Vilnius University
Faculty of Physics
Laboratory of Atomic and Nuclear Physics

Experiment No. 14

INVESTIGATION OF RADIOACTIVE EQUILIBRIUM AND MEASURING THE HALF-LIFE OF ^{137m}Ba

by Andrius Poškus
(e-mail: andrius.poskus@ff.vu.lt)

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Contents

The aim of the experiment

Investigate the phenomenon of radioactive equilibrium, test the law of radioactive decay.

1. Tasks

1. Using a ^{137}Cs / ^{137}Ba isotope generator, obtain the eluate containing atoms of the nuclide ^{137m}Ba.
2. Using a Geiger-Müller counter, measure the time dependence of the eluate activity and the time dependence of the isotope generator activity after creating the eluate.
3. Plot the measured dependences.
4. Calculate the half-life and average lifetime of the second excited state of the ^{137}Ba nucleus, and their errors.
5. Compare the measurement results with theoretical predictions.

2. Control questions

1. Explain the law of radioactive decay.
2. Explain the origin of nuclear gamma radiation and the concept of nuclear energy levels.
3. Explain the concept of radioactive equilibrium.
4. Explain the operation of an isotope generator and its applications in medical diagnostics.

3. Theoretical background

3.1. *The law of radioactive decay*

It is known that the number of atoms of a radioactive nuclear species (***nuclide***) decreases exponentially with time:

$$N(t) = N_0 \mathrm{e}^{-\lambda t}, \tag{3.1}$$

where $N(t)$ is the number of atoms (nuclei) at time t, N_0 is the initial number of nuclei (at time $t = 0$), and λ is the ***decay constant***. The decay constant determines the rate at which the number of radioactive nuclei decreases: the larger the decay constant, the faster the radioactive decay. Another way to specify the rate of decrease of the number of radioactive nuclei is by specifying the ***half-life***, which is related to the decay constant as follows:

$$T_{1/2} = \frac{\ln 2}{\lambda}. \tag{3.2}$$

The half-life is the time needed for the number of radioactive nuclei to decrease by half. Yet another quantity that can be used to describe the decay rate is the average lifetime of the nuclide. It is equal to the average time that has to pass until a given nucleus undergoes the decay. The average lifetime (τ) is the inverse of the decay constant:

$$\tau = \frac{1}{\lambda}. \tag{3.3}$$

Equation (3.1) is the formulation of the so-called "law of radioactive decay". An equivalent way to formulate it is to take time derivatives of both sides of the equation:

$$\frac{\mathrm{d}N}{\mathrm{d}t} = -\lambda N. \tag{3.4}$$

This is the first-order differential equation whose solution corresponding to initial condition $N(0) = N_0$ is (3.1). The left side of this equation is opposite to the rate of decay (the number of atoms decaying per unit time). The rate of decay is the so-called ***activity*** of the sample. Since N decreases with time, from Eq. (3.4) it follows that activity also decreases exponentially with time:

$$\Phi(t) \equiv -\frac{\mathrm{d}N}{\mathrm{d}t} = \lambda N(t) = \lambda N_0 \mathrm{e}^{-\lambda t}. \tag{3.5}$$

The logarithm of this function is a straight line (see Fig. 1). Its intercept gives the logarithm of the initial activity $\ln(\lambda N_0)$, and its slope is the opposite of the decay constant $(-\lambda)$. Thus, analysis of the decay curve is a simple method of determining the decay constant λ and the half-life $T_{1/2}$.

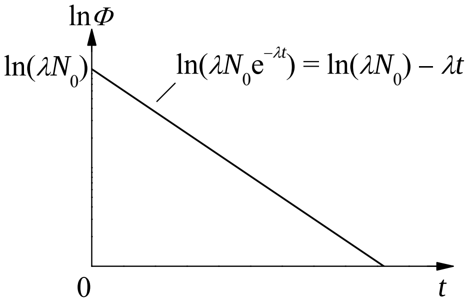

Fig. 1. Time dependence of the natural logarithm of activity

3.2. *Nuclear gamma radiation*

Gamma rays or ***gamma radiation*** (denoted as γ) are electromagnetic radiation of high frequency (very short wavelength). As a rule of thumb, the term "gamma radiation" is usually applied when the wavelength is much less than the distance between atoms of solid matter (i. e. much less than 10^{-8} cm). From the point of view of quantum mechanics, electromagnetic radiation can be interpreted as a flux of elementary particles called the ***photons***. The photons of gamma radiation are frequently called "γ quanta" (here, "quanta" is the plural form of the noun "quantum"). The wavelength range of gamma radiation partially overlaps with the wavelength range of X-ray radiation (e. g., see Table 1 in Section 3 of descriptions of Experiments No. 15 and No. 16). This is because in practice the distinction between those two types of electromagnetic radiation is based mainly on their physical origin and not on their wavelength. If the electromagnetic radiation is caused by quantum transitions between nuclear energy levels or by particle annihilation reactions, then it is called “gamma radiation”. If the electromagnetic radiation is caused by quantum transitions between atomic energy levels corresponding to innermost electron shells of an atom or by deceleration of fast electrons in matter, then it is called “X-ray radiation” (a more detailed information about X-ray radiation is in Section 4 of the description of Experiment No. 15).

The physical mechanism of nuclear gamma radiation is the following. A nucleus that is formed as a product of radioactive decay or of a nuclear reaction may be in an excited energy state (this means that the nucleus has excess internal energy). In such a case it eventually undergoes a quantum transition to the main (lowest-energy) state. During this transition, the excess energy is usually emitted in the form of a photon (γ quantum). This transition only causes a decrease of internal energy of the nucleus; it is not accompanied by a change of nuclear composition (i. e. of the numbers of protons and neutrons inside the nucleus). The energies of γ quanta emitter by a nucleus form a discrete sequence (i. e. there are several well-defined energy values instead of a continuous energy spectrum). This discrete nature of nuclear energy spectrum is a quantum phenomenon caused by localization of the constituent particles of a system in a small region of space (the nuclear constituent particles are protons and neutrons). In energy diagrams, those discrete energy values are shown as horizontal lines (e. g. see Fig. 2) and are called ***energy levels***. The lowest energy level is called the ***ground energy level***. The corresponding state of the nucleus is called the “ground state” or “principal state”. All other levels are called ***excited levels*** and correspond to the ***excited states*** of the nucleus. The typical interval between energy levels of a quantum system is mainly determined by spatial dimensions of the systems: the smaller the system, the larger typical differences of energy levels. That is why the typical intervals between nuclear energy levels (100 to 1000 keV) are 10^2 to 10^6 times larger than the typical intervals between atomic energy levels, which vary from a few eV (for valence electrons) to a few keV (for inner-shell electrons).

Thus, ***nuclear γ radiation*** is electromagnetic radiation caused by quantum transitions of excited nuclei to lower energy levels (in Fig. 2, one such quantum transition is shown by a vertical arrow). Sometimes the transition of a nucleus to the ground level proceeds in stages, as a series of transitions to lower excited levels.

The opposite transitions (from lower to higher energy levels) are also possible. However, since those transitions involve an increase of the internal energy of the nucleus, they can not happen spontaneously (unlike the transitions to lower levels, which do not require any external influence). They can only happen as a result of an external event that supplies energy to the nucleus, such as a collision with an energetic subatomic particle. A process that is the inverse of the process leading to nuclear γ radiation is absorption of a photon.

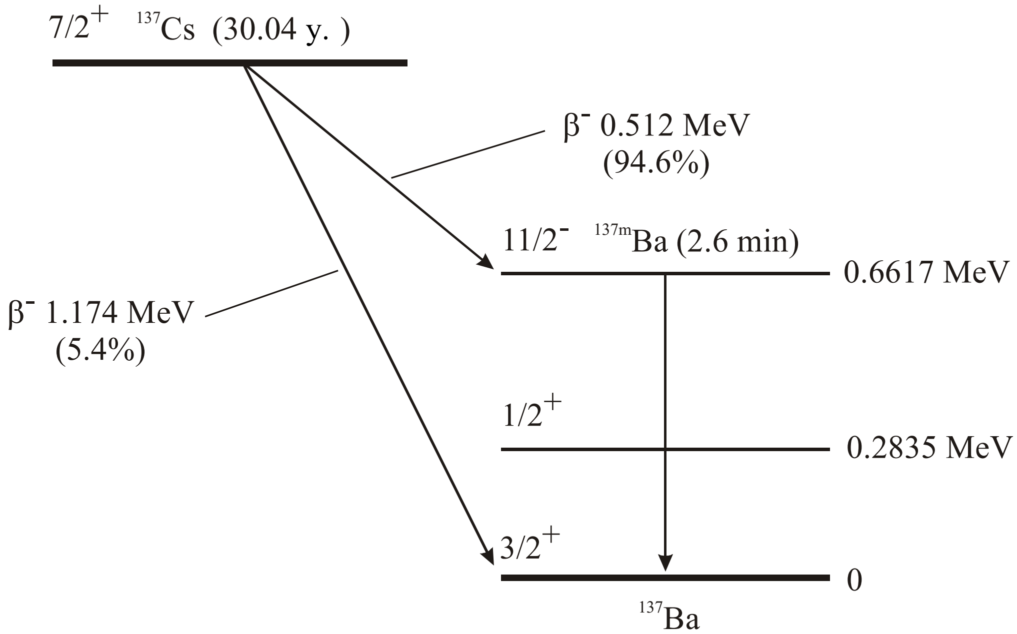

Fig. 2. The diagram of the radioactive decay ^{137}Cs → ^{137}Ba (cesium-137 → barium-137). The diagram shows the decay half-lives, the maximum energies of the β particles (i. e. of the electrons emitted from the nucleus), the probabilities of the two β-decay channels, the lowest energy levels of the ^{137}Ba nucleus and the most probable quantum transition between ^{137}Ba energy levels.

The energy of the photon that is emitted when the nucleus drops from the higher energy level E_1 to a lower energy level E_2 (or absorbed when the nucleus undergoes the opposite transition) is equal to the difference of the two energy levels:

$$h\nu = E_1 - E_2, \tag{3.6}$$

where ν is the frequency of the radiation, and h is the ***Planck constant***: $h = 6.626 \cdot 10^{-34}$ J·s. Thus, in order for a nucleus to be able to absorb a photon, the photon's energy must be equal to the difference of two energy levels.

The average time until the spontaneous transition of an excited nucleus to the ground level is called the "lifetime" of the excited state of the nucleus. If is usually of the order of $(10^{-14} - 10^{-6})$ s. However, some nuclei stay in an excited state for a much longer period of time. For example, the average lifetime of a ^{137}Ba nucleus in the second excited level (which is formed as a result of the β decay of ^{137}Cs) is 221 s. Such unusually long-lived excited states are called the ***metastable states*** and denoted by a letter "m" after the nuclear mass number (for example, "$^{137\mathrm{m}}$Ba").

Emission of a photon is usually the most probable mechanism by which an excited nucleus loses the excess energy, but it is not the only possible one. Two other mechanisms of energy loss are possible, too: internal conversion and internal pair creation. ***Internal conversion*** is a process whereby an excited nucleus transfers the excess energy directly to an electron of the atom to which is belongs. Since the energy transferred to the electron is much larger than the binding energy of an atomic electron, the electron is ejected from the atom with a kinetic energy that is approximately equal to the nuclear excitation energy. In the case of the previously-mentioned second excited level of ^{137}Ba, there is about 10 % probability of energy loss by means of internal conversion. ***Internal pair creation*** is process whereby the excitation energy of a nucleus is transformed into total relativistic energy of two particles that did not exist prior the nuclear transition to the ground state. Those two particles are the electron and the positron (the positron is the electron's antiparticle). From the definition of the total relativistic energy

$$E = mc^2, \tag{3.7}$$

where m is the relativistic mass of the particles, it follows that internal pair creation only becomes possible when the excitation energy is larger than $2m_0c^2 = 1.02$ MeV, where m_0 is the rest mass of the electron.

Gamma radiation is also emitted during some annihilation reactions. ***Annihilation*** is process that occurs when a subatomic particle collides with its respective antiparticle. As a result of this process, both initial particles disappear (hence the term "annihilation", which is defined as "total destruction" or "complete obliteration" of an object). Since energy and momentum must be conserved, the particles are not actually made into nothing, but rather into new particles. Some or all of the new particles may be photons. Since their energies are usually of the order of hundreds of keV or more, those photons are also classified as γ radiation. The annihilation reaction that is easiest to achieve in practice is the annihilation of an electron and a positron.

3.3. *Radioactive equilibrium in a system of two nuclides*

Let us discuss a mixture of two radioactive nuclides, where one nuclide (the "primary nuclide") decays into another radioactive nuclide (the "secondary nuclide"). An example of such a system is a mixture of ^{137}Cs and $^{137\mathrm{m}}$Ba (see Fig. 2). In this case, the primary nuclide is ^{137}Cs, which decays into $^{137\mathrm{m}}$Ba (the secondary nuclide), which then decays into the stable ground state of ^{137}Ba. This decay chain can be written as follows:

$$N_1 \xrightarrow{\lambda_1} N_2 \xrightarrow{\lambda_2} N_3, \tag{3.8}$$

where N_1 is the number of the nuclei of the primary nuclide (in this case, ^{137}Cs), N_2 is the number of the nuclei of the secondary nuclide (in this case, $^{137\mathrm{m}}$Ba), and N_3 is the number of the nuclei of the secondary nuclide's decay product (in this case, ground state of ^{137}Ba). The change of the numbers N_1 and N_2 with time is described by a system of two differential equations:

$$\frac{\mathrm{d}N_1}{\mathrm{d}t} = -\lambda_1 N_1 \quad \text{and} \quad \frac{\mathrm{d}N_2}{\mathrm{d}t} = \lambda_1 N_1 - \lambda_2 N_2. \tag{3.9}$$

The first equation is simply the law of radioactive decay for the primary nuclide (see Eq. 3.4). The second equation includes an additional positive term ($\lambda_1 N_1$) on the right-hand side. This term reflects the fact that, in addition to the radioactive decay of the secondary nuclide (this process decreases N_2), there is another process that influences N_2: production of the secondary nuclide due to radioactive decay of the primary nuclide (this process increases N_2, therefore the corresponding term on the right-hand side of the second equation is positive). The solution of the system of differential equations (3.9) depends on initial conditions, i. e. on initial values of N_1 and N_2. For example, if values of N_1 and N_2 at the initial moment of time ($t = 0$) are $N_1(0)$ and 0, respectively, then the solution is

$$N_1(t) = N_1(0)\mathrm{e}^{-\lambda_1 t}, \tag{3.10a}$$

$$N_2(t) = N_1(0)\frac{\lambda_1}{\lambda_2 - \lambda_1}\left(\mathrm{e}^{-\lambda_1 t} - \mathrm{e}^{-\lambda_2 t}\right). \tag{3.10b}$$

If ($\lambda_2 > \lambda_1$), then the second exponential function in parentheses decreases faster than the first one. After a time that is much larger than $1 / (\lambda_2 - \lambda_1)$, the second term becomes much smaller than the first one, therefore the second term may be ignored. Then $N_2(t)$ becomes proportional to a single exponential function $\mathrm{e}^{-\lambda_1 t}$, as is $N_1(t)$. Thus, the ratio of quantities of both nuclides becomes constant:

$$\frac{N_2}{N_1} = \frac{\lambda_1}{\lambda_2 - \lambda_1}. \tag{3.11}$$

Such a state of a mixture of radioactive nuclides belonging to a single decay chain, when the ratios of nuclide quantities remain constant, is called ***radioactive equilibrium***. If the decay constant of the secondary nuclide (λ_2) is much larger than the decay constant of the primary nuclide (λ_1), then Eq. 3.11 can be approximately re-written as

$$\lambda_2 N_2 \approx \lambda_1 N_1. \tag{3.12}$$

I. e., the decay rate of the secondary nuclide ($\lambda_2 N_2$) is practically equal to its production rate $\lambda_1 N_1$, which, in turn, is equal to the decay rate of the primary nuclide (i. e. to activity of the primary nuclide). This condition is satisfied in the system of ^{137}Cs/$^{137\text{m}}$Ba, because the $^{137\text{m}}$Ba decay half-life (153 s) is about $6 \cdot 10^6$ times smaller than decay half-life of ^{137}Cs (30 years), i. e. the $^{137\text{m}}$Ba decay constant is about $6 \cdot 10^6$ times larger than decay constant of ^{137}Cs (see the relation between decay constant and decay half-life, Eq. 3.2).

If the radioactive equilibrium is artificially disturbed by removing a part of the secondary nuclide from the system, then the amount of the secondary nuclide begins to grow until the equilibrium is restored. This increase of N_2 can be derived from Eq. 3.10b, taking into account that $\lambda_2 >> \lambda_1$:

$$N_2(t) = N_1(0)\frac{\lambda_1}{\lambda_2 - \lambda_1}\left(\mathrm{e}^{-\lambda_1 t} - \mathrm{e}^{-\lambda_2 t}\right) = N_1(0)\mathrm{e}^{-\lambda_1 t}\frac{\lambda_1}{\lambda_2 - \lambda_1}\left(1 - \mathrm{e}^{-(\lambda_2 - \lambda_1)t}\right) \approx N_1\frac{\lambda_1}{\lambda_2}\left(1 - \mathrm{e}^{-\lambda_2 t}\right). \tag{3.13}$$

Since in practice it is much easier to count the number of particles emitted by radioactive nuclei per unit time than to measure the number of those nuclei directly, is it more useful to formulate the latter result in terms of *activities* of the two nuclides. This transformation is done using the fact that current activity of a particular nuclide is equal to the current number of its nuclei multiplied by its decay constant (see Eq. 3.5). Thus, a removal of $^{137\text{m}}$Ba nuclei from the mixture ^{137}Cs/$^{137\text{m}}$Ba causes a decrease of the number γ photons emitted per second (because all photons are emitted due to decay $^{137\text{m}}\text{Ba} \rightarrow {}^{137}\text{Ba}$), but then it begins to grow according to the equation

$$A_2(t) \equiv \lambda_2 N_2(t) = \lambda_1 N_1\left(1 - \mathrm{e}^{-\lambda_2 t}\right) \equiv A_1\left(1 - \mathrm{e}^{-\lambda_2 t}\right), \tag{3.14}$$

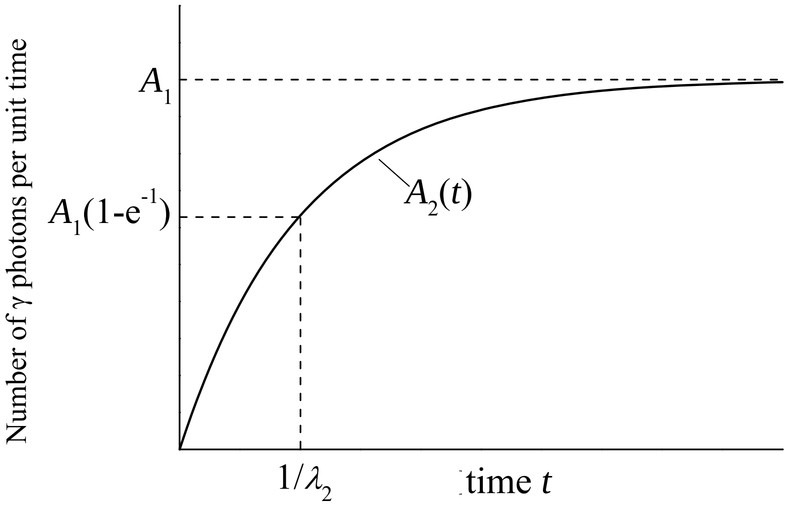

Fig. 3. Time dependence of γ radiation intensity after removing a part of the secondary nuclide from the mixture consisting of a long-lived primary nuclide and a short-lived secondary nuclide (it is assumed that only the secondary nuclide emits γ photons). A_1 is activity of the primary nuclide, which is practically constant

where A_1 is activity of the primary nuclide ^{137}Cs (see Fig. 3). At equilibrium, the number of photons emitted per second is equal to the primary nuclide's activity A_1, which is practically constant. Therefore, A_1 should be measured either before removing $^{137\text{m}}$Ba from the mixture or after complete restoration of radioactive equilibrium. Having measured A_1, the decay constant of $^{137\text{m}}$Ba can be determined from the measured values of the difference $A_1 - A_2(t)$, because

$$A_1 - A_2(t) = A_1 \mathrm{e}^{-\lambda_2 t}. \tag{3.15}$$

4. Experimental setup and procedure

The equipment used for this experiment consists of the following devices:

1. A ^{137}Cs/$^{137\text{m}}$Ba isotope generator.
2. A Geiger-Müller counter (the dead time of that counter is **$\tau_d \approx 0.001$ s $\approx 1.7 \cdot 10^{-5}$ min**).
3. ***Isotrak*** ratemeter (its user's manual is given in Section 6).

A general view of the equipment is shown in Fig. 4.

Fig. 4. General view of the equipment

An ***isotope generator*** is a device for chemical separation of a short-lived secondary nuclide from a long-lived primary nuclide. The separation of the two nuclides is achieved by passing a special solution through the generator. In the case of a ^{137}Cs/$^{137\text{m}}$Ba isotope generator, the mentioned solution consists of 0.9 % NaCl and $4 \cdot 10^{-6}$ % HCl. In Fig. 4, the isotope generator is seen as small cylinder placed in front of a Geiger-Müller detector (also called a "Geiger-Müller tube"). The electrons ("beta particles") emitted during the decay of the primary nuclide (e. g. see Fig. 2) are much less penetrating than γ rays, therefore those electrons are completely absorbed in the plastic cover of the isotope generator. Only the γ photons produced by quantum transitions of the secondary nuclide escape from the isotope generator.

Inside the ^{137}Cs/$^{137\text{m}}$Ba isotope generator, a material containing ^{137}Cs is deposited onto a special organic polymer layer (called "the substrate"), which facilitates transfer of Ba ions into a solution containing a sufficiently high concentration of Na (sodium) ions. I. e., the Na ions take up the positions on the substrate previously occupied by Ba ions, whereas Ba ions enter the solution. Such a chemical reaction is called the ***ion exchange***. In this way, a majority of ^{137}Ba atoms can be removed from the ^{137}Cs/$^{137\text{m}}$Ba isotope generator with the solution. The same principle is used in ^{99}Mo/$^{99\text{m}}$Tc (molybdenum-99 / technetium-99m) generators for medical applications of radioactivity. In those generators, the primary nuclide is ^{99}Mo, which has a decay half-life of 66 hours, and the secondary nuclide is $^{99\text{m}}$Tc (a metastable excited state of ^{99}Tc), which has a decay half-life of 6 hours. Molibdenum-99 (as well as cesium-137) is a product of nuclear fission (which takes place, e. g., in nuclear reactors). The solution containing short-lived secondary nuclide obtained from such a generator is administered to a patient (for example, by adding it to a medication, which is then ingested by a patient), and then its γ radiation is recorded. In this way, the nuclide's distribution inside the patient's body can be determined, which, in turn, may provide information about various medical conditions (such as tumors). One of the reasons why the isotope generators are useful in medicine is that the long half-life of the primary nuclide makes transportation over long distances easy, whereas the short half-life of the secondary nuclide helps to reduce the risk to the patient caused by exposure to radiation (the activity of the secondary nuclide decays rapidly and therefore does not do any significant damage to the biological tissues).

The measurement procedure

1. Switch on and prepare the ratemeter with the Geiger-Müller tube. In this experiment, the automatic counting mode must be used, with the gate time of 60 s (see the ratemeter's user's manual in Section 6). Since the measurement results must be stored in the ratemeter's memory, which can contain no more than 50 counts, it is important to erase any data stored in the memory before starting the experiment (this is done by pressing the "Start/Stop" and "Reset" buttons simultaneously). The state of the ratemeter after preparing it for the measurements must be such that it is enough to press the "Start/Stop" button to start a sequence of counts.

Warning! Wear disposable gloves when manipulating the radioactive eluate. After the experiment, drop the gloves into a bin.

2. Extract the nuclide ^{137m}Ba is from the ^{137}Cs/^{137m}Ba generator. This must be done in the following way:
   a) attach a plastic tube to a syringe, then take approximately 2 ml of the eluting solution into the tube, then detach the tube from the syringe,
   b) unscrew the protective cap of the isotope generator (it is at the side of the generator where the label "Isotope generator" is) and screw the syringe with the solution in its place,
   c) take off the protective cap that is at the opposite side of the isotope generator (there is no screw thread there, so that it is enough to pull the cap) and place the isotope generator together with the syringe above an empty flask,
   d) push the plunger of the syringe until all solution passes through the generator and is collected in the flask (this step should take about 10 or 20 s),
   e) put both protective caps back on the isotope generator and place it in front of the Geiger-Müller tube as shown in Fig. 4,
   f) place the flask with the radioactive eluate at a distance of 2 m or more from the counter (so that it does not influence the measurement results),
   g) start the sequence of counts by pressing the button "Start/Stop" on the ratemeter.
3. Do 30 counts, each count 1 minute-long. The results must be written down in the form of a table.
4. Empty the flask with the eluate (which is no longer radioactive at this point) into a sink. Repeat Step 2, but now place the flask with the radioactive eluate in front of the detector, whereas the isotope generator must be placed at a distance of at least 2 m from the detector.
5. Do 10 counts, each count 1 minute-long. Again, write down the results in the form of a table.
6. Place the flask at a distance of at least 2 m from the detector. Do 10 counts of the background.
7. Empty the flask with the eluate into a sink, rinse the flask with water.
8. Show the measurement results (in table format) to the laboratory supervisor for signing.

5. Analysis of measurement results

1. Correct the counts taking into account the so-called "dead time" of the detector (it is the time that has to pass after detecting a particle until the next particle can be detected). I. e., if N' is the measured number of particles in 1 min and τ_d is the dead time (in minutes), then the true number of particles in 1 min is
$$N = \frac{N'}{1 - N'\tau_\mathrm{d}} .$$
2. Calculate the average background.
3. Subtract the average background from the corrected results corresponding to the radioactive eluate.
4. Plot the time dependence of the natural logarithm of the numbers obtained in previous step (it should be plotted as a scatter graph). Then calculate the decay constant λ of $^{137\mathrm{m}}$Ba and its error, using the method of linear fitting, and plot the fitting line in the same graph. Calculate the average lifetime and decay half-life of $^{137\mathrm{m}}$Ba and their errors (use Eqs. (3.2) and (3.3)). Linear fitting can be done using various computer programs, or it can be done "by hand", using a calculator (see Section 7). Errors (i.e., uncertainties) must be calculated using the following general expression of uncertainty of quantity f, which is a function of another quantity x:
$$\Delta f = \left|\frac{\mathrm{d}f}{\mathrm{d}x}\right| \Delta x ,$$
where Δx is uncertainty of x. In this case, "f" has the meaning of decay half-life $T_{1/2}$ or average lifetime τ, and "x" has the meaning of the decay constant λ.
5. Calculate the average value of the last 10 counts of the ^{137}Cs/$^{137\mathrm{m}}$Ba isotope generator (i. e. measurements No. 21 – 30). At this stage of the measurements, intensity of γ radiation of the isotope generator has been completely restored, therefore this average is equal to the initial γ radiation intensity of the generator (prior to nuclide separation).
6. Subtract each of the first ten counts of the ^{137}Cs/$^{137\mathrm{m}}$Ba isotope generator from the initial intensity obtained in previous step. Then plot and analyze the resulting ten numbers in the same way as in Step 4 of this section.
7. Compare the calculation results of Step 4 and Step 6 with each other and with the exact value of $^{137\mathrm{m}}$Ba half-life (153 s).
8. Discuss the general shape of both measured time dependences.

6. The Geiger-Müller counter user's manual

Below are four pages scanned from an Isotrak booklet that is included with the measurement equipment. The Geiger-Müller counter consists of a detector (the Geiger-Müller tube) and a ratemeter. The ratemeter has a built-in the adjustable power supply for the detector, an LCD display for showing the counts, and buttons for selection of various counting modes.

1. Front panel of G-M counter

Caution! Before connecting the tube to the G-M counter make sure that the tube voltage has been set correctly (see also section 2.2).

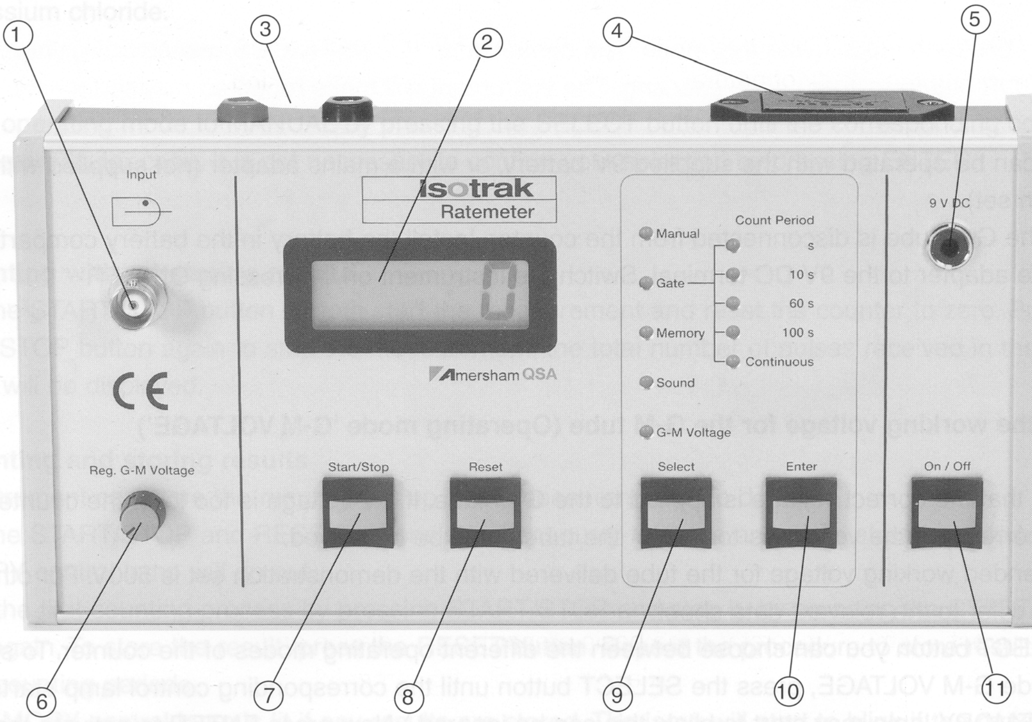

Figure 1: Front panel of isotrak Ratemeter

(1) INPUT terminal for the G-M tube
(2) Display
(3) TTL compatible interface
(4) Compartment for 9V battery
(5) 9V DC terminal for mains adapter
(6) G-M tube voltage control
(7) START/STOP button to start/stop the counting process
(8) RESET button to clear the display only.
When the RESET and START/STOP buttons are pressed simultaneously, the memory will also be cleared.
(9) SELECT button to choose operating modes and options
(10) ENTER button to enter and leave operating modes
(11) ON/OFF button

Control lamps: The selected operating mode is indicated by a lamp. The MEMORY lamp is lit when data have been stored, and starts blinking to indicate a memory overflow. During the measurement, the GATE lamp blinks rapidly to indicate that the counter is operating.

2. Operating modes of the G-M counter

This chapter describes how to operate the G-M counter and learn its operating modes by carrying out simple experiments. The experiments described in sections 2.4 to 2.6 are also suitable for introducing the subject of radioactivity to students.

2.1 Start-Up

The counter can be operated with the supplied 9V battery, or with a mains adapter (not supplied with the demonstration set).
Ensure that the G-M tube is disconnected from the counter. Install the battery in the battery compartment, or connect the adapter to the 9V DC terminal. Switch the instrument on by pressing ON/OFF.

2.2 Setting the working voltage for the G-M tube (Operating mode 'G-M VOLTAGE')

It is essential that the correct voltage is applied to the G-M tube. If the voltage is too low, the counter will not function correctly, if the voltage is too high, the tube could be destroyed.
The recommended working voltage for the tube delivered with the demonstration set is 500V. For other tube types, please refer to the relevant data sheet.
With the SELECT button you can choose between the different operating modes of the counter. To set the operating mode G-M VOLTAGE, press the SELECT button until the corresponding control lamp starts to blink. (The MEMORY lamp may start to blink; this can be ignored.) Now press ENTER to activate the currently selected operating mode. When the G-M VOLTAGE control lamp is lit, the tube voltage is indicated in the display. The only thing you have to do now is to set the voltage to the recommended voltage for your G-M tube by turning the REG. G-M VOLTAGE control. A small difference (a few volts) between the set voltage and the recommended voltage doesn't matter. If you want to leave the operating mode, press ENTER or simply switch off the instrument.

2.3 Audible indication of pulses (Operating mode 'SOUND')

Connect the lead from the G-M tube to the INPUT terminal. Leave the plastic cap over the fragile window of the G-M tube to prevent damage.
Press SELECT until the control lamp SOUND starts blinking and confirm by pressing ENTER. With SELECT you can now start or stop the audible indication of pulses (the display shows ON or OFF, respectively). With the G-M tube connected, you should now be able to hear the pulses caused by background radiation as short beeps. If you want to stop this operating mode, press ENTER or simply switch off the instrument.

2.4 Counting pulses (Operating mode 'MANUAL')

This section describes different methods to count the number of pulses measured in given time periods. For effective demonstrations, the pulse rate should be higher than the rate from background radiation. An increase in the pulse rate can be achieved by measuring the gamma radiation from the radioactive isotope K-40. All natural potassium compounds contain 0.01% K-40. Just insert the G-M tube into the supplied flask of potassium chloride.

Set the operating mode to MANUAL by pressing the SELECT button until the corresponding control lamp is lit. In this particular case, it is not necessary to confirm your selection by pressing ENTER.

a) Counting without storing results

Press the START/STOP button to both start the measurement and reset the counter to zero. Press the START/STOP button again to stop the measurement; the total number of pulses received in the time interval will be displayed.

b) Counting and storing results

The instrument can store the results from up to 50 measurements. To do this:
Press the START/STOP and RESET buttons simultaneously to erase any data already stored. The MEMORY control lamp will go out.
Initiate the first counting process by pressing START/STOP and stop it by pressing the START/STOP button again. To store the result, press the RESET button. Repeat the procedure to store results from further counting periods.
The MEMORY control lamp is lit if any results are stored. The lamp will start to blink if the maximum number of results (50) is exceeded (storage overflow).

2.5 Reading stored results (Operating mode 'MEMORY')

Press the SELECT button repeatedly until the MEMORY control lamp starts blinking (even if storage overflow has already been indicated!). The display will show the last value stored. Previous data points can be recalled by pressing ENTER. The stored data points can therefore be retrieved in reverse order; at the end of the list the display shows three dashes.
If required, the stored data can be called off a second time by repeating these steps.

2.6 Counting pulses for predefined periods (Operating mode 'GATE')

Press the SELECT button until the GATE control lamp starts blinking and press ENTER to confirm the selection. Press SELECT again to step through the available count times and press ENTER when the required time is indicated by a control lamp. The control lamp marked CONTINUOUS will start blinking. The instrument can now be either set to count for one time period, or set to run automatically.

a) One time period
Press SELECT and then START/STOP to start data collection; the instrument will count the number of pulses registered in the chosen count time. Press RESET to store the result in the memory.

b) Automatic counting (Operating mode 'CONTINUOUS')
In this mode, the instrument will store the results from repeated, consecutive, measurements.

Press the ENTER button to activate this mode. The control lamp (CONTINUOUS) will stop blinking. Press the START/STOP button to commence counting. The instrument will make repeated measurements, each for the selected time period. To stop the sequence, press the START/STOP button again.

At the end of each count period, the display pauses for about 5 seconds to show the number of pulses received, so that the result can be written down if needed. There is no delay between the counting periods, but pulses registered during the first 5 seconds of a count period are not shown; the display is instead used to show the result from the previous period.
There is an easier method to record the results, rather than writing them down between measurements: the results are also automatically stored in the memory. Up to 50 points can be stored if the memory has been cleared by pressing START/STOP and RESET simultaneously before starting the measurements. The results can then be retrieved following the method given in section 2.5.

7. Linear fitting

The aim of linear fitting is determination of the least squares estimates of the coefficients A and B of the linear equation

$$y = A + B \cdot x, \tag{7.1}$$

The essence of the method of least squares is the following. Let us assume that a data set consists of the argument values $x_1, x_2, \ldots, x_{n-1}, x_n$ and corresponding values of the function $y(x)$. A typical example is a set of measurement data. In such a case, n is the number of measurements. The measured function values will be denoted $y_1, y_2, \ldots, y_n$. The "theoretical" value of y at a given argument value x_k is a function of the unknown coefficients A and B (see (7.1)), hence we can write $y(x_k) = y(x_k; A,B)$ ($k = 1, 2, \ldots, n$). The problem of estimating the coefficients A and B is formulated as follows. The most likely values of A and B are the values that minimize the expression

$$F(A,B) \equiv \sum_{k=1}^{n} \left[y\left(x_k; A, B\right) - y_k \right]^2 . \tag{7.2}$$

Expression (7.2) is the sum of squared deviations of theoretical values from the measured ones (hence the term "least squares"). That sum is also called "the sum of squared errors" (SSE). This expression always has a minimum at certain values of A and B. However, even if the form of the theoretical function $y(x)$ correctly reflects the true relationship between the measured quantities y and x, those "optimal" values of A and B, which correspond to the minimum SSE, do not necessarily coincide with the true values of A and B (for example, because of measurement errors). The method of least squares only allows estimation of the most likely values of A and B.

Everything that was stated above about the method of least squares also applies to the case when the theoretical function is nonlinear. Regardless of the form of that function and of the number of unknown coefficients, a SSE expression of the type (7.2) must be minimized. However, when $y(x)$ is the linear function (7.1), this problem can be solved analytically (i.e., A and B can be expressed using elementary functions), but when $y(x)$ is nonlinear, this problem can only be solved numerically (applying an iterative procedure).

If $y(x)$ is the linear function (7.1), then the SSE expression (7.2) can be written as follows:

$$F(A,B) \equiv \sum_{k=1}^{n} (A + Bx_k - y_k)^2 = nA^2 + B^2 \sum_{k=1}^{n} x_k^2 + \sum_{k=1}^{n} y_k^2 + 2AB \sum_{k=1}^{n} x_k - 2A \sum_{k=1}^{n} y_k - 2B \sum_{k=1}^{n} x_k y_k . \tag{7.3}$$

It is known that partial derivatives of a function with respect to all arguments at a minimum point are zero. After equating to zero the partial derivatives of the expression (7.3) with respect to A and B, we obtain a system of two linear algebraic equations with unknowns A and B. Its solution is

$$B = \frac{n \sum_{k=1}^{n} x_k y_k - \left(\sum_{k=1}^{n} x_k \right) \left(\sum_{k=1}^{n} y_k \right)}{n \sum_{k=1}^{n} x_k^2 - \left(\sum_{k=1}^{n} x_k \right)^2} , \tag{7.4}$$

$$A = \frac{1}{n} \sum_{k=1}^{n} y_k - \frac{B}{n} \sum_{k=1}^{n} x_k . \tag{7.5}$$

The B coefficient is called the "slope" of the straight line, and the A coefficient is called the "intercept". The standard deviations (or "errors") of those two coefficients are calculated according to formulas

$$\Delta A = \sqrt{\frac{F_{\min}}{n(n-2)} \left(1 + \frac{\overline{x}^2}{D_x} \right)} , \tag{7.6}$$

$$\Delta B = \sqrt{\frac{F_{\min}}{n(n-2) D_x}} , \tag{7.7}$$

where $F_{\min}$ is the minimum value of the SSE (7.3), i.e., the value of SSE when A and B are equal to their optimal values (7.4) and (7.5), $\overline{x}$ is the average argument value:

$$\overline{x} = \frac{1}{n} \sum_{k=1}^{n} x_k , \tag{7.8}$$

and D_x is the variance of the argument values:

$$D_x = \frac{1}{n} \sum_{k=1}^{n} (x_k - \overline{x})^2 = \frac{1}{n} \sum_{k=1}^{n} x_k^2 - \overline{x}^2 . \tag{7.9}$$