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Abstract
This paper describes the dynamics of solitonic pulse propagation in a closed loop quantum system comprised of five atomic energy levels. In this system four higher energy levels with four control laser fields form a closed ring-shape loop which is coherently coupled to a ground level state by a tunable probe laser field. Although the propagation of an intense probe pulse suffers losses and broadening in linear regime, there exist specific conditions where the probe pulse is found to become robust during its propagation. We attribute this regime to the formation of an optical soliton with a slow group velocity because of the balance between the dispersion and Kerr optical nonlinearity. Results obtained are based on the theoretical model using the coupled Maxwell–Bloch equations for the nonlinear propagation.

Keywords: electromagnetically induced transparency, slow light optical soliton, Kerr nonlinearity

(Some figures may appear in colour only in the online journal)

1. Introduction

The propagation of optical pulses through nonlinear media coupled coherently to one or several laser fields has been an active subject of research in various branches of physics [1–7], ranging from optical communications [8] to quantum nonlinear optics [3, 9]. The pioneering studies of the nonlinear interactions in the atomic media have opened up many new aspects in wave propagation such as slow light [10–12], coherent control of absorption and dispersion [13, 14], and large enhancement of optical nonlinearities [15–19]. In addition, a range of other remarkable optical phenomena have been investigated, such as optical bistability [20–23], quantum entanglement [24–29], storage and retrieval of light pulses [30–35], four-wave mixing [36–38], giant Kerr nonlinearity [39–43], stable optical solitons [44, 45] and so on [46–50].

Due to the nonlinearity of the medium the propagation of a light pulse may lead to a significant change in its temporal and spatial profile. However, under specific circumstances, when the balance between linear and nonlinear effects takes place, the shape of the optical pulse can be preserved over the long propagation distances. A special category of stable shape-preserving waves propagating through the nonlinear media (called solitons) can be formed as a result of balance between Kerr nonlinearity and dispersion [51]. Over the recent years, the subject of solitons has been investigated in various types of matter including cold-atoms [48, 52, 53], Bose–Einstein condensates [54, 55], and other nonlinear media [56].

Following the report of ultra slow optical solitons in a highly resonant atomic medium by Wu and Deng [44], these type of wave packets have recently attracted much attention [32, 45, 57–67]. For instance, ultra slow optical solitons, their storage and retrieval were reported in ultracold three-level Rydberg atoms [32]. The dynamics of ultra slow optical soliton in a cold, highly resonant three-state atomic system under Raman excitation has been investigated by Huang and collaborators [61]. Hang et al showed that a large enhancement of Kerr nonlinearity can be realized due to the appearance of a gain induced by the quantum interference in a four-level atomic system; hence a stable long distance propagation of optical solitons can be obtained with superluminal propagating velocity [66]. Due to the high potential of
shape-preserving optical packets in the applications like optical information processing, more experimental studies are expected to be carried out in the near future.

So far, most of the previous studies have addressed the soliton formation using the basic atomic configurations comprising few atomic energy levels. The three basic configurations are $\Lambda-$, $V-$, and the ladder-type atom-light coupling schemes \[12, 17\]. However, in order to achieve greater control and flexibility over the distortionless pulse propagation, novel practical schemes are required exhibiting much richer quantum interference and coherence mechanisms. Indeed, each extra atomic level impels new physical features for a particular multilevel atomic system. For instance, four-level N-type \[6\] and tripod \[68, 69\] systems have already manifested a fascinating usage in the demonstration of soliton light as well as shape-preserving optical pulses \[44\]. More complicated level structures suggest even more charming effects because of the quantum interference and coherence induced by the extra atomic levels and the coupling light fields \[11, 14, 59, 63\]. In this regard, five level systems look quite promising due to both the increase in flexibility yet maintaining the reasonable complexity for the theoretical description.

To this end, a five-level toy-model atom-light coupling scheme is proposed here in which four control laser components couple a pair of internal states to another pair of states in all possible ways to establish a closed-loop setup of the atom-light interaction. A probe laser field then couples the ring-coupling structure to a ground level state. Such a scheme was first proposed by Kobrak and Rice \[70\] for establishing a complete population transfer \[71\] to a single target of a degenerate pair of states. Subsequently it has been employed to show the advantages of the measurement in coherent control of atomic or molecular processes \[72\]. Moreover, by using intense laser fields a new quantum measurement has been introduced in the Kobrak–Rice five-level system \[73\]. The Kerr nonlinearity \[43\] and atom localization \[74\] behaviors of such a configuration have been also investigated.

In our paper, we show that the nonlinear origin of the coupled Maxwell–Bloch equations describing the interaction of radiation field with such an atomic medium leads to significant effects on the behavior of pulse propagation. When the probe pulse is weak, it propagates without any significant loss and pulse shape distortion through the medium. In contrast, intense probe pulse intensity results in remarkable absorption of the probe pulse intensity through the medium accompanied by a pulse broadening after a very short propagation distance. Analytical solutions are given to elucidate such a behavior. Following this theoretical description, we investigate a different regime where a shape-preserving probe pulse propagation (soliton pulse) can be achieved. The corresponding nonlinear wave equation of the pulse propagation and the possible practical implementation of such a regime are discussed.

2. Model and dynamics of light propagation

Let us consider the propagation of a probe pulse by the electric field of the form

$$\hat{E}_p(z, t) = e_p E_p(z, t) e^{ik_z z - \omega_p t} + c.c.$$. \hspace{1cm} (1)

The total electric field of control laser fields is defined as

$$\hat{E}_c(z, t) = e_1 E_1(z, t) e^{ik_1 z - \omega_1 t} + e_2 E_2(z, t) e^{ik_2 z - \omega_2 t} + e_3 E_3(z, t) e^{ik_3 z - \omega_3 t} + e_4 E_4(z, t) e^{ik_4 z - \omega_4 t} + c.c.$$  \hspace{1cm} (2)

which acts as control on the propagation of probe pulse. Here, $e_i$ and $E_i (i = p, 1, 2, 3, 4)$ are the unit polarization vector as well as the envelope of the probe and control fields, respectively. The wave number of the probe and control field is denoted by $k_i = \omega_i / c$.

We consider a level structure as illustrated in figure 1. The system consists of the excited state $|1\rangle$, two non-degenerate metastable lower states $|3\rangle$ and $|5\rangle$ as well as two intermediate degenerate states $|4\rangle$ and $|2\rangle$. Four strong laser driving components couple a pair of atomic internal states $|1\rangle$ and $|3\rangle$ to another pair of states $|2\rangle$ and $|4\rangle$ in all possible paths making a ring-coupling structure. Such a level scheme is equivalent to a consequently coupled cyclic chain of four states $|1\rangle$, $|2\rangle$, $|3\rangle$, and $|4\rangle$ resulting in a closed-loop diamond-shape subsystem. Four Rabi-frequencies $\Omega_{21} = (e_1, d_{12}) E_1 / h$, $\Omega_{23} = (e_2, d_{32}) E_2 / h$, $\Omega_{41} = (e_3, d_{41}) E_3 / h$, and $\Omega_{43} = (e_4, d_{34}) E_4 / h$ are the dipole matrix element corresponding to the transition $|i\rangle \leftrightarrow |j\rangle$ are introduced to the transitions $|2\rangle \leftrightarrow |1\rangle$, $|2\rangle \leftrightarrow |3\rangle$, $|1\rangle \leftrightarrow |4\rangle$, and $|3\rangle \leftrightarrow |4\rangle$, respectively. An additional tunable coherent probe field with Rabi-frequency $\Omega_p = (e_p, d_{35}) E_p / h$ couples the diamond-shape subsystem to a ground (or metastable) state $|5\rangle$ through the dipole-allowed optical transition $|3\rangle \leftrightarrow |5\rangle$.  

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{Schematic diagram of the five-level quantum system.}
\end{figure}
The total Hamiltonian of the system $H_{TS}$ can be written as

$$H_{TS} = -\hbar(\Omega_p|3\rangle\langle 5| + \Omega_4|1\rangle\langle 4| + \Omega_{21}|2\rangle\langle 1| + \Omega_{13}|3\rangle\langle 2| + \Omega_{43}|4\rangle\langle 3| + \text{h.c.}),$$

(3)

where $\phi = \phi_4 + \phi_3 + \phi_{32} + \phi_{23}$ denotes the relative phase accumulated after completing a ring, while $\phi_{ij}$ describes the initial phase of each laser field.

The atomic motion is modeled by the density operator

$$\rho = -\frac{i}{\hbar}[H_{TS}, \rho] + L_p,$$

(4)

where $L_p$ stands for the damping operator represented the decay of the system. Substituting equation (3) into equation (4), the optical Bloch equations for density matrix elements describing the evolution of the system are obtained and are presented in appendix A.

Maxwell equation controls the evolution of the electric field $\vec{E} = \vec{E}_p + \vec{E}$, in the system

$$\nabla^2 \vec{E} - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \vec{E} = \frac{1}{\varepsilon_0 c^2} \frac{\partial^2}{\partial t^2} \vec{P},$$

(5)

with the optical polarization

$$\vec{P} = N(d_{35} \rho_3 \rho_5 e^{i(kz-\omega t)} + d_{12} \rho_1 \rho_2 e^{i(kz-\omega t)} + d_{32} \rho_3 \rho_2 e^{i(kz-\omega t)} + d_{43} \rho_4 \rho_3 e^{i(kz-\omega t)} + c.c.),$$

(6)

with $N$ being the atomic density.

Working in the slowly varying envelope approximation, one can obtain the temporal and spatial evolution of the probe pulse as

$$\frac{\partial \Omega_p}{\partial z} + \frac{\partial \Omega_p}{\partial \omega} + \frac{iq\rho_{35}}{c} = 0,$$

(7)

where $q = 2N\omega_d |d_{35}|^2 / \hbar c$ is the propagation coupling constant. The solution of Maxwell–Bloch equations (4) and (7) describes the propagation dynamics of probe pulse inside the medium.

### 3. Linear pulse propagation

In the following, we discuss the linear evolution of probe field propagating through the five-level atomic system. The solution of the nonlinearity coupled equations (4) and (7) enables us to model also the nonlinear propagation of the probe pulse and provide a rather precise picture of interplay between the dispersion and nonlinearities inside such a five-level system which will be studied in next section. However, before solving these equations for the nonlinear regime we first examine the linear behavior of the system by presenting some numerical results. This may give some effective hints for the study of nonlinear pulse propagation discussed later. In doing so, we assume that the atom is initially in ground state $|5\rangle$, and that the Rabi-frequency of the probe pulse is small enough (compared to the Rabi-frequencies of control fields) so that we can neglect the depletion of the ground level. Therefore, we can apply the perturbation expansion $\rho_{ij} = \rho_{ij}^{(0)} + \rho_{ij}^{(1)} + \rho_{ij}^{(2)} + \ldots$ where $\rho_{ij}^{(n)}$ is the $n$th-order of $\rho_{ij}$ in terms of probe pulse $\Omega_p$. The zeroth order solution is $\rho_{ij}^{(0)} = 1$ while other elements being zero. Keeping up to the first order of $\Omega_p$, the set of consequential equations from Maxwell–Bloch equations (4) and (7) can be reduced under linearization to the form

$$\rho_{15}^{(1)} = -g_1 \rho_{15}^{(1)} + i\Omega_4 e^{i\omega_p t} \rho_{45}^{(1)} + i\Omega_{21} \rho_{25}^{(1)},$$

(8a)

$$\rho_{25}^{(1)} = -g_2 \rho_{25}^{(1)} + i\Omega_2 \rho_{15}^{(1)} + i\Omega_2 \rho_{35}^{(1)},$$

(8b)

$$\rho_{35}^{(1)} = -g_3 \rho_{35}^{(1)} + i\Omega_4 \rho_{45}^{(1)} + i\Omega_{32} \rho_{15}^{(1)} + i\Omega_4 \rho_{45}^{(1)},$$

(8c)

$$\rho_{45}^{(1)} = -g_4 \rho_{45}^{(1)} + i\Omega_4 \rho_{15}^{(1)} + i\Omega_{43} \rho_{35}^{(1)},$$

(8d)

and

$$\frac{\partial \rho_{ij}}{\partial t} + \frac{\partial \rho_{ij}}{\partial \omega} = i\rho_{ij}^{(1)},$$

(9)

Performing the time Fourier transform of equations (8) and (9), we get

$$G_1(\omega) Y_{15}^{(1)} + i\Omega_4 e^{i\omega_p} Y_{45}^{(1)} + i\Omega_{21} Y_{25}^{(1)} = 0,$$

(10a)

$$G_2(\omega) Y_{25}^{(1)} + i\Omega_2 Y_{15}^{(1)} + i\Omega_2 Y_{35}^{(1)} = 0,$$

(10b)

$$G_3(\omega) Y_{35}^{(1)} + i\Omega_4 Y_{45}^{(1)} + i\Omega_{32} Y_{25}^{(1)} + \omega_p = 0,$$

(10c)

$$G_4(\omega) Y_{45}^{(1)} + i\Omega_4 e^{-i\omega_p} Y_{15}^{(1)} + i\Omega_{43} Y_{35}^{(1)} = 0,$$

(10d)

and

$$\frac{\partial \omega_p}{\partial z} - i\omega_p = i\rho_{15}^{(1)},$$

(11)

where $G_1(\omega) = [\omega + (\Delta_4 + \Delta_2 + \Delta_p) + i(\gamma_4 + \gamma_2)],$ $G_2(\omega) = [\omega + (\Delta_3 - \Delta + \Delta_p) + i\gamma_3)],$ $G_3(\omega) = [\omega + i\gamma_2 + \Delta_p],$ and $G_4(\omega) = [\omega - (\Delta_4 + \Delta_p) - i\gamma_4)].$ Note that $\omega_p$ and $Y_p^{(1)}$ represent the Fourier transform of $\Omega_p$ and $\rho_{ij}^{(1)}$, respectively, while $\omega$ is the Fourier transform variable.

Equations (10a)–(10d) can be solved analytically, yielding

$$Y_{35}^{(1)} = \frac{\omega_p}{L_1(\omega)} L_4(\omega),$$

(12a)

$$Y_{45}^{(1)} = -\frac{\omega_p}{L_1(\omega)} L_3(\omega),$$

(12b)

$$Y_{25}^{(1)} = -\frac{\omega_p}{L_1(\omega)} L_3(\omega),$$

(12c)

$$Y_{15}^{(1)} = \frac{\omega_p}{L_1(\omega)} L_4(\omega),$$

(12d)

where

$$L_1(\omega) = G_1(\omega) G_2(\omega) G_4(\omega) - G_2(\omega) \Omega_{21}^2 - G_2(\omega) \Omega_{41}^2,$$

(13a)

$$L_2(\omega) = \Omega_{23} \Omega_{21} \Omega_{41} e^{-i\omega} - \Omega_{43} \Omega_{21}^2 + \Omega_{43} G_1(\omega) G_2(\omega),$$

(13b)

$$L_3(\omega) = \Omega_{43} \Omega_{21} e^{i\omega} - \Omega_{32} \Omega_{21}^2 + \Omega_{32} G_1(\omega) G_2(\omega),$$

(13c)

$$L_4(\omega) = \Omega_{32} \Omega_{21} G_4(\omega) + \Omega_{43} \Omega_{41} e^{i\omega} G_2(\omega),$$

(13d)

and
L(ω) = \Omega_2 G_1 G_2(ω) - \Omega_2^2 \Omega_4^2 - \Omega_4^2 \Omega_2^2 \\
+ \Omega_2^2 G_1 G_2(ω) + \Omega_2^2 G_1 G_2(ω) G_3(ω) \\
+ \Omega_2^4 G_2(ω) G_3(ω) - G_1(ω) G_2(ω) G_3(ω) G_4(ω) \\
+ 2\Omega_4 \Omega_5 \Omega_2 \Omega_4 \cos \phi. \quad (14)

A solution of equation (11) is plane wave of the form
\[ \varphi_0(\tau, \omega) = \varphi_0(0, \omega)e^{j\kappa(\omega)\tau}, \]  
where
\[ \kappa(\omega) = \frac{\omega}{c} - \frac{L_1(\omega)}{L(\omega)}, \]  
(16)
describes the linear dispersion relation of the system. Expanding \( \kappa(\omega) \) in power series around the center frequency of the probe pulse and taking the first three terms, we get
\[ \kappa(\omega) = \kappa_0 + \kappa_1 \omega + \kappa_2 \omega^2. \]  
(17)
with
\[ \kappa_0 = -\frac{L_1}{L}, \]  
(18a)
\[ \kappa_1 = \frac{1}{c} - q\left( \frac{Q}{L} - \frac{L_1 B}{2 L^2} \right), \]  
(18b)
\[ \kappa_2 = \frac{Q}{L} \left( -\frac{1}{SL^2} + \frac{2}{BL^2} \right) + \frac{R}{L} - \frac{2Q}{BL^2} \]  
(18c)
where the detailed expressions for \( Q, B, S \) and \( R \) are given in appendix B, while \( L \) and \( L_1 \) can be obtained by substituting \( \omega = 0 \) into equations (13a) and (14) for \( L(\omega) \) and \( L_1(\omega) \), respectively. In equation (17), \( \kappa_i = \frac{\partial^2 \kappa(\omega)}{\partial \omega^2} \bigg|_{\omega=0} \) gives the dispersion coefficients in different orders. In general, for \( \kappa_0 = \Phi + i\alpha/2 \) the real part corresponds to the phase shift \( \Phi \) per unit length, while the imaginary part describes the absorption \( \alpha \) of the probe pulse. The group velocity is \( v_g = 1/\kappa_0 \), and the nonlinear term \( \kappa_2 \) is associated with group velocity dispersion and leads to pulse distortion during the propagation.

4. Influence of Kerr nonlinearity
The linear response of the atomic medium is now obtained through neglecting higher-order terms of the probe field and under the weak-field approximation. However, it is known that for the intense probe pulses the nonlinear factors due to the inherent Kerr nonlinearity of the system must be also taken into account. It is due to the significant impact of the nonlinearity of the system in high powers of probe pulse which may lead to probe attenuation and distortion. To have a better understanding of this, the propagation problem is simulated by solving simultaneously the coupled Maxwell–Bloch equations for the five-level closed loop system. We work in the retarded frame with the traveling coordinates \( t' = t - z/c, z' = z \). This choice of coordinates reduces the partial differential equation (7) with respect to the independent variable \( z' \),
\[ \frac{\partial \Omega_p}{\partial z'} = iq\rho_{35}. \]  
(19)

We assume the propagation of a Gaussian probe pulse
\[ \Omega_p(0, t') = \Omega_0^p e^{-t'/\tau_0^2}, \]  
(20)
where \( \tau_0 \) is the temporal width of the input pulse (its time duration).

In order to investigate the impact of nonlinearities on pulse propagation stemming from strong power of probe pulse, we assume the four control fields as continuous waves and all the Rabi-frequencies are the same
\[ \Omega_43 = \Omega_23 = \Omega_41 = \Omega_21 = \Omega. \]  
(21)

In what follows, we present our numerical results. Without loss of generality, we take \( \gamma_43 = \gamma_23 = \gamma_12 = \gamma_4 = \gamma_35 = \gamma \) [74]. Figure 2 shows the resulting propagation dynamics of a weak Gaussian probe pulse through the five-level quantum system for different values of relative phase \( \phi \) while \( \Omega_0^p = 0.01 \gamma \). It can be seen from figures (a)-(c) that the relative phase \( \phi \) can seriously affect the absorption of the pulse. The case with \( \phi = 0 \) (figure (a)) gives rise to the loss of the weak probe pulse intensity through the medium after almost a short propagation distance. In contrast and as can be observed from figures (b) and (c), when \( \phi = \pi/2 \) and \( \phi = \pi \) the weak probe pulse is transmitted through the phase-sensitive system almost without any substantial absorption and broadening while it can still keep its shape for quite a long propagation distance.

Such a phase-sensitive behavior can be elucidated through the following analytical model.

Without the ground state [5] in equation (3), the Hamiltonian of the atom-light interaction for the remaining atomic four-level closed-loop level structure of the ring-coupling subsystem \( H_{SS} \) simplifies to \( \hbar \leq 1 \):
\[ H_{SS} = -\Omega \left[ |1\rangle e^{i\phi} |4\rangle + \sum_{j=1}^{3} |j + 1\rangle \langle j | + h.c. \right]. \]  
(22)

In writing equation (22) the Rabi-frequencies are chosen according to equation (21).

Following [43, 74], the eigenenergies for the Hamiltonian equation (22) can be expressed as
\[ E_n = -2\Omega \cos \lambda_n, \]  
(23)
with \( \lambda_n = \frac{n\pi}{2} - \frac{\pi}{2} - \frac{\phi}{4} \), where \( n = 1, 2, 3, 4 \).

Let us now inspect the eigenenergies in the limiting cases depending on the relative phase \( \phi \).

(i) The case where \( \phi = 0 \). In this situation, equation (23) reduces to
\[ E_0 = -2\Omega \sin \left( \frac{n\pi}{2} \right). \]  
(24)

Equation (24) results in three eigenenergies \( E_3 = E_4 = 2\Omega, \) and \( E_2 = E_4 = 0 \). In this case the
The lowest energy eigenstate corresponding to $E_i = -2\Omega$ is not degenerate.

(ii) The case where $\phi = \pi/2$. If $\phi = \pi/2$, then

$$E_n = -2\Omega \sin \left(\frac{n\pi}{2} - \frac{\pi}{8}\right),$$

resulting in $E_3 = -E_1 = 4\Omega \cos \frac{\pi}{8}$, and $E_4 = -E_2 = 4\Omega \sin \frac{\pi}{8}$.

(iii) The case where $\phi = \pi$. In this case, equation (23) changes to

$$E_n = -2\Omega \sin \left(\frac{n\pi}{2} - \frac{\pi}{4}\right),$$

giving rise to two pairs of degenerate eigenenergies $E_1 = E_2 = -\sqrt{2}\Omega$ and $E_3 = E_4 = \sqrt{2}\Omega$ separated by the energy $2\sqrt{2}\Omega$.

Figure 3 illustrates the density plot of probe absorption against the probe detuning $\Delta_p$ and the relative phase $\phi$. One can see that three, four and two absorption peaks appear for $\phi = 0$, $\phi = \pi/2$, and $\phi = \pi$, respectively. Obviously, the medium experiences an absorption peak at zero probe detuning for $\phi = 0$ which may result in pulse attenuation during its propagation, as was observed in figure 2(a). However, for cases $\phi = \pi/2$ and $\phi = \pi$, the medium is transparent at $\Delta_p = 0$, leading to a distortionless transmission of light pulse (see also figures 2(b) and (c)) [43]. This indicates an optical switching between absorption and transparency through relative phase of applied fields [43].

Now, we investigate the propagation of probe pulses with higher intensities. This is the situation where the effect of nonlinearities becomes important. The propagation dynamics of an intense probe pulse with $\Omega^p = \gamma$ is plotted in figure 4 for different values of $\phi$ for the same parameters as used in figure 2. Clearly, the intense probe pulse is strongly attenuated by the medium even more rapidly with respect to figure 2(a) (figure 4(a)). The interesting results are obtained for the cases $\phi = \pi/2$ (figure 4(b)) and $\phi = \pi$ (figure 4(c)).
Evidently, in both situations the intense probe pulse suffers strong absorption and broadening during the propagating through the closed-loop medium.

In a linear regime and for a Gaussian shape incoming probe pulse of the form

$$W = W_0 e^{-((t - \tau_0^2)/\tau_0^2)}$$

with a duration \( \tau_0 \), the time evolution of probe pulse after carrying out an inverse Fourier transform is

$$W = W_0 e^{-((t - \tau_0^2)/\tau_0^2)}$$

where

$$k = -\frac{\omega}{c}$$

and

$$\lambda_{NL} = -g e^{-i\omega_0} \rho^{(1)}_{35} \sum_{i=1}^{4} \rho^{(1)}_{i5}$$

and \( \rho^{(1)}_{5j} \) can be obtained by taking \( \omega = 0 \) in equations (12a)–(12d). Reminding \( \kappa = v_g^{-1} \) and after carrying out inverse transform, equation (28) takes the form

$$-i \left[ \frac{\partial}{\partial \zeta} + v_g^{-1} \frac{\partial}{\partial \eta} \right] \Omega_p + \kappa_2 \frac{\partial^2}{\partial \eta^2} \Omega_p = \lambda_{NL}.$$

Defining new coordinates \( \zeta = z \) and \( \eta = t - z/v_p \), we arrive at the nonlinear wave function of slowly varying envelope \( \Omega_p \) as

$$i \frac{\partial}{\partial \zeta} \Omega_p - \kappa_2 \frac{\partial^2}{\partial \eta^2} \Omega_p = Ne^{-i\omega t} \Omega_p,$$

with \( \alpha = 2\text{Im}(\kappa_0) \) being the absorption coefficient. The nonlinear coefficient \( N \) of probe pulse proportional to

5. Soliton regime

In what follows we turn to study the nonlinear evolution of the probe pulse through the phase sensitive atomic structure. In order to investigate the nonlinear propagation of light, one needs to evaluate the nonlinear effects induced by Kerr nonlinearity, which is due to nonlinear terms up to the third order of \( \Omega_p \). According to [44], we take a trial function

$$\varpi_p(z, \omega) = \varpi_p(z, \omega) e^{i\omega t}.$$

Substituting equation (27) into wave equation (11) and using equation (17) we obtain

$$\frac{\partial \varpi_p}{\partial t} = i(\kappa_1 \omega + \kappa_2 \omega^2) \varpi_p,$$

where we only kept terms up to the order \( \omega^2 \) in expanding the dispersion relation \( \kappa(\omega) \). Note that here we have replaced \( \varpi_p \) with \( \varpi_p \), for the sake of convenience. Since we are interested in nonlinear evolution of light, we must take into account the nonlinear polarization of the probe pulse by the form

$$i \eta_p^{(1)} \rho^{(1)}_{35} + i \lambda_{NL} \text{where the nonlinear term is }$$

$$\lambda_{NL} = -g e^{-i\omega_0} \rho^{(1)}_{35} \sum_{i=1}^{4} \rho^{(1)}_{i5},$$

and \( \rho^{(1)}_{5j} \) can be obtained by taking \( \omega = 0 \) in equations (12a)–(12d). Reminding \( \kappa = v_g^{-1} \) and after carrying out inverse transform, equation (28) takes the form

$$-i \left[ \frac{\partial}{\partial \zeta} + v_g^{-1} \frac{\partial}{\partial \eta} \right] \Omega_p + \kappa_2 \frac{\partial^2}{\partial \eta^2} \Omega_p = \lambda_{NL}.$$

Defining new coordinates \( \zeta = z \) and \( \eta = t - z/v_p \), we arrive at the nonlinear wave function of slowly varying envelope \( \Omega_p \) as

$$i \frac{\partial}{\partial \zeta} \Omega_p - \kappa_2 \frac{\partial^2}{\partial \eta^2} \Omega_p = Ne^{-i\omega t} \Omega_p,$$

with \( \alpha = 2\text{Im}(\kappa_0) \) being the absorption coefficient. The nonlinear coefficient \( N \) of probe pulse proportional to
nonlinear Kerr susceptibility is defined by
\[ N = -\frac{q}{\hbar |\mathcal{F}|^2} (G_1 G_2 G_3 - G_1 \Omega_{31}^2 - G_2 \Omega_{41}^2) \times (\Psi_1 + \Psi_2 + \Psi_3 + \Psi_4), \]
where the coefficients \( \Psi_1, \Psi_2, \Psi_3 \) and \( \Psi_4 \) are presented in appendix C.

Equation (31) has generally complex coefficients. However, the absorption coefficient \( \alpha \) may be very small for some specific parametric values, i.e., \( \alpha \approx 0 \), and the real parts of the complex coefficients may become much larger than the corresponding imaginary parts
\[ \kappa_2 = \kappa_{2R} + i\kappa_{2I} \approx \kappa_{2R}, \]
and
\[ N = N_R + iN_I \approx N_R. \]

In this case, we get
\[ i\frac{\partial}{\partial t} \Omega_p = \kappa_{2R} \frac{\partial^2}{\partial \chi^2} \Omega_p = N_R |\Omega_p|^2 \Omega_p. \]

Equation (35) is the well-known nonlinear Schrodinger equation (NSE) which governs the nonlinear pulse propagation, which is well studied in fiber optics [75]. Similar equation was deduced, for example, in [44, 45] in cold three- and four-state atomic structures. Although the five-level atomic structure consider in this work and the three- and four-level schemes in [44, 45] reduce to the same NSE, however, our proposed scheme has some advantages over the previously studied configurations. A main advantage of our proposed model is the phase sensitivity of our proposed scheme, as discussed in details in section 4. In addition, as a result of the quantum interference and coherence induced by the extra atomic levels and the coupling light fields, such a five-level scheme can result in larger magnitude of the Kerr nonlinearity with respect to the three and four-level counterparts [43].

Depending on the sign of \( \kappa_{2R} N_R \), equation (35) allows bright and dark soliton solutions. A bright soliton is obtained when \( \kappa_{2R} N_R \) is positive
\[ \Omega_p = \Omega_{p0} \text{sech}[\eta/\tau] \exp(-i\zeta \Omega_p^2 \Omega_p), \]

while a dark soliton is expected when \( \kappa_{2R} N_R \) is negative
\[ \Omega_p = \Omega_{p0} \tanh[\eta/\tau] \exp(-i\zeta \Omega_p^2 \Omega_p). \]

It should be pointed out that \( \Omega_{p0} = (1/\tau)(\kappa_{2R}/N_R)^{1/2} \) denotes the typical Rabi-frequency of the probe field and \( \tau \) is the typical pulse duration.

Next we give a numerical example for the formation of the shape preserving optical solitons. The proposed level structure can be experimentally implemented using the \(^{87}\text{Rb} \) atoms. The ground level [5] can be assigned to the state \(^5\text{S}_1/2\). The level [3] can be attributed to the \(^5\text{P}_3/2\). Two intermediate levels [2] and [4] can be assigned to either the fine structure of the \(^4\text{D}_{3/2}\) or the \(^4\text{D}_{5/2}\) sub-states, as long as the dipole transition selection rules on the \( F \) quantum number is satisfied (the same \( F \) quantum number for the intermediate states). The top level [1] can be chosen to the \(^6\text{P}_3/2\) state. Taking

\[ \gamma_{21} = \gamma_{23} = \gamma_{31} = \gamma_{32} = \gamma = 3.7 \times 10^7 \text{ s}^{-1} [74], \]
\[ \tau = 10^{-9} \text{ s}, \]
\[ q = 10^{10} \text{ cm}^{-1} \text{ s}^{-1}, \]
\[ \Omega_{21} = \Omega_{41} = \Omega_{43} = \Omega_{32} = \Omega = 2.59 \times 10^8 \text{ s}^{-1}, \]
\[ \Delta_{12} = 2.96 \times 10^8 \text{ s}^{-1}, \]
\[ \Delta_{43} = 1.11 \times 10^9 \text{ s}^{-1}, \]
\[ \Delta_{43} = 1.85 \times 10^9, \]
\[ \Delta_{32} = 1.48 \times 10^9, \]
\[ \Delta_p = 0, \quad \phi = \pi, \]

one can obtain
\[ \kappa_1 = (9.44 + 7.7 \times 10^{-2}i) \times 10^{-10} \text{ cm}^{-1} \text{ s}^{-1}, \]
\[ \kappa_2 = (1.02 - 6.5 \times 10^{-2}i) \times 10^{-10} \text{ cm}^{-1} \text{ s}^{-1}, \]
\[ N = (2.39 - 6.95 \times 10^{-2}i) \times 10^{-20} \text{ cm}^2. \]

Clearly, the imaginary parts of the complex coefficients are much smaller than the corresponding real parts. Thus, the conditions (33) and (34) are satisfied. In this situation, the standard NSE (35) with \( \kappa_{2R} N_R > 0 \) is well characterized, leading to the generation of bright solitons in the proposed system. Such a soliton has a width and amplitude satisfying \( |\Omega_{p0}| \approx 2.06 \). With the above system parameters, one can find \( \eta_p \approx 3.5 \times 10^{-2} \text{cm} \), indicating that the soliton propagates with a slow propagating velocity. As can be observed from figure 5, such a bright soliton remains fairly stable during propagation, which may be due to the balance between the group velocity dispersion and Kerr-type nonlinearity.

6. Concluding remarks

In conclusion, the problem of pulse propagation has been theoretically investigated for a five-level toy-model atom-light coupling system in which the probe laser beam couples a ground level to the ring-coupling subsystem consisting of four atomic energy levels. We have found that the distortionless propagation of the probe pulse without significant absorption and broadening is possible in the linear regime when the intensity of the probe pulse is sufficiently weak. In contrast, for higher intensities, the probe pulse will be attenuated by the medium after almost a short propagation distance. We then presented an analytical model to elucidate the origin of such behavior. By employing a simple theoretical model for the nonlinear pulse propagation, we found the regime of stable distortionless probe pulses propagation through such a medium. Based on the coupled Maxwell–Bloch equations, a nonlinear equation governing the
evolution of the probe pulse envelope has been obtained showing the existence of stable optical solitons with a slow propagating velocity. The formation of these shape-preserving optical solitons has been attributed to the balance between dispersion and Kerr nonlinearity of the system. A possible experimental realization of the investigated model has been proposed for the $^{87}\text{Rb}$ atomic medium with suitable parameters.

As a result of the quantum interference and coherence effects induced by the extra atomic levels and the coupling light fields in such a five level atomic structure, the proposed atomic model has potential applications for establishing a complete population transfer to a single target of a degenerate pair of states [71], or high precision and high resolution localization of an atom [74]. The obtained results may be helpful for guiding experimental investigations of linear and nonlinear optical properties of atomic systems and for applications of optical information processing and engineering.

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Appendix A. Explicit expressions of the optical Bloch equation

Under the rotating wave approximation, the equation of motion (4) for the density operator describing the atomic system reduces to

$$\dot{\rho}_{15} = -g_1\rho_{15} + i\Omega_{41}\rho_{45} - i\Omega_{21}\rho_{25},$$

$$\rho_{25} = -g_2\rho_{25} + i\Omega_{21}\rho_{15} + i\Omega_{32}\rho_{35} - i\Omega_{32}\rho_{35},$$

$$\dot{\rho}_{35} = -g_3\rho_{35} + i\Omega_{43}\rho_{45} + i\Omega_{32}\rho_{25} + i\Omega_{\rho}\rho_{55} - i\rho_{33},$$

$$\rho_{45} = -g_4\rho_{45} + i\Omega_{41}\rho_{41} - i\rho_{15},$$

$$\dot{\rho}_{41} = -g_4\rho_{41} + i\Omega_{41}\rho_{41} - i\Omega_{21}\rho_{25},$$

$$\rho_{55} = -g_5\rho_{55} + i\Omega_{43}\rho_{43} - i\rho_{35},$$

$$\dot{\rho}_{43} = -g_3\rho_{43} + i\Omega_{43}\rho_{43} - i\rho_{33},$$

$$\rho_{32} = -g_2\rho_{32} + i\Omega_{21}\rho_{21} + i\Omega_{21}\rho_{21},$$

$$\dot{\rho}_{21} = -g_2\rho_{21} + i\Omega_{21}\rho_{21} + i\Omega_{32}\rho_{32} - i\Omega_{32}\rho_{32},$$

$$\rho_{23} = -g_8\rho_{23} + i\Omega_{21}\rho_{13} + i\Omega_{21}\rho_{13} - i\Omega_{32}\rho_{32},$$

$$\dot{\rho}_{21} = -g_8\rho_{21} + i\Omega_{21}\rho_{13} + i\Omega_{21}\rho_{13} - i\Omega_{21}\rho_{21},$$

$$\rho_{31} = -g_{10}\rho_{31} + i\Omega_{32}\rho_{32} + i\Omega_{32}\rho_{32} - i\Omega_{32}\rho_{32},$$

$$\dot{\rho}_{11} = -2(\gamma_{14} + \gamma_{12})\rho_{11} + i\Omega_{41}(e^{\gamma_{14}}\rho_{14} - e^{-\gamma_{14}}\rho_{14} - i\Omega_{21}(\rho_{12} - \rho_{21}),$$

$$\dot{\rho}_{22} = 2\gamma_{12}\rho_{11} - 2\gamma_{23}\rho_{22} + i\Omega_{32}(\rho_{32} - \rho_{23}) + i\Omega_{21}(\rho_{12} - \rho_{21}),$$

$$\dot{\rho}_{33} = 2\gamma_{43}\rho_{44} + 2\gamma_{23}\rho_{22} - 2\gamma_{23}\rho_{23} + i\Omega_{43}(\rho_{43} - \rho_{23}) + i\Omega_{32}(\rho_{32} - \rho_{23}) - i\Omega_{\rho}(\rho_{35} - \rho_{33}),$$

$$\dot{\rho}_{44} = 2\gamma_{14}\rho_{11} - 2\gamma_{34}\rho_{44} - i\Omega_{43}(\rho_{33} - \rho_{44}) - i\Omega_{41}(e^{\gamma_{14}}\rho_{14} - e^{-\gamma_{14}}\rho_{14}),$$

$$\rho_{11} + \rho_{22} + \rho_{33} + \rho_{44} + \rho_{55} = 1, \quad \text{(A15)}$$

with $g_1 = [-i(\Delta_{44} + \Delta_{43} + \Delta_{p}) + (\gamma_{14} + \gamma_{12})], \quad g_2 = [-i(\Delta_{32} - \Delta_{23} + \gamma_{14}],\quad g_3 = (\gamma_{43} - i\Delta_{p}), \quad g_4 = [i(\Delta_{43} + \Delta_{p}) - (\gamma_{43} + \gamma_{12})], \quad g_5 = [-i(\Delta_{14} - \Delta_{43}) + (\gamma_{43} + \gamma_{12})], \quad g_7 = [-i(\Delta_{32} - \Delta_{23} + \gamma_{14} + \gamma_{12}], \quad g_9 = [-i(\Delta_{42} + \Delta_{14} + \gamma_{14} + \gamma_{12})], \quad g_{10} = [i(\Delta_{44} + \Delta_{43}) + (\gamma_{43} + \gamma_{12})].$ The one-photon resonance detuning parameters for transitions $|4\rangle \leftrightarrow |3\rangle, |2\rangle \leftrightarrow |3\rangle, |1\rangle \leftrightarrow |4\rangle, |2\rangle \leftrightarrow |1\rangle$ and $|3\rangle \leftrightarrow |5\rangle$ are $\Delta_{23} = \omega_1 - \omega_{43}, \quad \Delta_{23} = \omega_2 - \omega_{23}, \quad \Delta_{41} = \omega_4 - \omega_1, \quad \Delta_{42} = \omega_2 - \omega_{43},$ and $\Delta_{p} = \omega_p - \omega_{35},$ while $\Delta = \Delta_{42} - \Delta_{43} + \Delta_{23} - \Delta_{23}$ and $\omega_i$ represent the multi-photon detuning and the central frequency of the corresponding laser field. The spontaneous decays from the excited state $|1\rangle$ to the lower levels $|3\rangle$ and $|5\rangle$ are ignored due to the assumption that the corresponding transitions are dipole forbidden. The spontaneous decay rates of upper level $|i\rangle$ to the lower level $|j\rangle$ is described by $2\gamma_{ij}.$

Appendix B. Explicit expressions of $Q, B, S$ and $R$

Expressions for $Q, B, S$ and $R$ read

$$Q = \Omega_{21}^2 + \Omega_{41}^2 - G_4G_1 - G_2G_1 - G_2G_4,$$  

$$B = \Omega_{32}^2G_4 + \Omega_{32}^2G_1 + \Omega_{32}^2G_2 + \Omega_{43}^2G_4 + \Omega_{43}^2G_1 + \Omega_{43}^2G_3 + \Omega_{12}^2G_3 + G_3G_1G_4 - G_2G_1G_3 - G_2G_4 + G_2G_4 - G_2G_4,$$  

$$S = -2(\Omega_{21}^2 + \Omega_{41}^2 + \Omega_{43}^2 + \Omega_{41}^2G_1 + G_1G_3 + \Omega_{12}^2G_3 + G_3G_1G_4 + G_2G_4 + G_3G_2 + G_3G_3 + G_4G_4),$$  

$$R = 2\Delta_{44} - 2\Delta + 2\Delta_{23} + 2\Delta_{p} + 2i(\gamma_{23} - \gamma_{33}).$$  

where $G_1, G_2, G_3, G_4, L$ and $L_4$ can be obtained by substituting $\omega = 0$ in coefficients $G_1(\omega), G_2(\omega), G_3(\omega), G_4(\omega), L(\omega)$ and $L_4(\omega),$ respectively.
Appendix C. Explicit expressions of $\Psi_1$, $\Psi_2$, $\Psi_3$ and $\Psi_4$

Expressions for $\Psi_1$, $\Psi_2$, $\Psi_3$ and $\Psi_4$ can be expressed as

$$
\Psi_1 = G_1^* G_2^* G_3^* G_4^* G_5^* - \Omega_1^2 G_1 G_2 G_4^*
- \Omega_2^2 G_2 G_3 G_5^* - \Omega_3^2 G_1 G_4 G_5^* + \Omega_5^2 G_1 G_2 G_3^*
+ \Omega_4^2 G_1 G_3 G_5^*.
$$

(C1)

$$
\Psi_2 = \Omega_2^2 G_2^* G_1 G_3 G_4 G_5^* - 2 \Omega_2^2 \Omega_3^2 G_1 G_4 G_5^* G_2 G_3
+ \Omega_3^2 G_2 G_1 G_5^* G_4^* - \Omega_4^2 G_1 G_2 G_3 G_5^*.
$$

(C2)

$$
\Psi_3 = \Omega_3^2 G_3^* G_1 G_2 G_4 G_5^* - 2 \Omega_3^2 \Omega_4^2 G_2 G_3 G_4 G_5^* G_1
+ \Omega_4^2 G_2 G_3 G_5^* G_4^* - \Omega_5^2 G_2 G_3 G_4 G_5^*.
$$

(C3)

$$
\Psi_4 = \Omega_4^2 G_4^* G_1 G_2 G_3 G_5^* - \Omega_4^2 \Omega_5^2 G_1 G_3 G_4 G_5^* G_2
+ \Omega_5^2 G_2 G_3 G_4 G_5^* G_1^* + \Omega_3^2 G_1 G_2 G_3 G_4^*.
$$

(C4)

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