda}{8\pi G}$.

If $\Lambda < 0$ we have $k = -1$ and $a = b^{-1}\sin bt$. 0.7 points

If $\Lambda > 0$ we have

for $k = -1$: $a = b^{-1}\sinh bt$

for $k = 0$: $a = b^{-1}e^{bt}$

for $k = +1$: $a = b^{-1}\cosh bt$

0.7+0.7+0.7 points

B Find the Killing vectors for the Robertson-Walker metric

$$\mathrm{d}s^2 = \mathrm{d}t^2 - a^2(t)\left[\frac{\mathrm{d}r^2}{1-kr^2} + r^2\mathrm{d}^2\Omega\right]$$

3 points

Homework: Spin — **due 2018/05/03, 12:00**

David Griffiths, Chapter 4, p. 139, n. 4.23 and inspired by David Griffiths, Introduction to Quantum Mechanics, p. 169, n. 4.38:

4.23. The extension of everything in Section 4.4 to higher spin is relatively straightforward. For spin 1 we have three state ($m_s = +1,\, 0,\, -1$), which can we may represent as column vectors

$$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix},$$

respectively. The only problem is to construct the 3×3 matrices $\hat{S}_x$, $\hat{S}_y$, and $\hat{S}_z$. The latter is easy:

(a) Construct $\hat{S}_z$ for spin 1. 0.5 POINTS

To obtain $\hat{S}_x$ and $\hat{S}_y$ it is easiest to start with the "raising" and "lowering" operators, $\hat{S}_\pm = \hat{S}_x \pm i\hat{S}_y$, which have the property

$$\hat{S}_\pm |sm\rangle = \hbar\sqrt{s(s+1) - m(m \pm 1)}\, |s(m \pm 1)\rangle$$

(b) Construct the matrices $\hat{S}_+$ and $\hat{S}_-$ for spin 1. 0.5 POINTS

(c) Using (b) determine the spin-1 matrices $\hat{S}_x$ and $\hat{S}_y$. 0.2 POINTS

(d) Do the same construction for spin $\frac{3}{2}$. 1.3 POINTS

"4.38" Consider a "boundstate" B, made out of two spin-$\frac{1}{2}$ states, f_1 and f_2. These states f_i are described by $\hat{S}^{(i)}$. The boundstate has a spinoperator

$$\hat{S} = \hat{S}^{(1)} + \hat{S}^{(2)}$$

with $\hat{S}^{(i)}$ acting on the ith constituent f_i.

(a) What are the possible eigenvalues of $\hat{S}_z$ for B? 0.5 POINTS

(b) What are the possible eigenvalues of $\hat{S}^2$ for B? 0.5 POINTS

(c) What are the possible eigenstates of B with respect to $\hat{S}^2$ and $\hat{S}_z$ in terms of the constituent eigenstates f_i? 0.5 POINTS

(d) How does that compare to the previous exercise 4.23 ? 0.5 POINTS

(e) How can you apply that to the two $SU(2)$ subgroups of the Lorentz group? 1.0 POINTS