SUPPLEMENTARY INFORMATION to Adiabatons in a double tripod coherent atom-light coupling scheme

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I. ADIABATONS IN Λ-TYPE ATOMIC SYSTEM

A. Equations of motion for atoms and fields

We consider a Λ-type atomic system, shown in Fig. 1, involving two metastable ground states $|0\rangle$ and $|1\rangle$, as well as an excited state $|e\rangle$. Laser fields with the Rabi frequencies Ω_0 and Ω_1 induce resonant transitions $|0\rangle \rightarrow |e\rangle$ and $|1\rangle \rightarrow |e\rangle$, respectively. Applying the rotating wave approximation (RWA), the atomic Hamiltonian in the rotating frame with respect to the atomic levels reads

$$H_\Lambda = -\frac{1}{2}\left(\Omega_0|e\rangle\langle 0| + \Omega_1|e\rangle\langle 1| + \text{H.c.}\right) + \delta|1\rangle\langle 1| - \frac{i}{2}\Gamma|e\rangle\langle e|, \tag{1}$$

where δ is a two-photon detuning. The losses in the Hamiltonian (1) are taken into account in an effective way by introducing a rate Γ of the excited state decay. In order to simplify the mathematical description of the system while keeping the relevant physical details, we characterize the state of an atom using a state vector $|\Psi\rangle = \psi_0|0\rangle + \psi_1|1\rangle + \psi_e|e\rangle$, as in Ref. [1, 2], instead of more complete description employed in Refs. [3, 4] that involves a density matrix.

The time-dependent Schrödinger equation $i\hbar\partial_t|\Psi\rangle = H_\Lambda|\Psi\rangle$ for the atomic state-vector $|\Psi\rangle$ yields the following

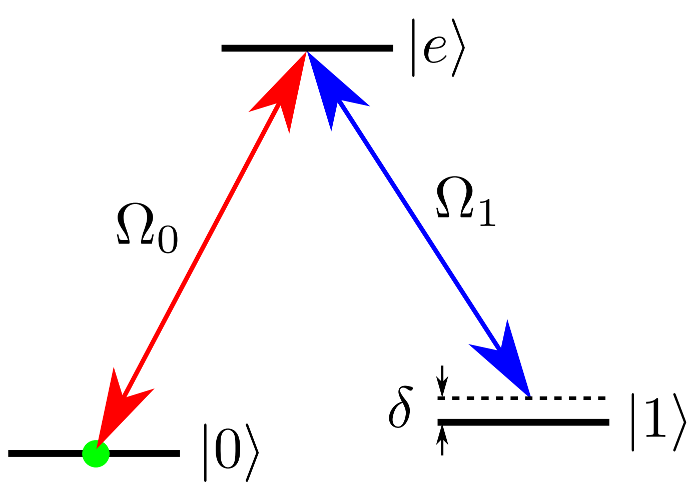

FIG. 1. Three level Λ-type atomic system. Two laser beams with the Rabi frequencies Ω_0 and Ω_1 act on atoms characterized by two hyperfine ground levels $|0\rangle$ and $|1\rangle$ as well as an excited level $|e\rangle$. Parameter δ denotes two-photon detuning from resonance. Atoms are initially in the ground level $|0\rangle$ as marked by green circle.

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equations for the atomic probability amplitudes ψ_0, ψ_1 and ψ_e:

$$i\partial_t\psi_0 = -\frac{1}{2}\Omega_0^*\psi_e\,, \tag{2}$$

$$i\partial_t\psi_1 = \delta\psi_1 - \frac{1}{2}\Omega_1^*\psi_e\,, \tag{3}$$

$$i\partial_t\psi_e = -\frac{i}{2}\Gamma\psi_e - \frac{1}{2}\Omega_0\psi_0 - \frac{1}{2}\Omega_1\psi_1. \tag{4}$$

On the other hand, the Rabi frequencies of the laser fields obey the propagation equations

$$\partial_t\Omega_l + c\partial_z\Omega_l = \frac{i}{2}g\psi_e\psi_l^*\,, \qquad l = 0,1 \tag{5}$$

where the parameter g characterizes the strength of coupling of the light fields with the atoms. It is related to the optical depth α as $g = c\Gamma\alpha/L$, where L is the length of the medium. For simplicity here we have assumed that the coupling strength g is the same for both laser fields.

B. Coupled and uncoupled states

The Hamiltonian (1) can be rewritten as [3]:

$$H_\Lambda = -\frac{\Omega}{2}\left(|e\rangle\langle\mathrm{C}| + |\mathrm{C}\rangle\langle e|\right) + \delta|1\rangle\langle 1| - \frac{i}{2}\Gamma|e\rangle\langle e|\,, \tag{6}$$

where

$$|\mathrm{C}\rangle = \frac{1}{\Omega}(\Omega_0^*|0\rangle + \Omega_1^*|1\rangle), \tag{7}$$

is a coupled state with

$$\Omega = \sqrt{|\Omega_0|^2 + |\Omega_1|^2}, \tag{8}$$

being a total Rabi frequency. Additionally we introduce an uncoupled state

$$|\mathrm{U}\rangle = \frac{1}{\Omega}(\Omega_1|0\rangle - \Omega_0|1\rangle), \tag{9}$$

which is orthogonal to the coupled state. The probability amplitudes to find an atom in the coupled and uncoupled states read

$$\psi_\mathrm{C} = \langle\mathrm{C}|\Psi\rangle = \frac{1}{\Omega}(\Omega_0\psi_0 + \Omega_1\psi_1)\,, \tag{10}$$

$$\psi_\mathrm{U} = \langle\mathrm{U}|\Psi\rangle = \frac{1}{\Omega}(\Omega_1^*\psi_0 - \Omega_0^*\psi_1)\,. \tag{11}$$

In terms of coupled and uncoupled states, the equations (2)–(4) for the atomic amplitudes take the form

$$i\partial_t\psi_\mathrm{U} = \Delta\psi_\mathrm{U} + \Omega_-^*\psi_\mathrm{C}\,, \tag{12}$$

$$i\partial_t\psi_\mathrm{C} = -\Delta\psi_\mathrm{C} + \Omega_-\psi_\mathrm{U} - \frac{1}{2}\Omega\psi_e\,, \tag{13}$$

$$i\partial_t\psi_e = -\frac{i}{2}\Gamma\psi_e - \frac{1}{2}\Omega\psi_\mathrm{C}\,, \tag{14}$$

with [3]

$$\Delta = i\frac{\Omega_0}{\Omega}\partial_t\frac{\Omega_0^*}{\Omega} + i\frac{\Omega_1}{\Omega}\partial_t\frac{\Omega_1^*}{\Omega} + \delta\frac{|\Omega_0|^2}{\Omega^2}\,, \tag{15}$$

$$\Omega_- = i\frac{\Omega_1}{\Omega}\partial_t\frac{\Omega_0}{\Omega} - i\frac{\Omega_0}{\Omega}\partial_t\frac{\Omega_1}{\Omega} - \delta\frac{\Omega_0}{\Omega}\frac{\Omega_1}{\Omega}\,. \tag{16}$$

Here Ω_- describes non-adiabatic losses and 2Δ represents the separation in energies between the uncoupled and coupled states.

C. Adiabatic approximation

Let us consider a situation where the total Rabi frequency Ω is sufficiently large so that the conditions presented below by Eqs. (20) and (24) hold. In this case the adiabatic approximation can be applied. To prepare for the derivation of approximate equations we express ψ_e from Eq. (13):

$$\psi_e = 2\frac{\Omega_-}{\Omega}\psi_{\rm U} - \frac{2}{\Omega}(i\partial_t + \Delta)\psi_{\rm C}\,. \tag{17}$$

On the other hand, Eq. (14) relates $\psi_{\rm C}$ to ψ_e as:

$$\psi_{\rm C} = -\frac{2i}{\Omega}\left(\partial_t + \frac{\Gamma}{2}\right)\psi_e\,. \tag{18}$$

Since the excited state decay rate Γ is considered to be large compared to the rate of change of the fields, we neglect the temporal derivative ∂_t in the above equation. Substituting Eq. (17) into Eq. (18) one gets

$$\psi_{\rm C} = -2i\frac{\Gamma}{\Omega^2}(\Omega_-\psi_{\rm U} - (i\partial_t + \Delta)\psi_{\rm C})\,. \tag{19}$$

We solve this equation iteratively with respect to $\psi_{\rm C}$, assuming that Ω is large compared to the rate of non-adiabatic transitions

$$\Omega \gg |\Omega_-|\,. \tag{20}$$

In the zeroth-order of the adiabatic approximation the coupled state is not populated, $\psi_{\rm C} \approx 0$, so Eq. (10) yields

$$\psi_0 \approx \frac{\Omega_1}{\Omega}\,, \quad \psi_1 \approx -\frac{\Omega_0}{\Omega}\,. \tag{21}$$

Non-zero ψ_C appears in the first-order approximation. Putting $\psi_{\rm C} \approx 0$ on the right hand side (r.h.s.) of Eq. (19), one arrives at the first order result for the amplitude of the coupled state

$$\psi_{\rm C} \approx -2i\frac{\Gamma}{\Omega}\frac{\Omega_-}{\Omega}\psi_{\rm U}\,. \tag{22}$$

Inserting this expression back in the r.h.s. of Eq. (19) we obtain the expression containing the second-order correction

$$\psi_{\rm C} = -2i\frac{\Gamma}{\Omega}\frac{\Omega_-}{\Omega}\psi_{\rm U} + 4\frac{\Gamma^2}{\Omega^2}(i\partial_t + \Delta)\frac{\Omega_-}{\Omega^2}\psi_{\rm U}\,. \tag{23}$$

The second order term should be much smaller than the first order one in the r.h.s. of the above equation. Since the atomic population is concentrated in the uncoupled state, $|\psi_{\rm U}| \approx 1$ and $|\psi_{\rm C}| \ll 1$, one arrives at the following condition

$$\frac{\Gamma|\Delta|}{\Omega^2} \ll 1\,, \qquad \frac{\Gamma|\Omega_-|}{\Omega^2} \ll 1\,. \tag{24}$$

Now let us present the adiabatic expansion of the excited state amplitude ψ_3. The zeroth-order approximation of Eq. (17) is $\psi_e \approx 0$. In the first order, taking into account Eq. (22) and the condition (24), we have

$$\psi_e \approx 2\frac{\Omega_-}{\Omega}\psi_{\rm U}\,. \tag{25}$$

Since the excited state should be weakly populated, the condition (20) is to be imposed.

Finally, inserting the first-order adiabatic result Eq. (22) relating $\psi_{\rm C}$ to $\psi_{\rm U}$ into Eq. (12) we obtain the equation for the amplitude of the uncoupled state

$$i\partial_t\psi_{\rm U} = \Delta\psi_{\rm U} - 2i\Gamma\frac{|\Omega_-|^2}{\Omega^2}\psi_{\rm U}\,. \tag{26}$$

The last term on the r.h.s. represents losses due to non-adiabatic corrections. Similarly, inserting Eqs. (25) and (22) into Eq. (5) we get the equations for the amplitudes of the radiation fields Ω_0 and Ω_1:

$$\partial_t\Omega_0 + c\partial_z\Omega_0 = g\left(i\frac{\Omega_-}{\Omega}\frac{\Omega_1^*}{\Omega} - 2\frac{\Gamma}{\Omega}\frac{|\Omega_-|^2}{\Omega^2}\frac{\Omega_0}{\Omega}\right)|\psi_{\rm U}|^2\,, \tag{27}$$

$$\partial_t\Omega_1 + c\partial_z\Omega_1 = g\left(-i\frac{\Omega_-}{\Omega}\frac{\Omega_0^*}{\Omega} - 2\frac{\Gamma}{\Omega}\frac{|\Omega_-|^2}{\Omega^2}\frac{\Omega_1}{\Omega}\right)|\psi_{\rm U}|^2. \tag{28}$$

Equations (26), (27) and (28) describe adiabatic propagation of the fields. The second terms on the r.h.s. of Eqs. (27) and (28) describe non-adiabatic losses.

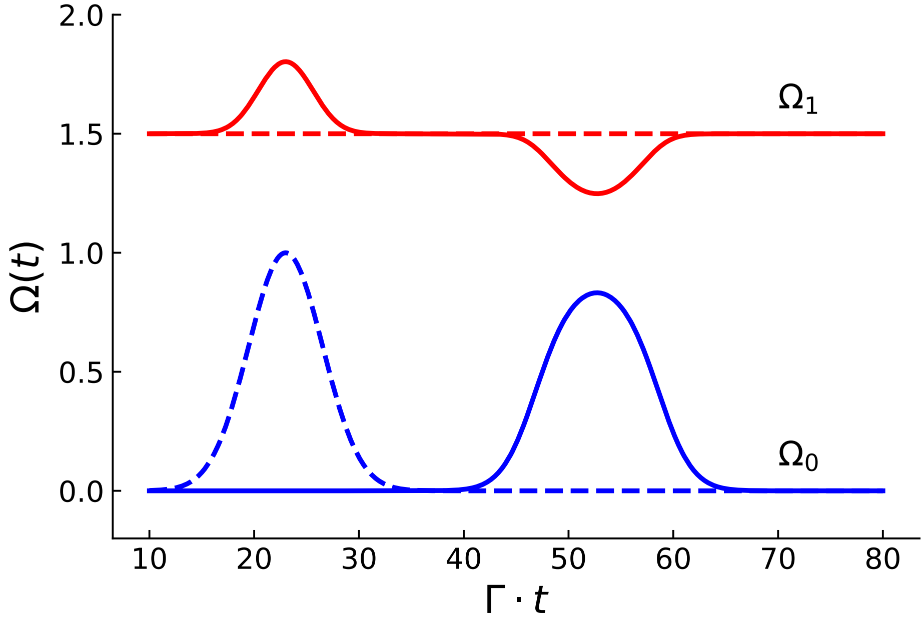

FIG. 2. Temporal dependence of pulse amplitudes in Λ system. Dashed lines correspond to pulse envelopes Ω_0 and Ω_1 at the input ($z = 0$), solid lines show pulse envelopes at propagation distance $z = 70L_{\rm abs}$. Here $L_{\rm abs} = L/\alpha$ is resonant absorption length. Amplitude shown on the vertical scale is measured in Γ.

D. Short duration of propagation

Let us now consider the case when the duration of the propagation $\tau_{\rm prop}$ is much smaller than the life time of the adiabatons: $\Gamma\frac{|\Omega_-|^2}{\Omega^2}\tau_{\rm prop} \ll 1$. The propagation duration is of the order of L/v_g, where $v_g = c\Omega^2/g$ is the group velocity. Expressing the atom-light coupling strength via the optical density α, $g = c\Gamma\alpha/L$, we obtain the condition

$$\left(\frac{\Gamma|\Omega_-|}{\Omega^2}\right)^2 \alpha \ll 1, \tag{29}$$

which should be satisfied. In this situation one can neglect the decay terms in Eqs. (26), (27) and (28). Furthermore, since $|\psi_{\rm U}| \approx 1$, Eqs. (27), (28) reduce to

$$\partial_t\Omega_0 + c\partial_z\Omega_0 = ig\frac{\Omega_-}{\Omega}\frac{\Omega_1^*}{\Omega}\,, \tag{30}$$

$$\partial_t\Omega_1 + c\partial_z\Omega_1 = -ig\frac{\Omega_-}{\Omega}\frac{\Omega_0^*}{\Omega}\,. \tag{31}$$

Combining Eqs. (30) and (31), the total Rabi frequency Ω obeys the equation

$$\partial_t\Omega + c\partial_z\Omega = 0\,. \tag{32}$$

On the other hand, Eqs. (30) and (31) provide the following equation for the ratio $\chi = \Omega_0/\Omega_1$:

$$\left(c^{-1} + \frac{g}{c\Omega^2}\right)\partial_t\chi + \partial_z\chi + i\delta\frac{g}{c\Omega^2}\chi = 0\,. \tag{33}$$

This equation has a similar form to the equation for the propagation of a weak probe field affected by a stronger control field when $|\Omega_0/\Omega_1| \ll 1$. Yet, in the present situation, such a condition is not imposed.

E. Solution

Equation (32) has the following solution satisfying the boundary condition at $z = 0$:

$$\Omega(z,t) \equiv \Omega(\tau) = \sqrt{|\Omega_0(0,\tau)|^2 + |\Omega_1(0,\tau)|^2}\,, \tag{34}$$

where $\tau = t - z/c$. To obtain a solution of Eq. (33) we change the variables t and z to $\tau = t - z/c$ and z, respectively. Then

$$\frac{g}{c\Omega^2(\tau)}\partial_\tau\chi + \partial_z\chi + i\delta\frac{g}{c\Omega^2(\tau)}\chi = 0\,. \tag{35}$$

Further we change the time variable τ to the stretched time

$$\zeta(\tau) = \frac{c}{g}\int_{-\infty}^{\tau}\Omega^2(\tau')d\tau'\,, \tag{36}$$

yielding

$$\partial_\zeta\chi + \partial_z\chi + i\delta\frac{g}{c\Omega^2(\zeta)}\chi = 0\,. \tag{37}$$

A particular solution of this equation is

$$\chi(z,\zeta) = \exp\left(ik(z-\zeta) - i\delta\frac{g}{c}\int_{-\infty}^{\zeta}\frac{d\zeta'}{\Omega^2(\zeta')}\right)\,. \tag{38}$$

When $\delta = 0$, a solution of Eq. (37) is an arbitrary function $f(\zeta - z)$, fixed by the boundary condition at $z = 0$:

$$f(\zeta(t)) = \frac{\Omega_0(0,t)}{\Omega_1(0,t)}, \tag{39}$$

or

$$f(z) = \frac{\Omega_0(0,\zeta^{-1}(z))}{\Omega_1(0,\zeta^{-1}(z))}\,, \tag{40}$$

where ζ^{-1} is a function inverse to the function ζ. It follows from Eq. (36) that if $\Omega(\tau)$ becomes constant after a certain time, the stretched time ζ becomes a linear function of τ. After that time the obtained solution $f(\zeta - z)$ describes a shape-preserving propagation [1].

We performed numerical investigation of the above analytical study by modeling field equations with optical fields of particular temporal shape. Here, the first optical field is described as a Gaussian pulse at the input: $\Omega_0(0,t) = A\exp[-(t-t_0)^2/\tau_0^2]$, where $\tau_0 = 5\Gamma^{-1}$, $t_0 = 23\Gamma^{-1}$ and $A = \Gamma$. The second one is simply a constant field: $\Omega_1(0,t) = 1.5\Gamma$. Results of the numerical solution of Eqs. (30) and (31) are depicted in Fig. 2 which shows the adiabaton propagation regime for the optical fields after some propagation time. Specifically, a shape-preserving combination of fields $\Omega_0(z,t)$ and $\Omega_1(z,t)$ propagating with the group velocity v_g can be seen on the right part of Fig. 2. The upward pulse from $\Omega_1(z,t)$ field seen in the left upper part of the figure is propagating with the speed of light.

[1] R. Grobe, F. T. Hioe, and J. H. Eberly, Formation of shape-preserving pulses in a nonlinear adiabatically integrable system, Phys. Rev. Lett. **73**, 3183 (1994).

[2] J. H. Eberly, Transmission of dressed fields in three-level media, Quantum and Semiclassical Optics **7**, 373 (1995).

[3] M. Fleischhauer and A. S. Manka, Propagation of laser pulses and coherent population transfer in dissipative three-level systems: An adiabatic dressed-state picture, Phys. Rev. A **54**, 794 (1996).

[4] Y.-L. Chuang, R.-K. Lee, and I. A. Yu, Optical-density-enhanced squeezed-light generation without optical cavities, Phys. Rev. A **96**, 053818 (2017).