Visual material for the course “Nuclear Physics”

Lecture 1

1. Basic facts and definitions

1.1. The nucleus and its constituents

Atomic nucleus consists of Z ***protons*** and N ***neutrons*** (they are collectively called ***nucleons***).
Z is called the ***atomic number***, and the total number of nucleons ($A = Z + N$) is called the ***mass number***.

The size of the nucleus is of the order of 10^{-15} m.

The basic properties of the atomic constituents:

	charge	mass (u)	spin ($\hbar$)	magnetic moment (J T^{-1})
proton	e	1.007276	1/2	1.411×10^{-26}
neutron	0	1.008665	1/2	-9.66×10^{-27}
electron	$-e$	0.000549	1/2	9.28×10^{-24}

Here, the mass is given in atomic mass units (symbol: “u”), which is defined as one twelfth of the mass of an unbound neutral atom of carbon-12:
1 u = $1.660538921 \times 10^{-27}$ kg.

1.2. Isotopes, isotones and isobars

Common notation of a nucleus: ${}^{A}_{Z}\mathrm{X}$, where X is a symbol of a chemical element (for example, notation of a nucleus of carbon-12 is ${}^{12}_{6}\mathrm{C}$). ***Isotopes*** are atoms whose nuclei have the same Z, but different N. For example, carbon has three naturally occurring isotopes: ${}^{12}_{6}\mathrm{C}$, ${}^{13}_{6}\mathrm{C}$ and ${}^{14}_{6}\mathrm{C}$. ***Isotones*** are atoms whose nuclei have the same N, but different Z. ***Isobars*** have the same mass A, but different Z and N.

1.3. Nuclear mass and energy

The very existence of a nucleus means that nucleons are bound together by a force that is strong enough to counteract the Coulomb repulsion of protons. The ***binding energy*** (B) of a nucleus is the energy (i.e., the work by external forces) required to separate it into its constituent nucleons.

The binding energy is the opposite of the total mechanical energy of the nucleons. Since the nucleons are bound together, the total mechanical energy is negative. Hence, $B > 0$.

Relationship between mass and energy:

$$E = mc^2$$

Hence, the negative term $-B$ in the total energy of a nucleus means that the mass of the nucleus is less than the sum of masses of the constituent nucleons by the amount equal to B / c^2:

$$M(A,Z) = Zm_{\mathrm{p}} + (A-Z)m_{\mathrm{n}} - B/c^2$$

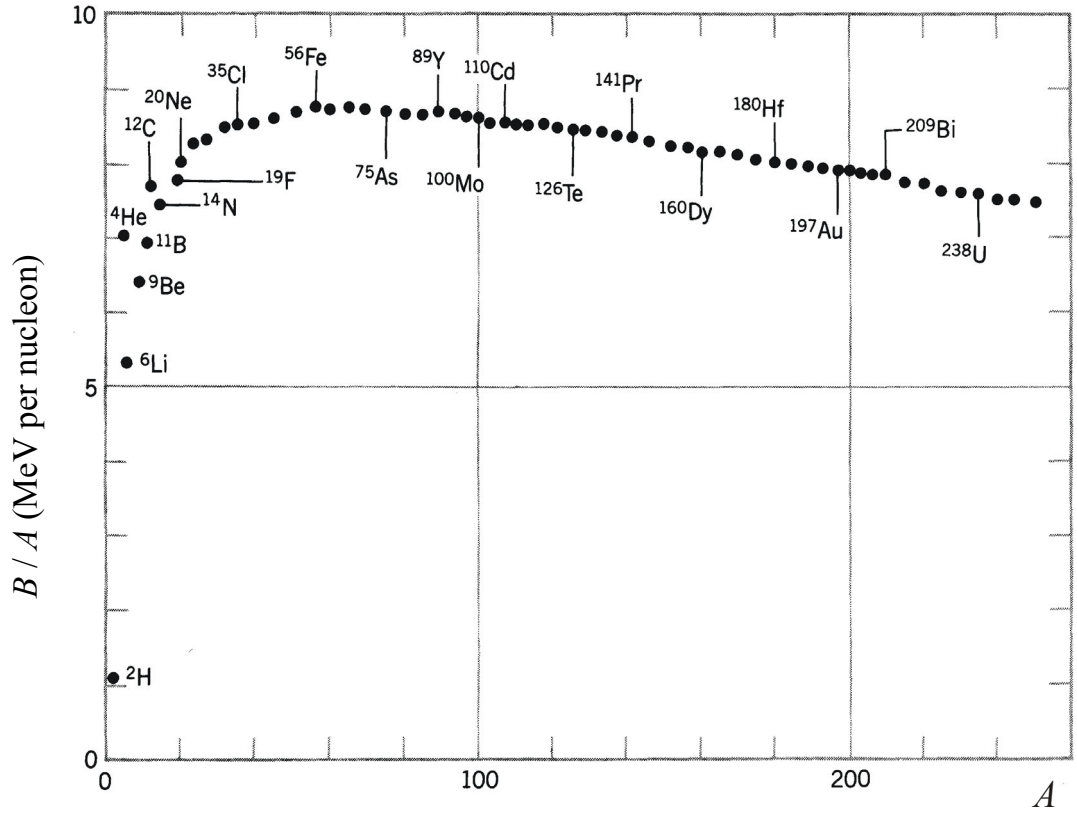


Fig. 1.1. Binding energy per nucleon as a function of mass number A

The shape of this curve is a result of the combined effect of the nuclear and electrostatic (Coulomb) forces. The nuclear force is short-range (~10^{-15} m), hence it binds a nucleon only to its nearest neighbours. The Coulomb force is long-range (each proton interacts with all other protons of the nucleus).

Decrease of B / A with increasing A at large values of A reflects an increase of the relative weight of the Coulomb interaction in larger nuclei. This decrease of B / A means that energy could be released by breaking a heavy nucleus into two, roughly equal fragments (***fission***).

Decrease of B / A with decreasing A at small values of A reflects the fact that in smaller nuclei a larger fraction of nucleons are on the surface of the nucleus. This decrease of B / A means that energy could be released by combining two light nuclei into a single nucleus (***fusion***).

2. Nuclear potential and energy levels

2.1. Nucleon states in a nucleus

A nucleon inside a nucleus is inside a ***potential well***:

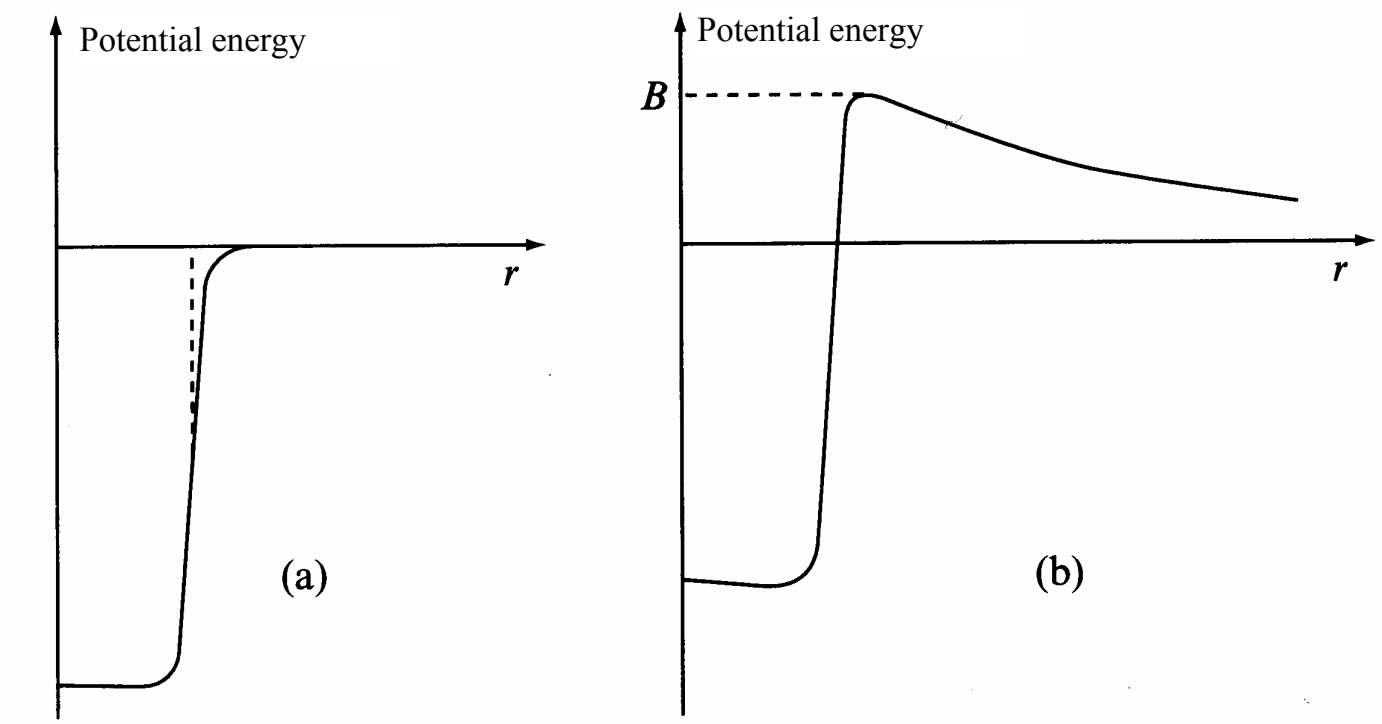


Fig. 2.1. Schematic representation of the potential energy experienced by (a) a neutron and (b) a proton, as a function of distance r from the center of a nucleus. Outside the nucleus, the proton experiences the electric force only and its potential energy is $Ze^2 / (4\pi\varepsilon_0 r)$.

Near the edge of the nucleus, the proton potential energy has a maximum B. Such a type of potential energy dependence on coordinate is called a ***potential barrier***. This particular potential barrier is called the ***Coulomb barrier***.

Quantum mechanics proves that the total mechanical energy E (i.e., potential energy + kinetic energy) of a particle inside a potential well can only be equal to discrete values called ***energy levels***. This statement is the result of solving the ***Schrödinger equation***:

$$\nabla^2\psi + \frac{2m}{\hbar^2}[E - U(x,y,z)]\psi = 0.$$

Here,

$$\nabla^2\psi \equiv \frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2}$$

The solutions of this equation are called ***wave functions*** of a particle. The meaning of a wave function: it is a complex-valued function whose absolute value squared is equal to ***probability density*** of finding a particle at a given point of space x, y, z:

$$|\psi|^2 = \frac{\mathrm{d}P}{\mathrm{d}V}$$

This implies the following ***normalization condition***:

$$\int_{\infty} |\psi|^2 \, \mathrm{d}V = 1$$

"Standard conditions" that must be satisfied by the wave function:

(1) the wave function must be finite;

(2) the wave function must be one-valued;

(3) the wave function must be continuous.

When a particle is inside a potential well, the Schrödinger equation only has such solutions at particular values of E. Those values are the particle ***energy levels***, and the corresponding solutions ψ are the particle wave functions when it is in a given energy level.

The simplest example of a potential well is a one-dimensional box. In this case, $|\psi|^2$ is *one-dimensional* probability density, i.e.,

$$\int_0^w |\psi|^2 \, \mathrm{d}x = 1.$$

Potential energy:

$$U(x) = \begin{cases} 0, & 0 \le x \le w; \\ \infty, & x < 0 \ \text{ir} \ x > w. \end{cases}$$

The corresponding Schrödinger equation:

$$\frac{\mathrm{d}^2\psi}{\mathrm{d}x^2} + \frac{2m}{\hbar^2} E\psi = 0 \qquad (0 \le x \le w).$$

The boundary conditions follow from the continuity requirement:

$$\psi(0) = 0, \quad \psi(w) = 0.$$

By solving this equation with given boundary conditions, we obtain the energy levels:

$$E = E_n = n^2 \frac{\pi^2\hbar^2}{2mw^2} = n^2 \frac{h^2}{8mw^2} \qquad (n = 1, 2, 3, \ldots).$$

and the corresponding wave functions:

$$\psi(x) = \psi_n(x) = \sqrt{\frac{2}{w}} \sin \frac{n\pi}{w} x.$$

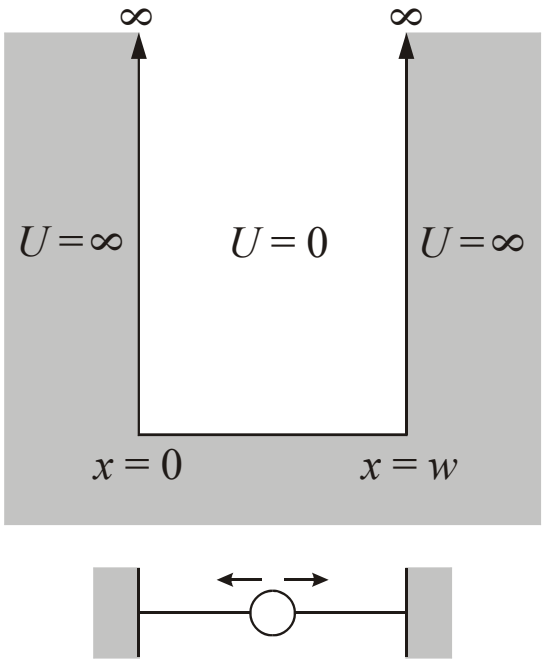


Fig. 2.2. A one-dimensional rectangular potential well and a physical example – a bead sliding without friction on a wire and bouncing elastically from the walls

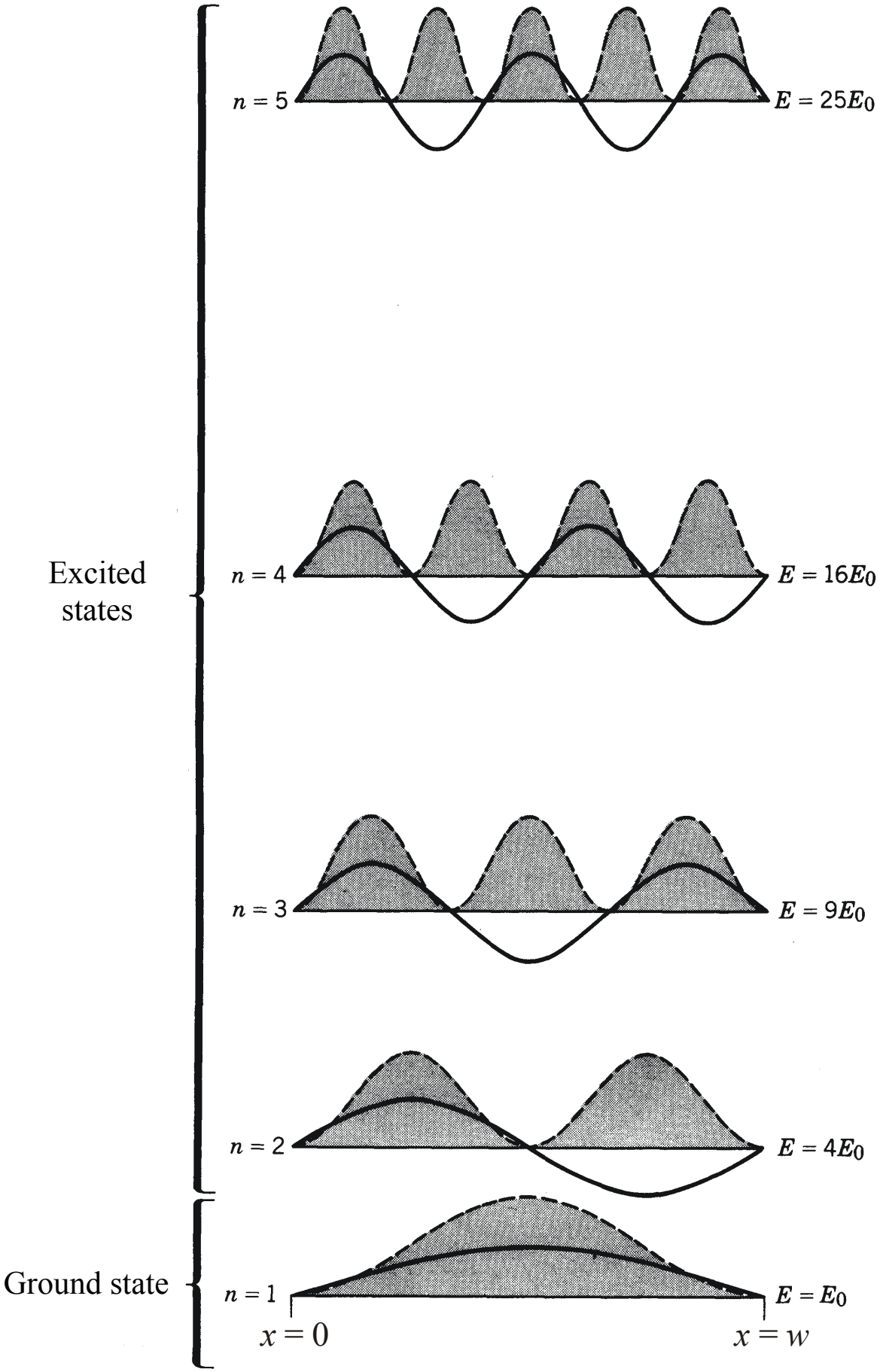


Fig. 2.3. Energy levels, wave functions (solid lines) and probability densities (dashed lines) of a particle inside a one-dimensional rectangular potential well. Energy E_0 is equal to $h^2/(8mw^2)$

The simplest model of a nucleon in a nucleus is that of a particle inside a cubic box of side w. The solution of the corresponding Schrödinger equation is similar to the one-dimensional case. The energy levels corresponding to a particle inside a cubic box:

$$E = E_n = \frac{h^2}{8mw^2}\left(n_x^2 + n_y^2 + n_z^2\right) \qquad (n_x, n_y, n_z = 1, 2, 3, \ldots),$$

The dimensionless quantities n_x, n_y, n_z are the example of so-called ***quantum numbers***, which are used in quantum mechanics to identify ***quantum states*** of a system. Each quantum number corresponds to a quantity that is conserved when there are no external influences. One of such conserved quantities is energy (E). Another one is ***angular momentum*** ($\boldsymbol{L}$, a vector quantity). When a particle is inside a spherical potential well (as a nucleon inside a nucleus, or an electron inside an atom), then the possible absolute values of the angular momentum vector are given by

$$|\boldsymbol{L}| = \hbar\sqrt{l(l+1)} \qquad (l = 0, 1, 2, \ldots)$$

Here, l is the quantum number of the angular momentum. The projection of the angular momentum vector to any given axis (L_z) is conserved, too:

$$L_z = m_l\hbar \qquad (m_l = -l, -l+1, \ldots l-1, l)$$

The states with $l = 0, 1, 2, 3, \ldots$ are denoted by letters s, p, d, f, …

The quantum numbers l and m_l quantize the *orbital* angular momentum (i.e., the angular momentum that is related to spatial motion of a nucleon around the center of the nucleus). In addition to the orbital angular momentum, a nucleon has an *intrinsic* angular momentum ($\boldsymbol{S}$), which is called ***spin***. The nucleon's spin quantum number is $s = 1 / 2$, i.e.,

$$|\boldsymbol{S}| = \hbar\sqrt{s(s+1)} = \frac{\sqrt{3}}{2}\hbar$$

$$S_z = m_s\hbar \qquad (m_s = \pm s = \pm 1/2)$$

If nucleon's orbital angular momentum and the direction of its spin were independent, then the quantum state of a nucleon would be completely characterized by four numbers:

- n – the energy quantum number (the number of a state among all states whose other three numbers are equal, in the direction of increasing energy),
- l – the orbital momentum quantum number,
- m_l – the orbital momentum projection quantum number,
- m_s – the spin projection quantum number.

However, because of ***spin-orbit interaction***, the projections of orbital and spin angular momenta of a nucleon are not conserved. The conserved quantities are the *total* angular momentum ($\boldsymbol{J} = \boldsymbol{L} + \boldsymbol{S}$) and its projection ($J_z$). The corresponding two quantum numbers are j and m_j:

$$|\boldsymbol{J}| = \hbar\sqrt{j(j+1)}$$

$$J_z = m_j\hbar \qquad (m_j = -j, -j+1, \ldots j-1, j)$$

a) when $l > 0$, $\quad j = l \pm \frac{1}{2}$;

b) when $l = 0$, $\quad j = \frac{1}{2}$

The quantum states of a nucleon are identified using the following four quantum numbers:

$$n, l, j, m_j.$$

The quantum number j is specified as a subscript. For example, "$2p_{3/2}$" means a nucleon with $n = 2$, $l = 1$ and $j = 3/2$.

2.2. Energy levels of nuclei

Neutrons and protons in a nucleus occupy energy states subject to the ***exclusion principle***:

No two identical nucleons can have the same set of quantum numbers.

In absence of external fields, nucleon's energy does not depend on m_j. Therefore, each energy level can contain up to $2j + 1$ nucleons, each having a different value of m_j. There are two independent systems of energy levels – proton levels and neutron levels.

Each arrangement of nucleons among the energy levels corresponds to a particular energy level of the nucleus as a whole.

The ***spin*** of the nucleus is defined as its *total* angular momentum (***J***), which is a sum of orbital and spin momenta of the constituent nucleons. As any other angular momentum, the nuclear spin is defined by a quantum number J:

$$|\boldsymbol{J}| = \hbar\sqrt{J(J+1)}$$

Nuclear energy levels are characterized by corresponding energies and spin quantum numbers J:

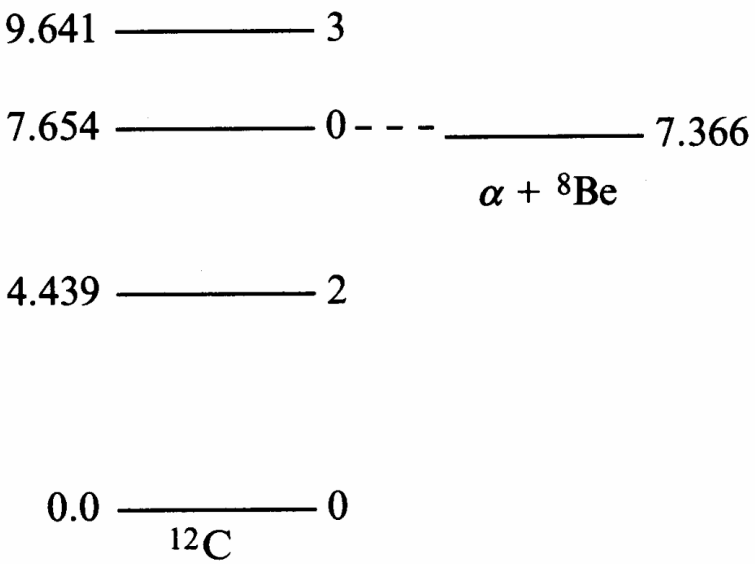


Fig. 2.4. Lowest energy levels of ^{12}C; each level is labeled by its energy in MeV and total angular momentum (or spin) quantum number. Also shown on the same energy scale is the state corresponding to separating ^{12}C into an α particle and a ^{8}Be nucleus.

A nucleus in an excited state normally remains there for a very short time. It loses the excess energy by emitting a particle or several particles. Usually the emitted particle is a ***photon*** (a quantum of electromagnetic radiation). This is the ***gamma radiation***. However, if the energy of the state is high enough, the nucleus may emit a heavy particle, such as an α particle (a ^{4}He nucleus).

A ***nuclide*** is an atomic species characterized by: **1)** chemical symbol (or Z); **2)** atomic mass A; **3)** energy level of the nucleus. Thus, if isotopes of the same element are in different energy levels, they are considered to be different nuclides.

2.3. Occurrence and stability of nuclei

For each value of the mass number A, there is a particular value of Z corresponding to the smallest mass of the nucleus. This value corresponds to the stable nucleus. If Z is larger (a "proton-rich" nucleus) or smaller (a "neutron-rich" nucleus), then the nucleus is unstable (***radioactive***). Such a nucleus eventually transforms into a stable nucleus by series of ***decays***. There are various types of decays. For example, a proton may be converted into a neutron, or vice versa (such a type of radioactivity is called ***beta radioactivity***). Alternatively, the nucleus may emit an α particle (***alpha radioactivity***).

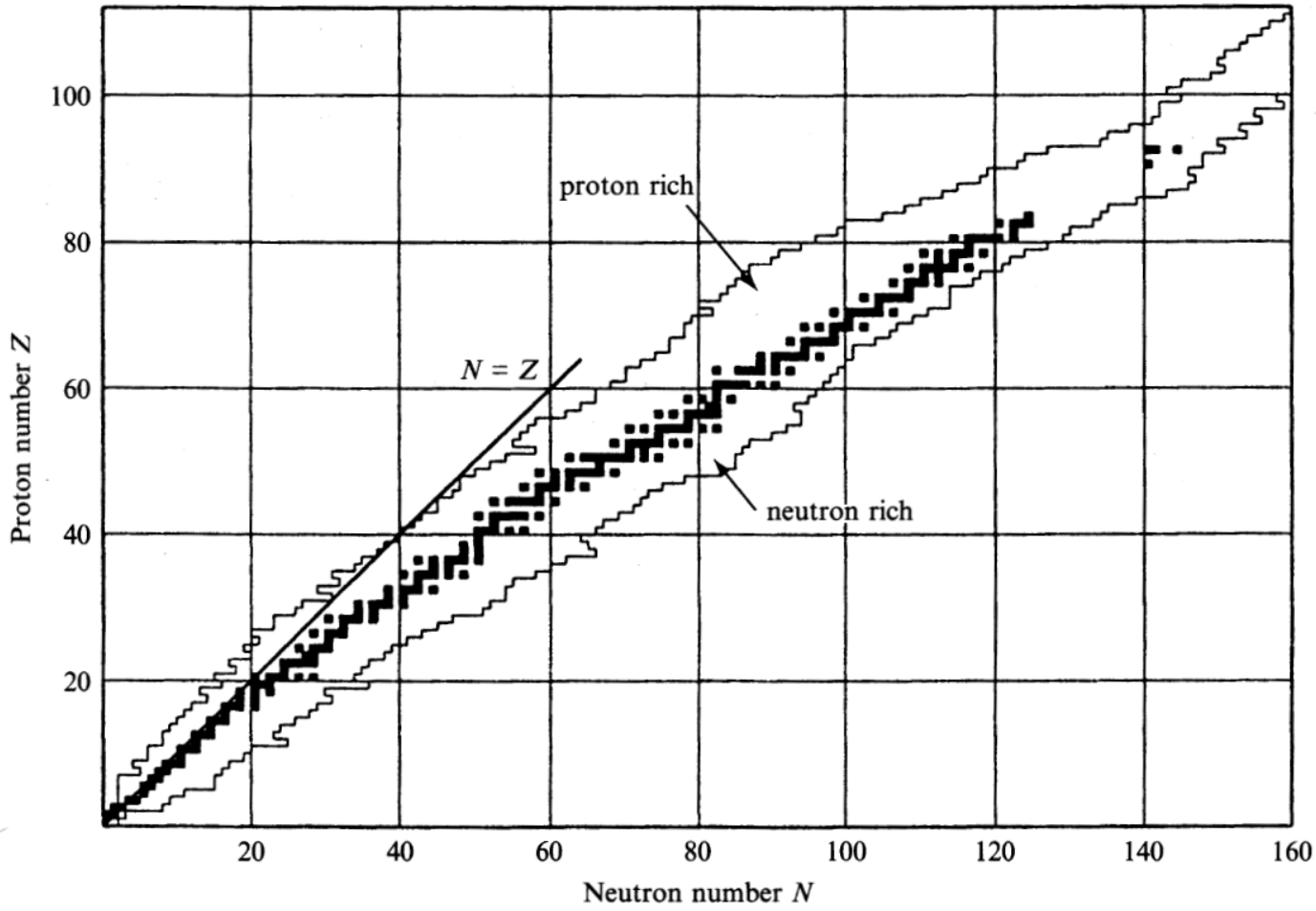


Fig. 2.5. Stable and unstable nuclei plotted according to proton number Z and neutron number N. Regions of known proton-rich and neutron-rich nuclei are indicated on either side of the band of stable nuclei (and very long-lived unstable nuclei), which are represented as black squares.

Additional reading for Lecture 1:

[1], p. 6 – 14,
[2], p. 4 – 7, 9 – 15, 20 – 22, 34 – 37, 65 – 67.

3. Radioactive decay

In radioactive decay, an unstable nucleus (called "parent") is transformed into a more stable nuclide (called the "daughter"). If the daughter nuclide is also radioactive, the process continues in a ***decay chain*** until a stable nuclide is reached.

Radioactivity is a random process. We cannot know exactly when a given unstable nucleus will decay and can only specify a probability per unit time that it will do so. That probability is called the ***decay constant***. It is frequently denoted by the Greek letter λ. Another quantity, which is related to the decay constant, is the decay ***half-life*** ($t_{1/2}$), which is the time taken for half the nuclei in a sample to decay:

$$t_{1/2} = \frac{\ln 2}{\lambda}$$

The mean time until the decay of a nucleus is called its mean ***lifetime***:

$$\tau = \frac{1}{\lambda}$$

If a given radioactive nuclide is not created (i.e., it is not a daughter of another nuclide, and it is not created in any nuclear reaction), then its amount decreases exponentially with time:

$$N(t) = N(0)\exp(-\lambda t) \equiv N(0)2^{-t/t_{1/2}}$$

Decay rate $-\mathrm{d}N / \mathrm{d}t$ is called ***activity***. Unit of activity is becquerel (Bq): 1 Bq = 1 s^{-1}.

All naturally occurring, and the majority of artificially produced, radioactive nuclei are either α active, β active, or both, and emit a combination of α, β and γ radiation. Artificially produced unstable nuclei may also decay by emitting protons, neutrons or even heavy ions.

3.1. Alpha decay

3.1.1. Main properties of alpha decay

During α emission, the parent nucleus loses both mass and charge:

$$(A, Z) \rightarrow (A - 4, Z - 2).$$

A generalized equation of α decay:

$${}^{A}_{Z}\mathrm{X} \rightarrow {}^{A-4}_{Z-2}\mathrm{Y} + {}^{4}_{2}\mathrm{He}$$

An example:

$${}^{226}_{88}\mathrm{Ra} \rightarrow {}^{222}_{86}\mathrm{Rn} + {}^{4}_{2}\mathrm{He}$$

Main properties of α decay:

1. $Z > 82$.
2. Discrete energies of α particles emitted by a particular nuclide.
3. Narrow energy range of emitted α particles: $E = (4 - 8{,}7)$ MeV.
4. Very strong dependence of the decay half-life on the particle energy E. It is given by the ***Geiger and Nuttall law*** of alpha decay:

$$\lg t_{1/2} = C + \frac{D}{\sqrt{E}}$$

Property No. 1 is related to the fact that α decay is caused by Coulomb repulsion of protons.

The emitted particle is an α particle (and not, e.g., a proton), because, when an α particle is emitted from a nucleus, the total rest mass of the system decreases. The decay energy (Q_α) is released in the form of kinetic energies of the daughter nucleus and the α particle:

$$Q_\alpha = (m_\mathrm{P} - m_\mathrm{D} - m_\alpha)c^2 = E_\mathrm{D} + E_\alpha$$

As evident from the table below, the decay energy is only positive for the case when the emitted particle is an α particle, hence this is the only possible type of spontaneous decay.

Table 3.1 Decay energy for various types of decay of the ^{232}U nucleus

Emitted particle	Decay energy (MeV)	Emitted particle	Decay energy (MeV)
n	–7,26	^{4}He	+5,41
^{1}H	–6,12	^{5}He	–2,59
^{2}H	–10,70	^{6}He	–6,19
^{3}H	–10,24	^{6}Li	–3,79
^{3}He	–9,92	^{7}Li	–1,94

Property No. 2 is caused by discrete energy levels of the daughter nucleus

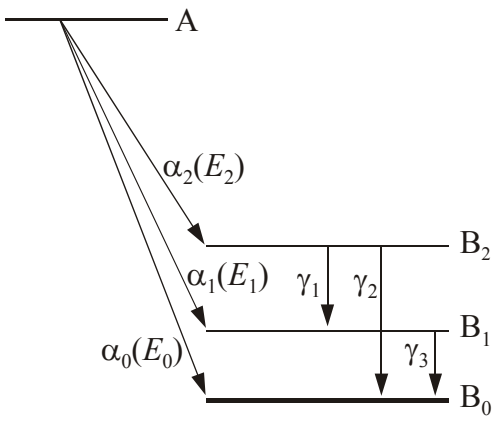


Fig. 3.1. Explanation of the discrete character of α particle velocity distribution

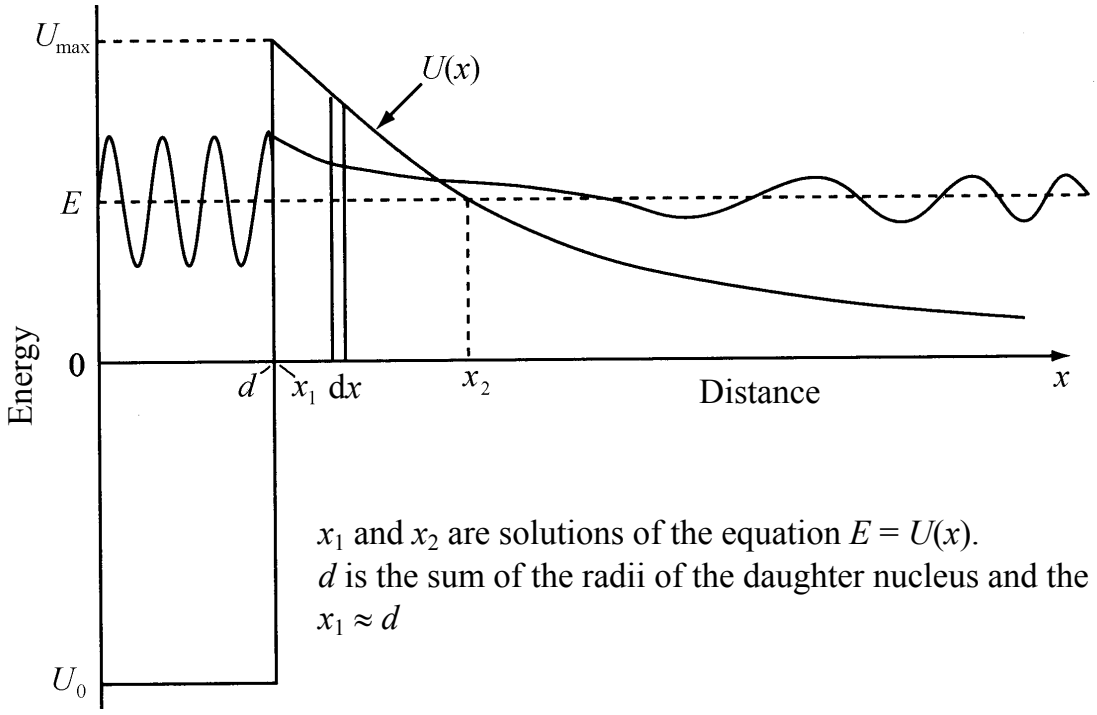
Properties No. 3 and 4 are explained by the semi-classical theory of α decay by G. Gamow (1928).

Potential energy of the alpha particle:

$$U(x) \approx \begin{cases} Ze^2/(2\pi\varepsilon_0 x), & \text{kai } x > d, \\ U_0 < 0, & \text{kai } x \le d. \end{cases}$$

Height of the Coulomb potential barrier:

$$U_{\max} \approx U(d) = \frac{Ze^2}{2\pi\varepsilon_0 d}$$



x_1 and x_2 are solutions of the equation $E = U(x)$.
d is the sum of the radii of the daughter nucleus and the α particle.
$x_1 \approx d$

Fig. 3.2. Dependence of the potential energy of the α particle and daughter nucleus on distance

3.1.2. Wave function of a free particle and quantum tunneling

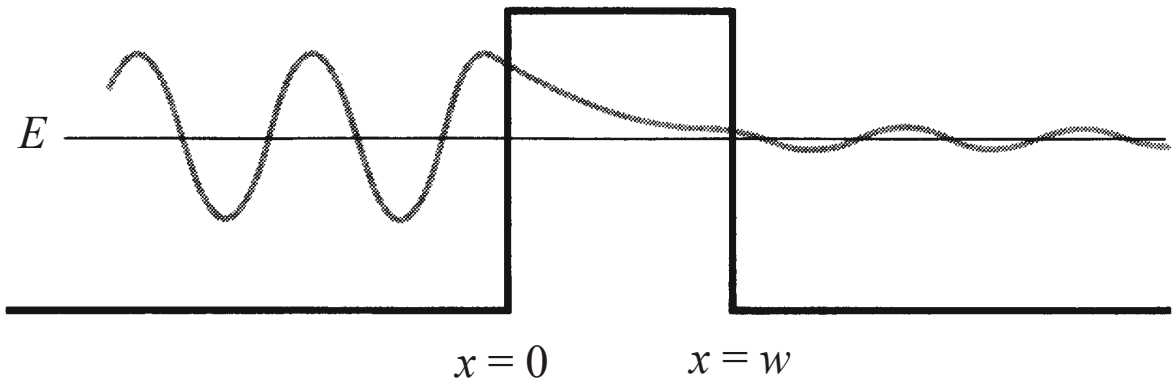


Fig. 3.3. Wave function of a particle when there is a potential barrier, whose height exceeds the particle energy E

One-dimensional Schrödinger equation:

$$\frac{\mathrm{d}^2\psi}{\mathrm{d}x^2} = \frac{2m}{\hbar^2}(U(x) - E)\psi$$

In the case of a rectangular potential barrier (see Fig.), the Schrödinger equation is:

$$\frac{\mathrm{d}^2\psi_1}{\mathrm{d}x^2} + k_1^2\psi_1 = 0 \qquad \text{(I region)},$$

$$\frac{\mathrm{d}^2\psi_2}{\mathrm{d}x^2} - k_2^2\psi_2 = 0 \qquad \text{(II region)},$$

$$\frac{\mathrm{d}^2\psi_3}{\mathrm{d}x^2} + k_1^2\psi_3 = 0 \qquad \text{(III region)}.$$

$$k_1 = \frac{\sqrt{2mE}}{\hbar}, \qquad k_2 = \frac{\sqrt{2m(U_0 - E)}}{\hbar}$$

The general solution:

$$\psi_1 = A\exp(\mathrm{i}k_1x) + B\exp(-\mathrm{i}k_1x) \qquad \text{(I region)},$$

$$\psi_2 = C\exp(k_2x) + D\exp(-k_2x) \qquad \text{(II region)},$$

$$\psi_3 = F\exp(\mathrm{i}k_1x) + G\exp(-\mathrm{i}k_1x) \qquad \text{(III region)}.$$

If the particle source is at $x = -\infty$, then

$$G = 0$$

A is the amplitude of the *incident* wave, B is the amplitude of the *reflected* wave, and F is the amplitude of the *transmitted* wave.

The amplitude (A, B, or F) defines the particle ***flux density***. E.g., if a particle is incident on the potential barrier, then its flux density is

$$i = N\frac{\hbar k}{m}|A|^2 .$$

The transmission probability is defined as the ratio of transmitted and incident flux densities:

$$S \equiv \frac{|F|^2}{|A|^2}.$$

By applying the continuity conditions to the functions ψ_1, ψ_2 and ψ_3 and assuming that

$$k_2 w >> 1,$$

(i.e., a high and wide potential barrier), the following expression of S is obtained:

$$S \approx \exp\left[-\frac{2}{\hbar}\sqrt{2m(U_0 - E)} \cdot w \right] << 1$$

Such effect when a particle tunnels through a potential barrier that it classically can not surmount is called ***quantum tunneling***.

A wide potential barrier of any shape can be constructed as a sequence of a large number N of thin potential rectangular barriers. Hence, the transmission probability of such a barrier is

$$S \approx \lim_{N\to\infty} \prod_{n=1}^{N} S_n \approx \exp\left[-\frac{2}{\hbar} \int_{x_1}^{x_2} \sqrt{2m(U(x) - E)}\,\mathrm{d}x \right]$$

3.1.3. Derivation of the Geiger-Nuttall law from the expression for S

Solutions of equation $U(x) = E$:

$$x_1 \approx d \approx 10^{-14}\ \mathrm{m}$$

$$x_2 = \frac{Ze^2}{2\pi\varepsilon_0 E}$$

We can imagine the α particle moving back and forth inside the nucleus with a speed υ and presenting itself at the barrier with a frequency (υ / d). Then the decay constant λ can be obtained by multiplying that frequency and the transmission probability S:

$$\lambda \approx \frac{\upsilon}{d} S$$

$E = M\upsilon^2/2$. When $E = 10$ MeV, $\upsilon \approx 2\cdot 10^7$ m/s. Therefore,

$$\lambda \approx 10^{21} \cdot S \ \ [\mathrm{s}^{-1}]$$

Since

$$t_{1/2} = \frac{\ln 2}{\lambda},$$

we obtain

$$\lg t_{1/2} \approx B - 0.434 \ln S,$$

where $t_{1/2}$ is expressed in seconds, and $B \approx -21$. Dependence of $\ln S$ on E is obtained on the basis of the simplifying assumption that $E << U$ inside the barrier:

$$-\int_{x_1}^{x_2} \sqrt{2m(U(x) - E)}\,\mathrm{d}x \approx -\int_{x_1}^{x_2} \sqrt{2mU(x)}\,\mathrm{d}x = -\int_{x_1}^{x_2} \sqrt{2m\frac{Ze^2}{2\pi\varepsilon_0 x}}\,\mathrm{d}x = const(\sqrt{x_1} - \sqrt{x_2}) \sim -\frac{1}{\sqrt{E}}$$

(because $x_2 \sim 1/E$, and x_1 is approximately constant). Hence,

$$\lg t_{1/2} = C + \frac{D}{\sqrt{E}}$$

3.2. Gamma emission

3.2.1. The concepts of gamma radiation and internal conversion

The ***gamma (γ) radiation*** is electromagnetic radiation whose wavelength is much less than the distance between atoms of a solid material, i.e., much less than 10^{-10} m (1 Å).

The most common physical mechanism of γ radiation is a quantum transition of a nucleus from an energy level E_a to a lower energy level E_b in the same nucleus. When this occurs, the transition energy ($E_a - E_b$) may appear in the form of a γ-ray photon:

$$E = h\nu = E_a - E_b,$$

where ν is the radiation frequency. Typical energies of γ-ray photons: (0.01 – 5) MeV.

Two other physical mechanisms by which a nucleus may lose excitation energy:

In the case of ***internal conversion***, an electron is ejected from one of the atomic orbits. The energy of the ejected electron is equal to the difference between the excitation energy E and the binding energy ε_r of the electron:

$$E_e = E - \varepsilon_r.$$

In the case of ***internal pair formation***, an electron-positron pair is created. In this case, the excitation energy is converted into the rest energy of two new particles – an electron and a positron – and to their kinetic energy. This process is usually weak, except for very high transition energies, and can only occur if the available energy exceeds that needed to create the pair, which is $2m_0c^2 = 1.022$ MeV, where m_0 is the rest mass of an electron.

Typical lifetimes of excited nuclei: (10^{-14}–10^{-6}) s. However, long-lived excited states with lifetimes in excess of 1 min are also possible. They are called ***metastable states***.

Gamma radiation can also be emitted during annihilation reactions. The most common example of such a reaction is annihilation of an electron and a positron:

$$e^- + e^+ \rightarrow \gamma + \gamma + 1.022 \text{ MeV}$$

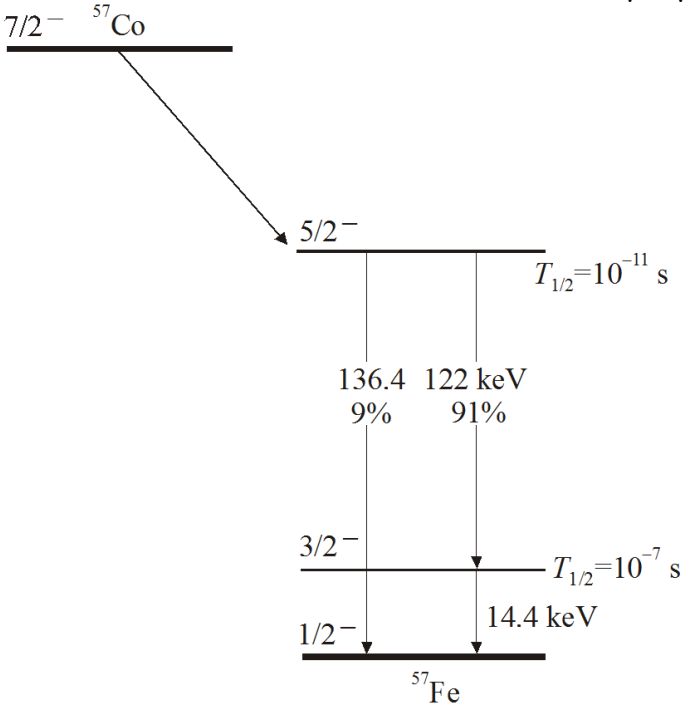


Fig. 3.4. The decay diagram of ^{57}Co (decay type – electron capture). During this decay, excited nuclei of ^{57}Fe are formed, which emit γ-ray photons of three energies

3.2.2. The concept of multipole radiation, its angular momentum and parity

Notation:
J_a – the spin quantum number of the initial quantum state of a nucleus;
J_b – the spin quantum number of the final quantum state of a nucleus;
l – the angular momentum quantum number of the photon emitted during transition $a \to b$.
The corresponding vectors of angular momenta will be denoted using bold font: $\boldsymbol{J}_a$, $\boldsymbol{J}_b$, $\boldsymbol{l}$.

$$|\boldsymbol{l}| = \hbar\sqrt{l(l+1)}$$

(similarly for the other angular momenta).

Conservation of angular momentum: $\boldsymbol{l} = \boldsymbol{J}_a - \boldsymbol{J}_b$

According to the general rule of addition of two angular momenta, the following values of l are possible:

$$|J_a - J_b| \le l \le |J_a + J_b| \qquad (\text{excluding } l = 0)$$

Another quantum number (m) gives the projection of the vector $\boldsymbol{l}$ to a given coordinate axis:

$$l_z = m\hbar \qquad (m = -l, -l+1, ..., l-1, l)$$

$$m = m_{Ja} - m_{Jb}$$

The possible values of m are:

$$m = -l, -l+1, ..., l-1, l$$

If both m_{Ja} and m_{Jb} are precisely known, then m is uniquely defined by

$$m = m_{Ja} - m_{Jb}$$

Usually, only the value of l is of interest (m is undefined). The quantum number l is referred to as ***multipolarity*** of the photon. Electromagnetic radiation having a definite value of l is called ***multipole radiation***. Specifically, radiation with $l = 1$ is called ***dipole radiation*** ($2^l = 2$), radiation with $l = 2$ is called ***quadrupole radiation*** ($2^l = 4$), radiation with $l = 3$ is called ***octupole radiation*** ($2^l = 8$), etc. The same term is applied to a transition that leads to the corresponding radiation (e.g., "dipole transition", etc.).

The quantum numbers l and m are not sufficient to define electromagnetic radiation completely (i.e., to describe the spatial distribution of electric and magnetic field vectors). For complete definition, the parity of the radiation field must be given.

The concept of ***parity*** is used to describe symmetry of any function $f(\boldsymbol{r})$ relative to the operation of inversion in which all coordinates are reflected through the origin, i.e., $\boldsymbol{r} \to -\boldsymbol{r}$:

even parity: $f(-\boldsymbol{r}) = f(\boldsymbol{r})$
odd parity: $f(-\boldsymbol{r}) = -f(\boldsymbol{r})$

Multipole radiation has a definite parity. The parity of the radiation field is usually treated as an additional quantum number, which can be either +1 (even parity), or –1 (odd parity).

For each value of l, two types of multipole radiation are possible, one having even parity and another having odd parity. Parity of electric field is opposite to parity of magnetic field. By convention, parity of radiation is synonymous with parity of magnetic field strength $\boldsymbol{H}$:

even-parity radiation: $\boldsymbol{H}(-\boldsymbol{r}) = +\boldsymbol{H}(\boldsymbol{r})$,

odd-parity radiation: $\boldsymbol{H}(-\boldsymbol{r}) = -\boldsymbol{H}(\boldsymbol{r})$.

An equivalent way to specify parity is by specifying the ***type*** of multipole radiation: electric or magnetic. ***Electric (E) radiation*** is generated by oscillating electric charge (in the context of a nucleus, it is the charge of protons). ***Magnetic (M) radiation*** is generated by varying electric current (in the context of a nucleus, it is the current caused by orbital motion of protons). In addition, magnetic radiation can be caused by oscillating internal magnetic moments of the nucleons (i.e., spin magnetic moments).

Relation between parity and type (i.e., "*E*" or "*M*"):

Parity of electric radiation with multipolarity l: $(-1)^l$,

Parity of magnetic radiation with multipolarity l: $-(-1)^l$.

The parity of dipole radiation is the same as the parity of its source, which can be either oscillating charge (for *E* radiation), o alternating current (for *M* radiation).

(a) (b)

Fig. 3.5. (a) Electric dipole moment formed by positive and negative charges separated by a distance $\boldsymbol{r}$. The dipole moment is equal to $q\boldsymbol{r}$. (b) Magnetic dipole moment created by a charge q moving with speed v in a circular loop of radius r. The dipole moment $\boldsymbol{\mu}$ is proportional to $q\boldsymbol{r}\times\boldsymbol{v}$

Wave functions of a nucleus that is in a definite quantum state also have a definite parity Π. In energy diagrams, it is usually indicated to the right of the spin quantum number of the nucleus (e.g., "3^-" or "2^+"). ***Conservation of parity***:

if $\Pi_a = \Pi_b$, then only even-parity radiation is emitted

if $\Pi_a = -\Pi_b$, then only odd-parity radiation is emitted

Table 3.2 Selection rules for gamma emission

Multipolarity	Dipole		Quadrupole		Octupole		…
Type of radiation	*E*1	*M*1	*E*2	*M*2	*E*3	*M*3	…
Parity change	Yes	No	No	Yes	Yes	No	…

Transition rate (i.e., radiation intensity) decreases rapidly with increasing multipolarity l. Transitions with the smallest allowed l are therefore the most likely.

3.3. Beta emission and electron capture

During β emission, the atomic number changes by ±1, and the mass number does not change:

$$(A, Z) \rightarrow (A, Z \pm 1).$$

There are three types of β decay:

β^- decay: an electron is emitted from the nucleus, Z increases by 1,

β^+ decay: a positron is emitted from the nucleus, Z decreases by 1,

electron capture: an atomic electron is captured into the nucleus, Z decreases by 1.

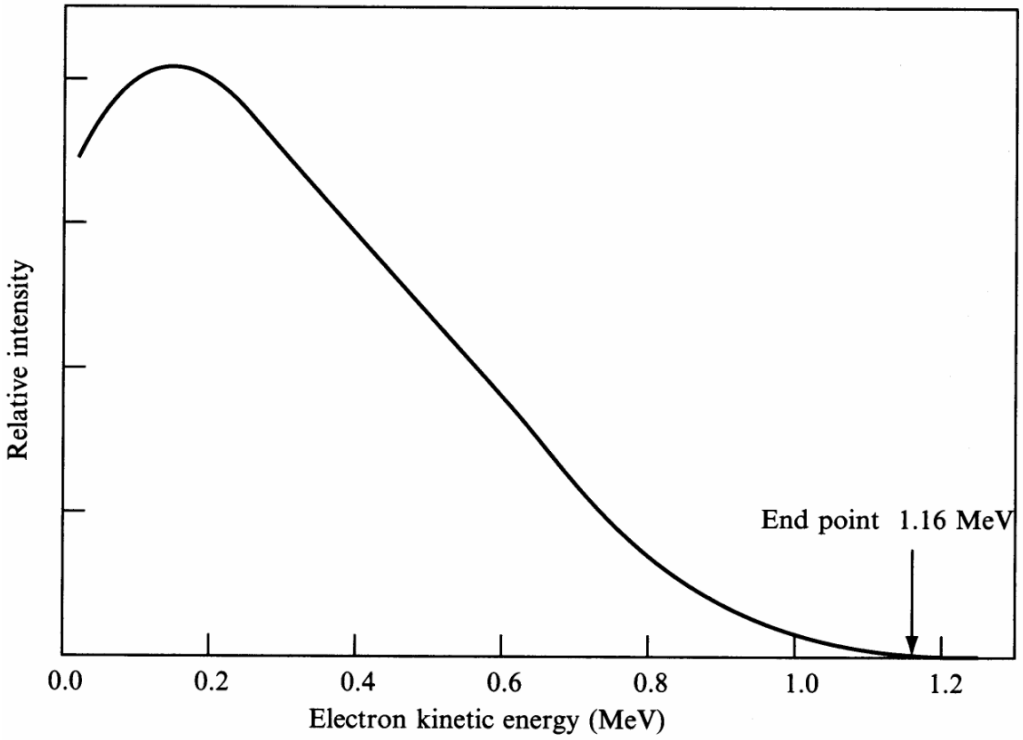


Fig. 3.6. Energy spectrum of β particles emitted by ^{210}Bi

Unlike the energy spectrum of α particles or gamma photons, the β particle energy spectrum is continuous. However, it is known that the total decay energy is uniquely defined (it is equal to the difference of the rest energies of the parent nucleus and daughter nuclei). This means that a part of the decay energy is carried away by another particle. That particle is neutrino or antineutrino.

During β^- or β^+ decay, one neutron inside the nucleus transforms into a proton or vice versa:

$$\mathrm{n} \rightarrow \mathrm{p} + \mathrm{e}^- + \bar{\nu}, \qquad \mathrm{p} \rightarrow \mathrm{n} + \mathrm{e}^+ + \nu$$

During electron capture, one proton inside the nucleus transforms into a neutron (as in β^+ decay):

$$\mathrm{p} + \mathrm{e}^- \rightarrow \mathrm{n} + \nu$$

3.4. Radioactive decay chains. Radioactive dating

… see [1], p. 19 – 22.

Additional reading for Lecture 2:
[1], p. 14 – 22, 65 – 67, 84 – 88.
[2], p. 160 – 165, 170 – 184, 246 – 254, 272 – 277.

4. Nuclear reactions

4.1. Introduction. The concept of a nuclear reaction

A ***nuclear reaction*** is any transformation of a nucleus caused by interaction ("collision") with an incident particle. If a nucleus X is bombarded with a particle "a" and transforms into a nucleus Y, emitting a particle "b", then such a reaction is written as follows:

$$\mathrm{a} + \mathrm{X} \rightarrow \mathrm{Y} + \mathrm{b} \quad \text{or} \quad \mathrm{X\,(a,\,b)\,Y}$$

A group of reactions which have the same incident and emitted particles ("a" and "b", respectively) is called "(a,b) reactions". Reaction energy, or Q value:

$$Q = (m_\mathrm{a} + m_\mathrm{X} - m_\mathrm{b} - m_\mathrm{Y})c^2$$

If $Q > 0$, then the reaction is ***exothermic***, and if $Q < 0$, then it is ***endothermic***.

4.2. The concept of the reaction cross-section

Reaction cross-section σ is an area whose value is chosen on the basis of this requirement: *the geometric probability for an incident particle to strike that area must be equal to the probability that the incident particle will induce the nuclear reaction*.

$$\mathrm{d}P = \frac{\mathrm{d}S'}{S} = \sigma n\,\mathrm{d}x$$

When several types of collision outcomes are possible,

$$\sigma = \sum_i \sigma_i$$

(σ_i is the cross-section of the reaction with i-th outcome). When several types of targets are present,

$$\sigma = \sum_i p_i \sigma_i$$

(σ_i is the cross-section of the reaction with targets of i-th type, and p_i is the fraction of those targets). Cross-sections are usually measured in barns. 1 b = 10^{-28} m^2.

Mean free path is the average distance traveled by the incident particle until the reaction event:

$$l = \frac{1}{\sigma n}$$

(n is the target concentration). Derivation of the mean free path expression: number of targets (e.g., atoms) inside the cylinder shown in Fig. 4.2 is

$$N = \sigma x n,$$

$$l = \frac{x}{N} = \frac{x}{\sigma x n} = \frac{1}{\sigma n}.$$

If several types of collisions are possible,

$$\frac{1}{l} = \sum_i \frac{1}{l_i}$$

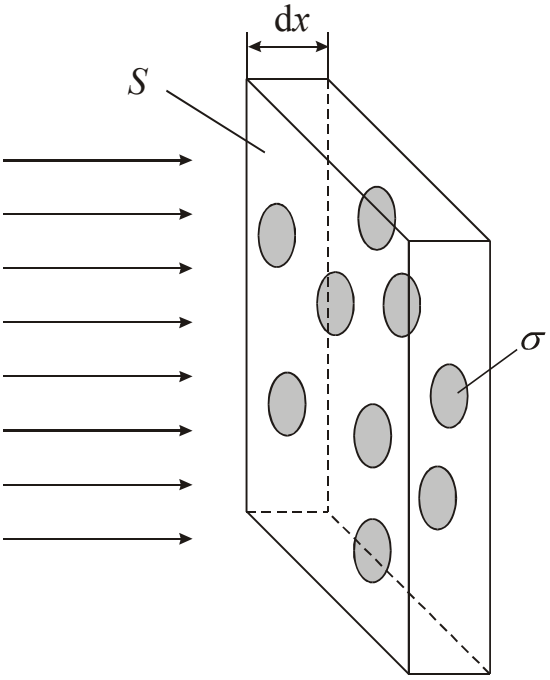


Fig. 4.1. Explanation of the reaction cross-section σ

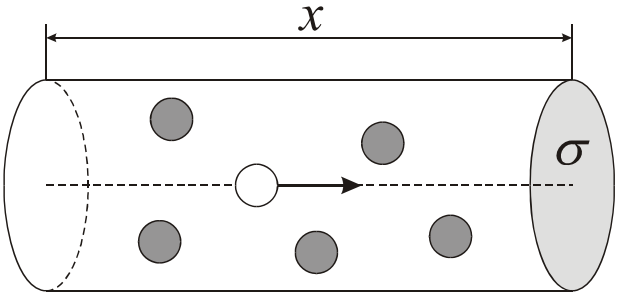


Fig. 4.2. For derivation of the mean free path

Macroscopic cross-section is the reaction cross-section corresponding to unit volume of the target material:

$$\Sigma = \frac{\mathrm{d}P}{\mathrm{d}x} = \sigma n = \frac{1}{l}$$

Reaction rate R is the number of reaction events per unit time in a given volume V. Derivation of the expression for R: from expression of dP it follows that

$$\frac{\mathrm{d}P}{\mathrm{d}t} = \sigma n \frac{\mathrm{d}x}{\mathrm{d}t} \equiv \sigma n \upsilon$$

$$R = \frac{\mathrm{d}P}{\mathrm{d}t} n_{\mathrm{inc}} V = \sigma n \upsilon n_{\mathrm{inc}} V = \sigma N \upsilon n_{\mathrm{inc}} = \sigma N j = \Sigma V j,$$

where υ is velocity of incident particles, n_{inc} is concentration of incident particles, $N = nV$ is the number of targets inside volume V, and $j = n_{\mathrm{inc}}\upsilon$ is the flux density of incident particles (i.e., number of particles per unit time and unit area).

A cross-section can be defined for any type of interaction. Usually, the interaction is defined by:

(1) types and energies of primary particles (e.g., incident and target particles);
(2) the reaction outcome, i.e., types and energies of secondary particles ("reaction products").

However, when defining the reaction outcome, we can be even more specific and analyze only the reactions when secondary particles move in specific directions. This is when the concept of "differential cross-section" is useful. Let us define $\mathrm{d}\sigma(\theta, \phi)$ as cross-section of such interaction when the secondary particle moves into an infinitesimal solid angle $\mathrm{d}\Omega$ in direction defined by polar and azimuthal angles (θ, ϕ). Then the ***differential cross-section*** is defined by

$$\sigma_\Omega = \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}$$

Unit of measurement of the differential cross-section is b/sr (barn per steradian).

Total cross-section:

$$\sigma = \int \sigma_\Omega \mathrm{d}\Omega \equiv \int_0^{2\pi} \mathrm{d}\phi \int_0^{\pi} \sigma_\Omega \sin\theta \mathrm{d}\theta$$

Usually, σ_Ω does not depend on ϕ. Then

$$\sigma = 2\pi \int_0^{\pi} \sigma_\Omega \sin\theta \mathrm{d}\theta$$

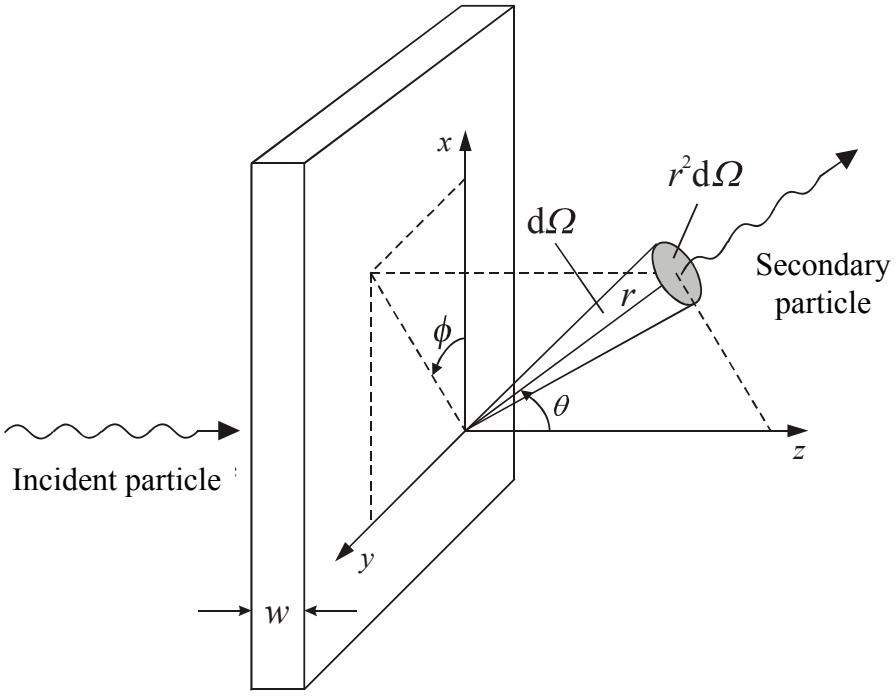


Fig. 4.3. For explanation of differential cross-section

4.3. Isotope production

… see [1], p. 25 – 26.

4.4. Examples of nuclear reactions

4.4.1. Elastic scattering

$$\mathrm{a} + \mathrm{X} \rightarrow \mathrm{X} + \mathrm{a}$$

A separate case of elastic scattering is elastic Coulomb scattering, or ***Rutherford scattering***.

$$F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{zZe^2}{r^2}$$

If we assume that the target nucleus is infinitely massive, then the Rutherford scattering differential cross-section is given by

$$\sigma_\Omega = \frac{d^2}{16\sin^4(\theta/2)} = \left(\frac{zZe^2}{4\pi\varepsilon_0}\right)^2 \left(\frac{1}{4E}\right)^2 \frac{1}{\sin^4(\theta/2)},$$

where d is the minimum distance of approach in the case of a central collision between a nucleus with charge $+Ze$ and an incident particle with charge $+ze$ and kinetic energy E:

$$d = \frac{zZe^2}{4\pi\varepsilon_0 E}$$

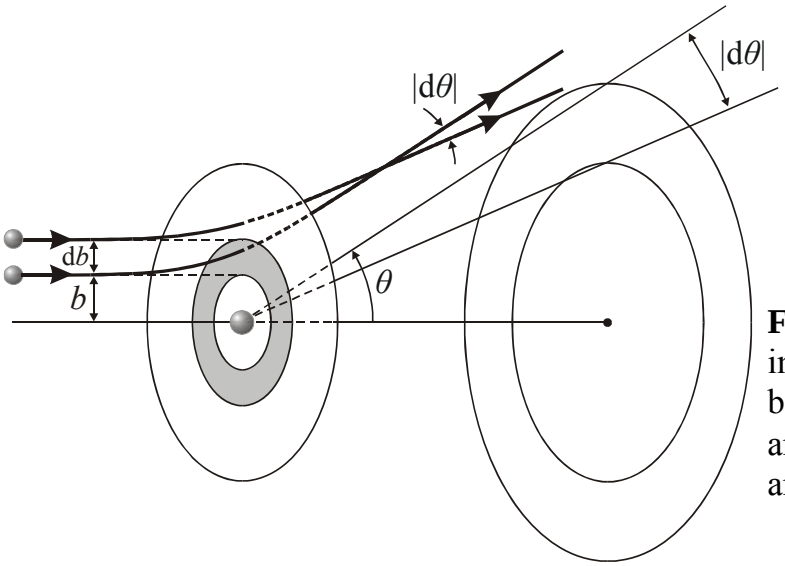


Fig. 4.4. Rutherford scattering: when the impact parameter of an incident particle is between b and $b + \mathrm{d}b$, then the scattering angle is between θ and $\theta + \mathrm{d}\theta$ ($\mathrm{d}b$ and $\mathrm{d}\theta$ are of opposite sign)

4.4.2. Direct reactions

A ***direct nuclear reaction*** is a reaction when the incident particle only interacts with a small number of nucleons, which are near the surface of the target nucleus.

Properties of direct reactions:

a) relatively small exchange of energy, momentum and mass,
b) extremely short duration of interaction (~10^{-22} s).

Examples of direct reactions:

Inelastic scattering:

$$\mathrm{a} + \mathrm{X} \rightarrow \mathrm{X}^* + \mathrm{a}$$

Transfer reactions:

stripping reaction (e.g., (d,p), (α,d) and (^{16}O, ^{12}C)),
pickup reaction (e.g., (p,d), (p,t) and (^{16}O, ^{17}O)).

4.4.3. Compound nucleus reactions

In the case of a central or near-central collision between an incident particle and the target nucleus, a so-called "compound nucleus reaction" is possible. It proceeds in two stages:

1) Both interacting particles merge into an intermediate ("compound") nucleus, which is in an excited state.
2) After 10^{-16} – 10^{-18} s, the compound nucleus loses the excess energy, emitting one or more particles.

Properties of the emitted particles depend only on properties of the compound nucleus and do not depend on the method of its formation. In this respect, the second stage is similar to radioactive decay. Sometimes, several decay types ("decay channels", or "decay branches") are possible. Likewise, several ways to form a given compound nucleus may exist.

The emitted particles may be:

- nucleons,
- nuclei,
- gamma photons,
- internal conversion electrons.

Emission of nucleons is similar to evaporation of a hot liquid. However, if excitation energy is less than nucleon binding energy or only slightly larger than the latter, then gamma emission or internal conversion is more likely. In such a case, the incident particle is "captured" into the target nucleus, hence such a reaction is called a ***capture reaction***.

If the incident particle is a nucleus (e.g., a proton), then formation of the compound nucleus is inhibited by the Coulomb potential barrier. However, if the incident particle is a neutron, then there is no Coulomb barrier, hence formation of the compound nucleus becomes possible even at low energies of the incident neutron. In such a case, reaction probability is inversely proportional to the neutron velocity (this is the so-called "1 / υ law").

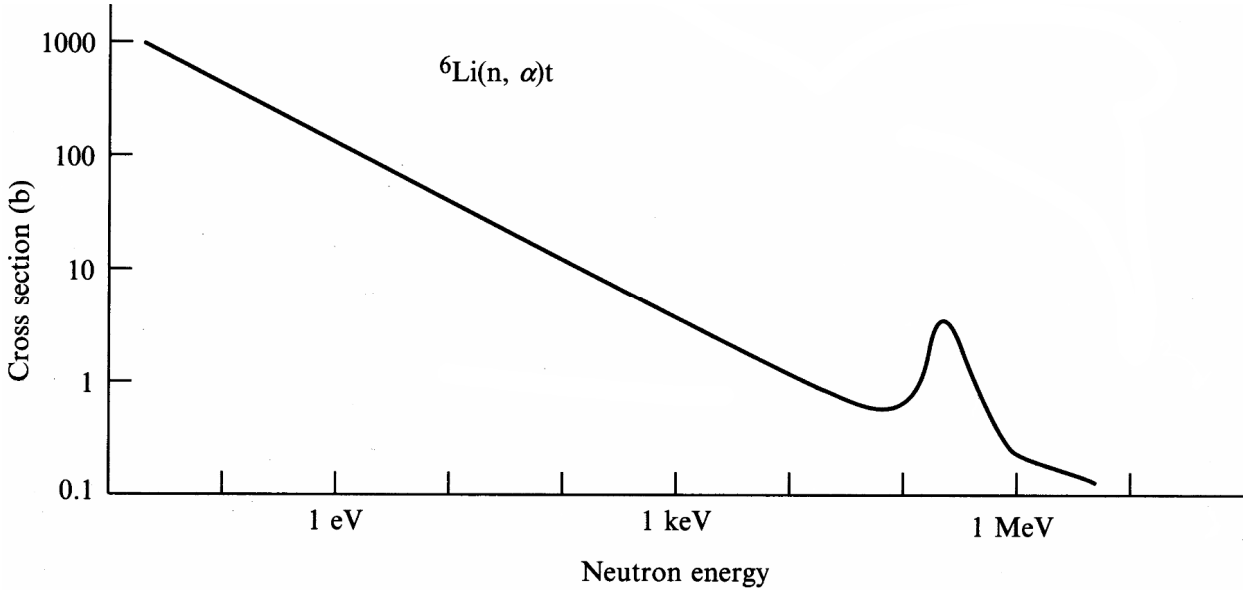


Fig. 4.5. Dependence of cross-section of nuclear reaction ^{6}Li(n,α)t on neutron energy. A resonance is evident at 250 keV

4.4.4. Resonance in compound nuclear reactions

Frequently, energy dependence of a compound-nucleus reaction cross-section has several maxima. They correspond to an increased probability of formation of the compound nucleus. Such an increase is observed when the excess energy supplied to the compound nucleus becomes exactly equal to the energy that is needed to excite the nucleus into one of its excited energy levels ("natural oscillation frequencies"). This phenomenon is called ***resonance***. Near the resonance energy, the reaction cross-section is approximately given by

$$\sigma(E) \sim \frac{1}{(E - E_r)^2 + (\Gamma / 2)^2}$$

(this is the so-called ***Lorentz function***, or ***Breit-Wigner function***).

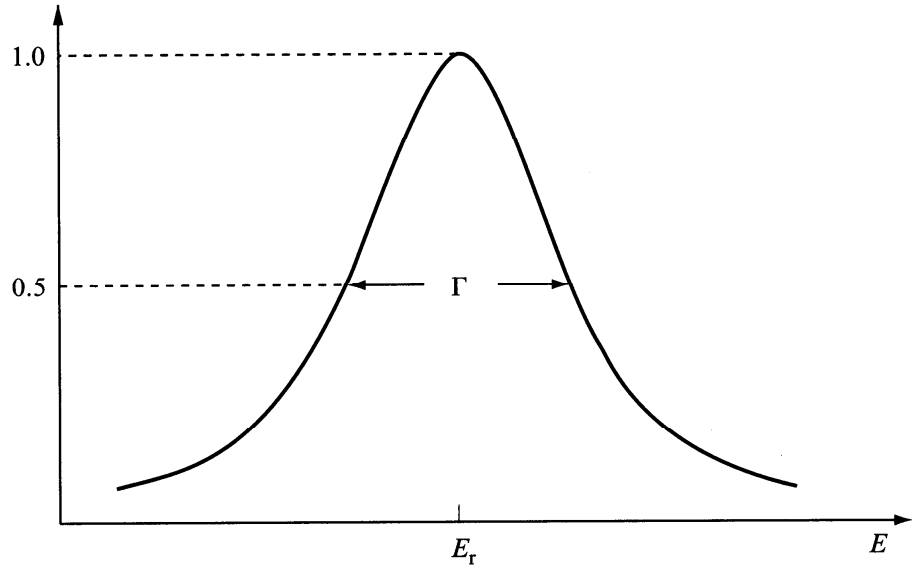


Fig. 4.6. The Breit-Wigner resonance curve

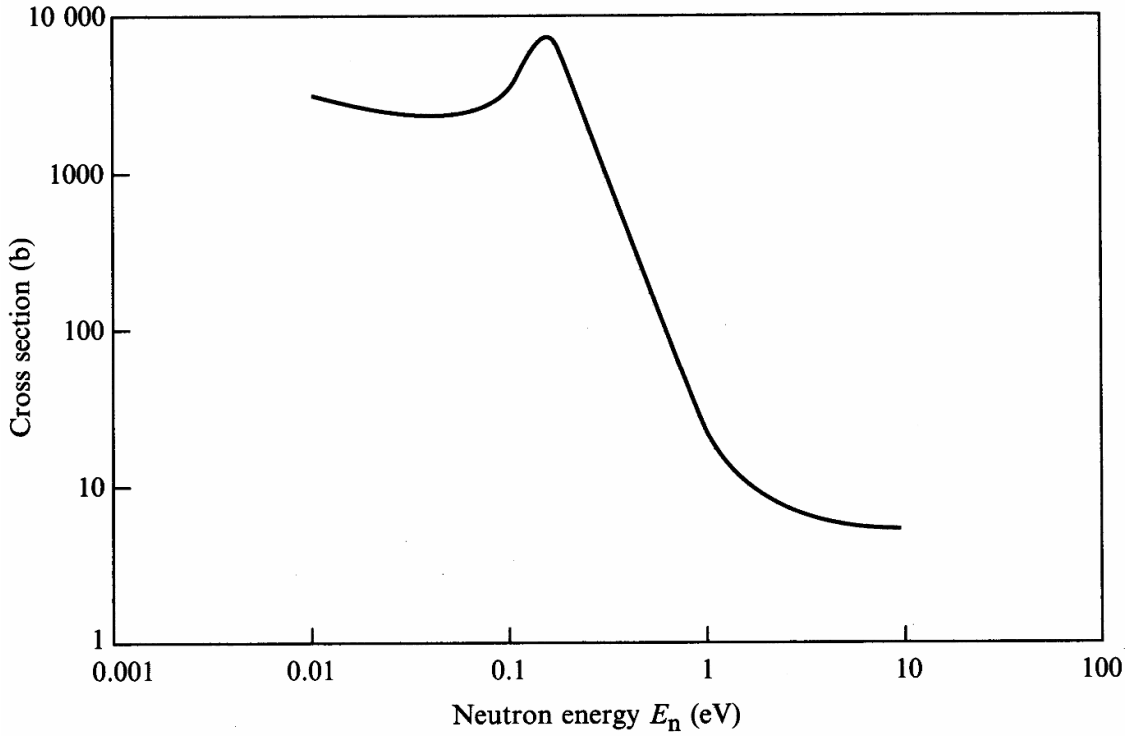


Fig. 4.7. The total cross-section of neutron interaction with an isotope of cadmium ^{113}Cd. A pronounced resonance at 0.17 eV is evident. The asymmetry of the peak is mainly a result of superposition of two curves: the "1 / v law" and the Breit-Wigner resonance curve.

5. Interaction of radiation with matter

5.1. Introduction

Nuclear radiation is *ionizing* radiation, i.e., it causes ionization of atoms of the material through which it passes. On the one hand, this fact makes nuclear radiation harmful (because ionizing action breaks chemical bonds between molecules of the material), but on the other hand, this fact facilitates detection of the radiation.

According to the manner by which particles interact with matter, four distinct groups of particles can be defined:

1) heavy charged particles (such as alpha particles and nuclei),
2) light charged particles (such as electrons and positrons),
3) photons (neutral particles with zero rest mass),
4) neutrons (neutral heavy particles).

5.2. Interaction of heavy charged particles with matter

Heavy particles are the particles whose mass is much larger than the mass of the electron (e.g., protons and nuclei).

Charged particles interact with electrons and nuclei of the atoms of the material due to Coulomb force. Interaction with electrons is much more likely than interaction with nuclei.

From conservation of energy and momentum, it follows that the maximum energy that can be imparted to an electron (mass m_e) by an incident non-relativistic particle of mass M is

$$\Delta E_{\max} = E\frac{4m_{\mathrm{e}}}{M} = 2m_{\mathrm{e}}\upsilon^2 << E,$$

where E is energy of the incident particle, υ is its velocity.

Since the number of interaction events is large, and the decrease of particle energy after each interaction is small, heavy charged particles have a well-defined ***range*** in matter (i.e., distance traveled until the loss of all initial energy). For the same reason, the path of heavy charged particles in matter is straight.

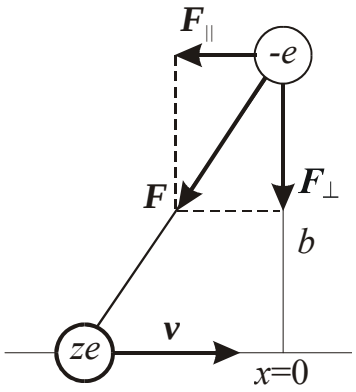


Fig. 5.1. The classical model of ionization of an atom due to Coulomb interaction of its electrons with an incident heavy charged particle. ze is the electric charge of the incident particle, $-e$ is the electron charge

Stopping power is defined as the average decrease of particle energy after traveling the unit distance ($-\mathrm{d}E / \mathrm{d}x$).

Formal relation between the range R and stopping power:

$$R = \int_0^{E_0} \frac{\mathrm{d}E}{-\mathrm{d}E/\mathrm{d}x}$$

For heavy charged particles, the stopping power is given by the Bethe formula (*H. Bethe*, 1930):

$$-\frac{\mathrm{d}E}{\mathrm{d}x} = \frac{1}{4\pi\varepsilon_0^2}\frac{z^2e^4n}{m_{\mathrm{e}}\upsilon^2}\left[\ln\frac{2m_{\mathrm{e}}\upsilon^2}{I(1-\beta^2)} - \beta^2\right],$$

where $\beta = \upsilon / c$, z is the charge number of the particle, I is the average ionization energy of the material, n is electron concentration:

$$n = \frac{Z\rho N_A}{A},$$

ρ is density of the material, Z is its atomic number, A is atomic mass of the material.

Mass stopping power: $-\mathrm{d}E/(\rho\,\mathrm{d}x)$. ***Mass range***: ρR (measurement unit is g/cm^2).

5.3. Interaction of electrons with matter

The physical mechanism of electron interaction with matter is the same as in the case of heavy charged particles (Coulomb force). However, due to the small mass of the electron, even a single collision can change its energy and direction of motion significantly. Therefore, its path in matter is random.

All charged particles emit electromagnetic radiation when they move with acceleration. Since intensity of this radiation is proportional to acceleration squared, it is much more pronounced for electrons and positrons than for heavy charged particles. The radiation emitted due to sudden changes in direction and speed when electrons or positrons are slowed down in matter is called ***bremsstrahlung*** (a German term, which means "braking radiation"). Thus, electron energy losses consist of two terms: ***ionization losses*** and ***radiation losses***. Accordingly, two stopping powers can be defined: ionization stopping power and radiation stopping power.

At small energies, ionization losses predominate. Radiation losses increase with increasing energy of the incident particle, and at higher energies they exceed ionization losses. The critical energy, when ionization and radiation losses become equal to each other, is

$$E_{\text{cr}} \approx \frac{1600 m_{\text{e}} c^2}{Z} \approx \frac{800}{Z}\,\text{MeV}$$

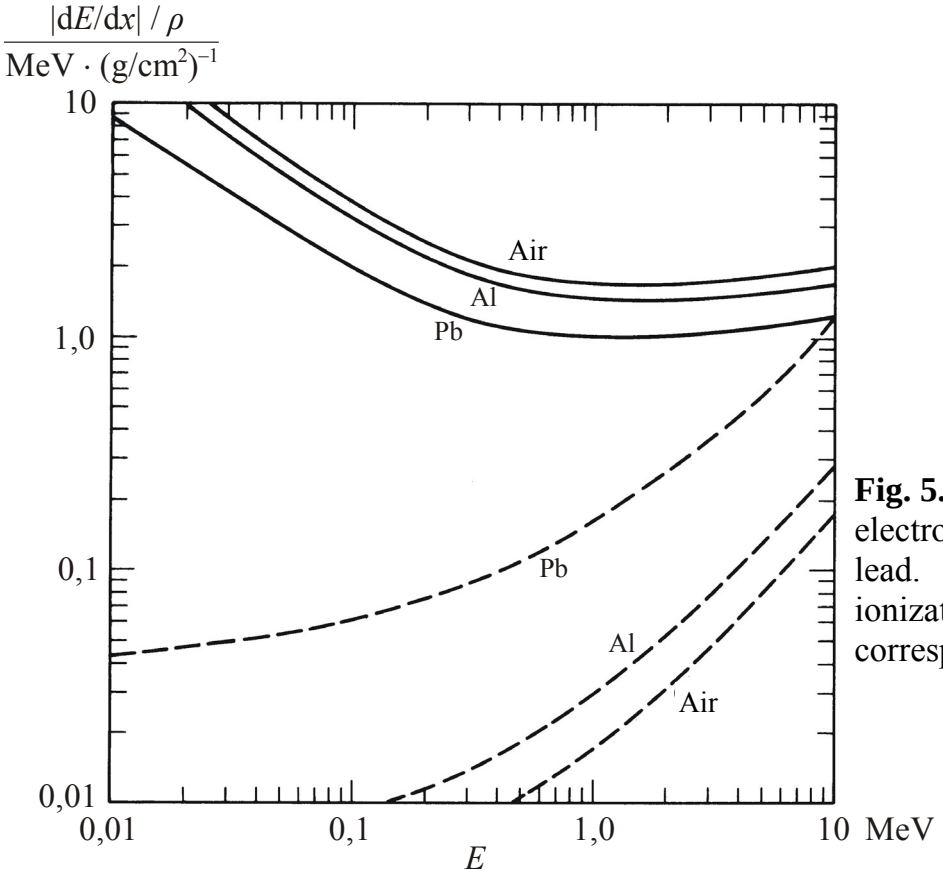


Fig. 5.2. Mass stopping powers of electrons in the air, aluminum and lead. Solid lines correspond to ionization losses, and dashed lines correspond to radiation losses

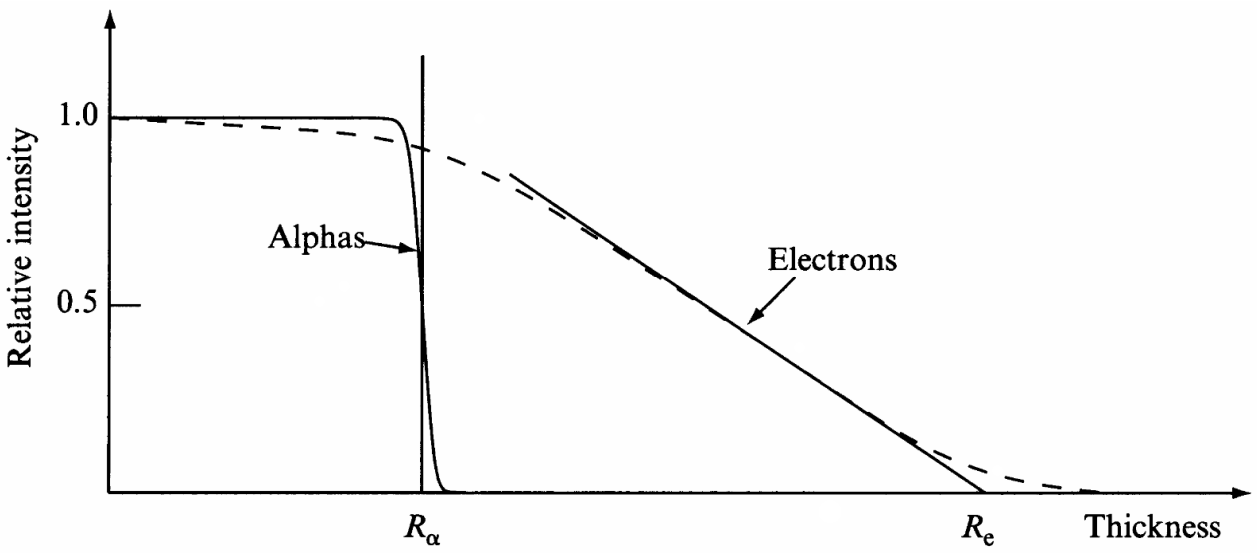


Fig. 5.3. Relative transmitted intensity of collimated beams of α particles and electrons as a function of absorber thickness. For α particles, the mean range R_α is the point where the intensity has fallen to half its initial value; R_e is the extrapolated range for electrons. The horizontal scale is not the same for the two types of radiation.

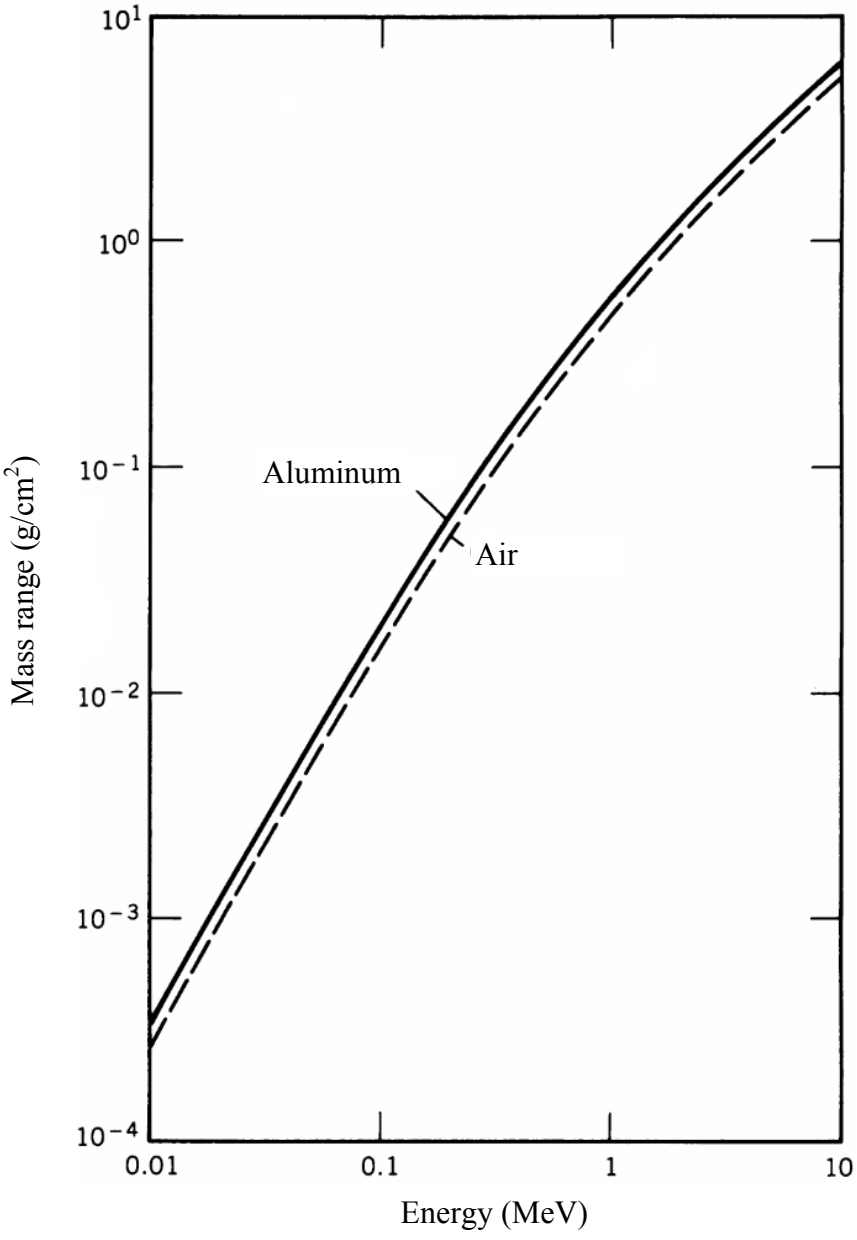


Fig. 5.4. The measured range-energy relationship for electrons in air and in aluminum. This curve is approximately the same for any other absorber, because in this energy range the mass range is only weakly dependent on atomic number Z of the absorber.

5.4. Interaction of gamma radiation with matter

Unlike charged particles, photons do not have rest mass, hence they can not be slowed down. They can only be scattered or absorbed. In ***scattering***, the photon changes its direction of motion (it may also lose a part of its energy, i.e., its wavelength may increase). In ***absorption***, all energy of the photon is transferred to the absorber material (i.e., the photon disappears).

Among the interaction mechanisms of gamma photons with matter, three mechanisms are the most important. They are described below.

5.4.1. Compton scattering

Compton scattering is a type of interaction of a photon with an atom that can be described as an elastic collision between the photon and an atomic electron (which can be assumed to be free). From the laws of conservation of energy and momentum it follows that the scattered photon has a smaller energy (i.e., larger wavelength) than the incident one, because a part of that energy is transferred to the electron. The ***Compton formula*** gives the increase of wavelength:

$$\Delta\lambda = \lambda_C (1 - \cos\theta),$$

where θ is the scattering angle and λ_C is the ***Compton wavelength***:

$$\lambda_C = \frac{h}{m_e c},$$

where m_e is the electron rest mass.

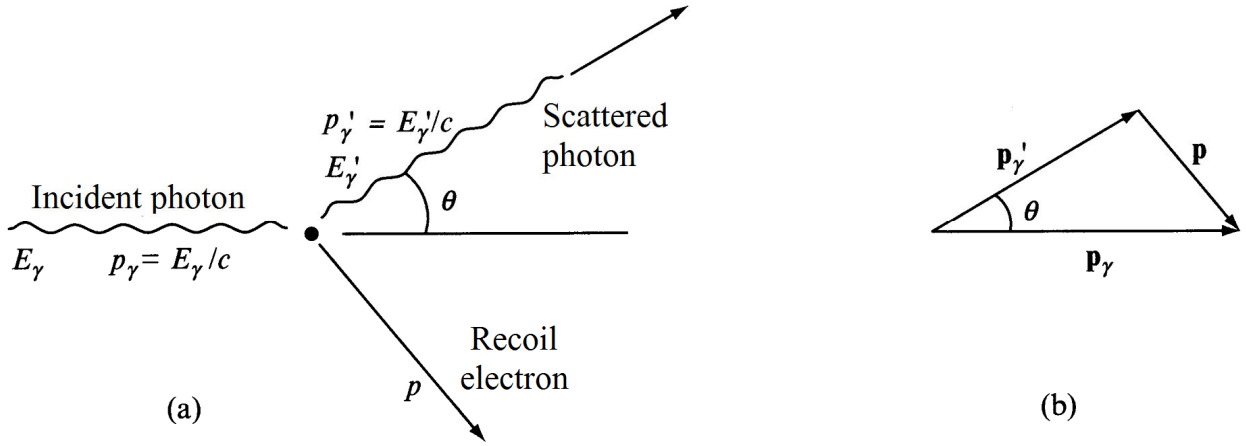


Fig. 5.5. (a) Geometry of Compton scattering; (b) the vector relationship between the three momenta

The atomic Compton scattering cross-section:

$$\sigma_C = Z\sigma_{Ce},$$

where Z is the atomic number (i.e., the number of electrons in the atom), and σ_{Ce} is the *electronic* Compton scattering cross-section, which only depends on photon energy.

5.4.2. Photoelectric absorption

Photoelectric effect is a type of interaction of a photon with an atom when the atom absorbs all energy of the photon (i.e., the photon disappears) and one of atomic electrons is removed from the atom. Usually, the electron is removed from the innermost electronic shell of the atom (the K shell).

When photon energy is of the order of 100 keV, the cross-section of photoelectric effect is approximately equal to

$$\sigma_f \approx 10^{-37} Z^5 / (h\nu)^{7/2},$$

where the cross-section σ_f is expressed in m^2, and $h\nu$ is the photon energy in MeV.

5.4.3. Electron-positron pair production

In the electric field of an atomic nucleus, a photon may stop existing by transforming all its energy into relativistic energy of two new particles – a free electron and a positron:

$$h\nu = m_+c^2 + m_-c^2,$$

where m_+c^2 are m_-c^2 the total relativistic energies of the positron and the electron (m_+ and m_- are the total relativistic masses of the positron and the electron). Pair production is only possible when photon energy is larger than two rest energies of an electron: $2m_0c^2 \approx 1{,}02$ MeV.

At photon energies less than 3 MeV, the cross-section of pair production is much smaller than the cross-section of Compton scattering.

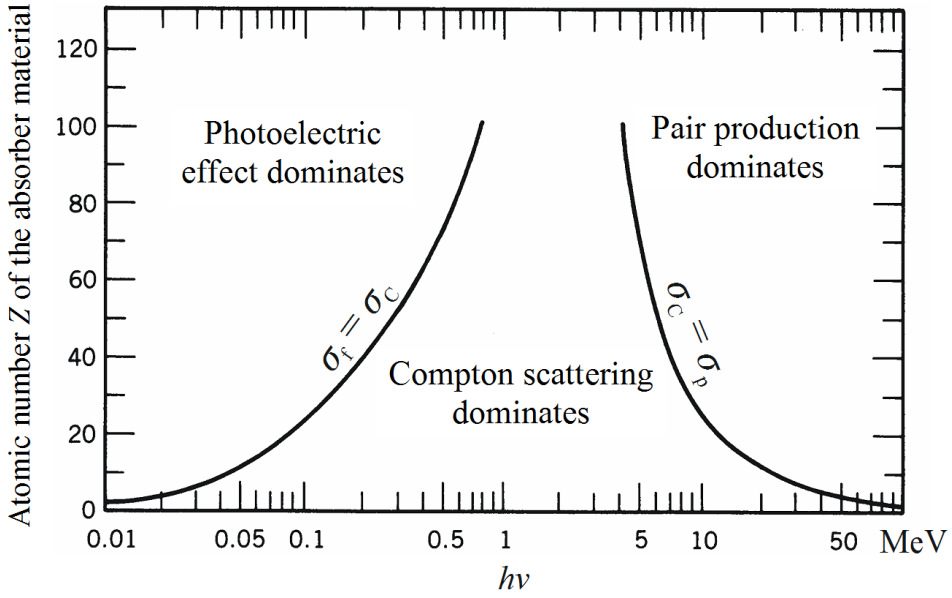


Fig. 1. The relative importance of various processes of gamma radiation interaction with matter

5.4.4. Attenuation coefficient

Change of the beam intensity (i.e., number of photons per second per unit area) after passing a layer of the absorber material with thickness dx:

$$dI = -n_a \sigma I dx,$$

where n_a is the concentration of absorber atoms and σ is the total interaction cross-section:

$$\sigma = \sigma_C + \sigma_f + \sigma_p$$

By integrating, one obtains:

$$I = I_0 \exp(-n_a \sigma x) = I_0 \exp(-\mu x),$$

where μ is the ***attenuation coefficient***:

$$\mu = n_a \sigma$$

Mass attenuation coefficient:

$$\mu_m = \frac{\mu}{\rho}$$

6. Measuring particle energies

6.1. Simplified detector model

1) Ionizing radiation creates free charge carrier in the detector medium (e.g., electrons and holes in a semiconductor, or electrons and positive ions in a gas);
2) due to motion of the created free charge carriers in the electric field (which exists in the detector), electric current $i(t)$ flows in the detector load circuit. That current is measured.

Two main operation modes of a detector are pulse mode and current mode. In ***pulse mode***, each detected particle creates a separate pulse of current, which is converted into a voltage pulse by a device called "preamplifier". In ***current mode***, only the average current $I(t)$ over a relatively long time interval is measured.

In order to measure properties of individual particles (such as particle energy), pulse mode must be used. The time integral of the detector current pulse gives the total charge Q created in the detector due to ionization:

$$\int_0^{t_c} i(t)dt = Q,$$

where t_c is the charge collection time (i.e., pulse duration).

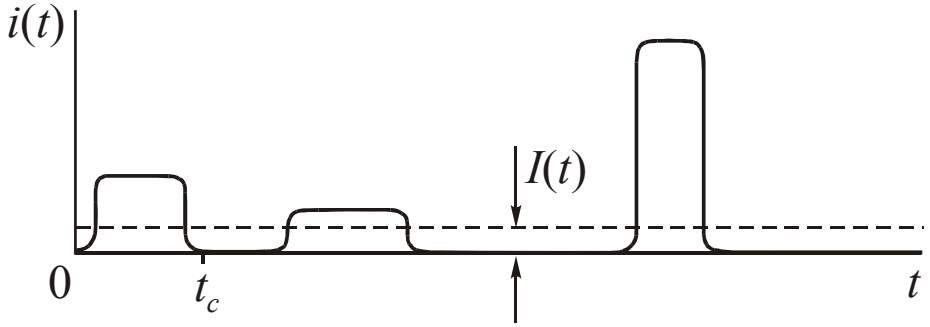


Fig. 6.1. Examples of detector current pulses. The dashed line indicates the time average of the detector current

In pulse mode, the height of each voltage pulse (H) is proportional to Q:

$$H = \frac{Q}{C},$$

where C is the sum of the detector capacitance and the effective capacitance of the load circuit of the detector.

Thus, the pulse height H can be used to measure the charge Q created in the detector due to ionization. The charge Q, in turn, can be used to measure the energy absorbed in the detector (see next section).

6.2. Detector pulse height spectrum

For detectors that are used to measure particle energies, the number of free charge carriers created due to absorption of particle's energy in the detector material is directly proportional to the absorbed energy. Since the height of each pulse is directly proportional to the charge of the mentioned charge carriers, the distribution of pulse heights approximately reflects the distribution of energy absorbed in the detector material. The distribution of pulse heights or energies is called the ***spectrum*** of pulse heights or energies, respectively.

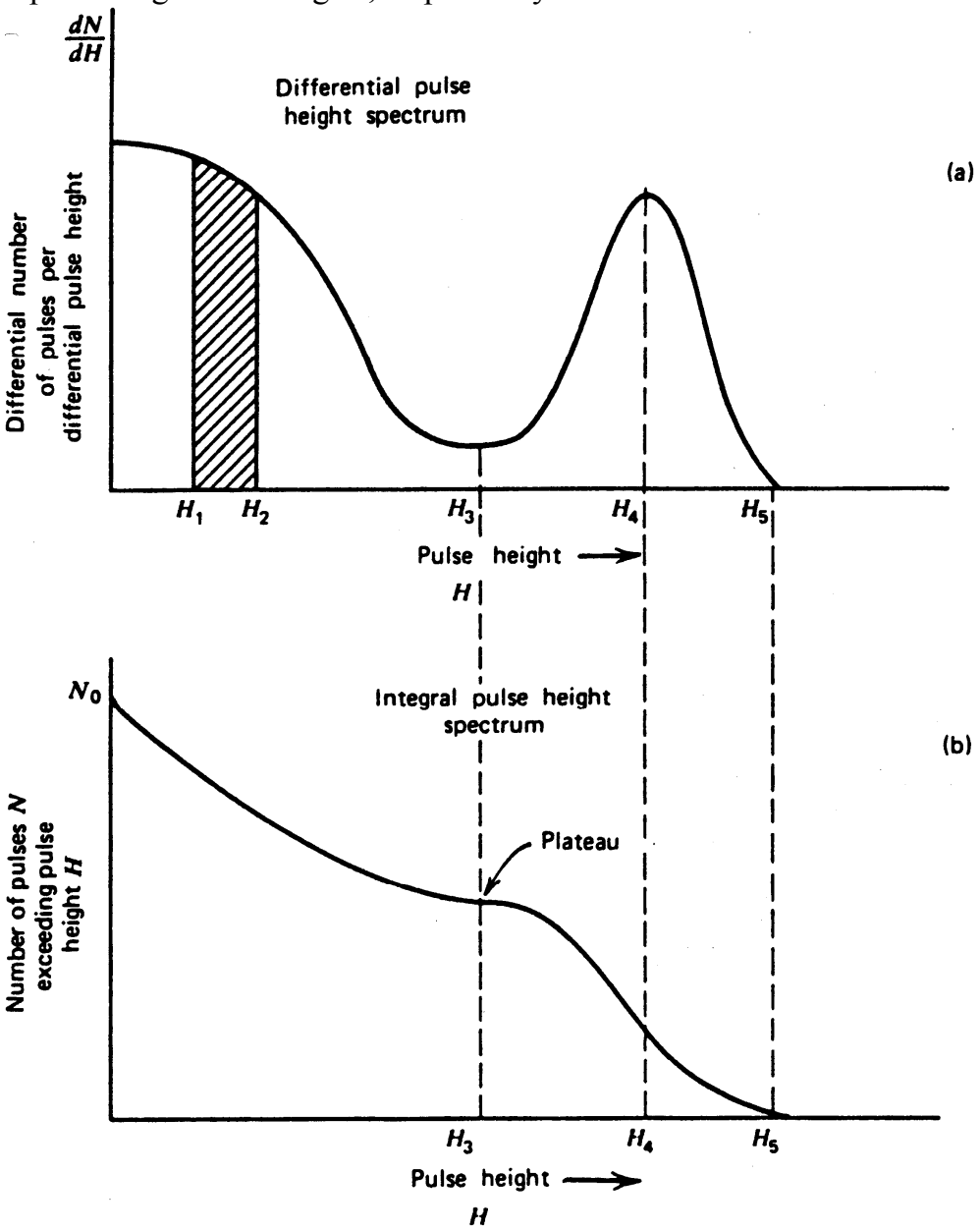


Fig. 6.2. Examples of differential and integral pulse height spectra

The number of pulses with heights between H_1 and H_2 can be determined by integrating the differential pulse height spectrum from H_1 to H_2:

$$N(H_1 < H < H_2) = \int_{H_1}^{H_2} \frac{\mathrm{d}N}{\mathrm{d}H}\mathrm{d}H.$$

The total number of pulses is equal to the integral of the entire differential spectrum:

$$N_0 = \int_0^{\infty} \frac{\mathrm{d}N}{\mathrm{d}H}\mathrm{d}H.$$

6.3. Detector energy resolution

The pulse height H_0 of an ideal detector is proportional to the absorbed energy E_0:

$$H_0 = const \cdot E_0$$

In the case of real detectors, even when the absorbed energy is the same for all particles, detector pulse heights will not be exactly equal to each other. The relation written above only applies to the *average* pulse height. The heights of individual pulses are randomly distributed about the average height. That distribution (when E_0 is exactly defined) is called the detector ***response function*** corresponding to the particle energy E_0. It is usually Gaussian in shape:

$$G(H;E_0) = \frac{N_0}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(H - H_0(E_0))^2}{2\sigma^2}\right),$$

where H_0 is the average pulse height, σ is the standard deviation of pulse height, and N_0 is the total number of pulses. The width of the response function at half maximum (FWHM: "full width at half maximum") is the measure of the detector's ability to resolve two close peaks in the spectrum. In the case of a Gaussian response function, FWHM is related to σ as follows:

$$\Delta H = 2.35\sigma.$$

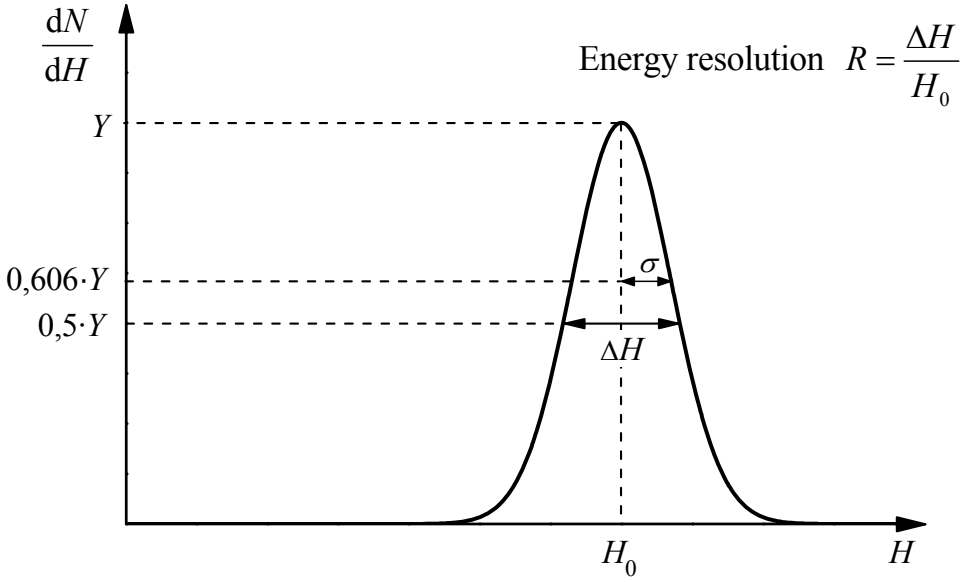


Fig. 6.3. Detector response function and definition of ***energy resolution***

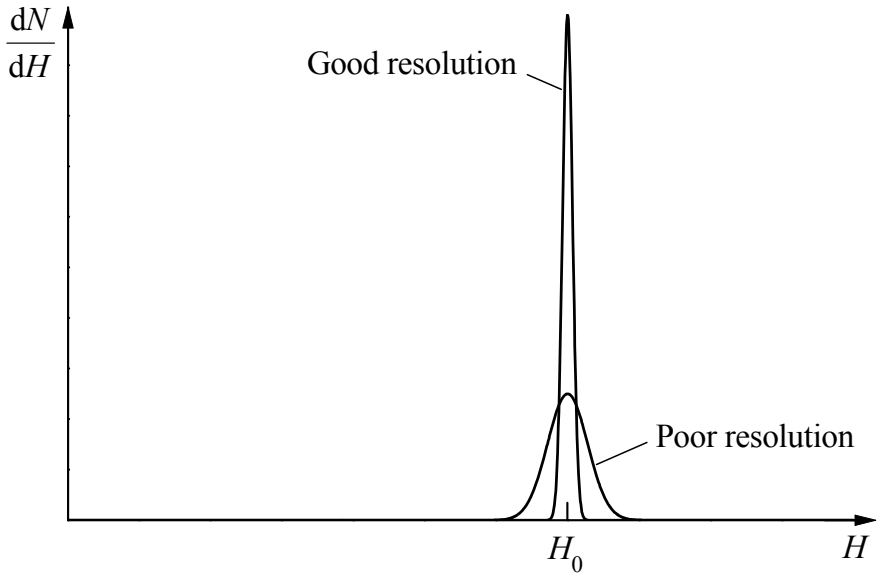


Fig. 6.4. Examples of response functions for detectors with relatively good resolution and relatively poor resolution

Additional reading for Lecture 3:
[1], p. 22 – 32, 129 – 131, 134 – 142.
[2], p. 378 – 380, 392 – 394, 192 – 204.
[3], p. 103 – 115.

References

1. Lilley J. Nuclear Physics: Principles and Applications. New York: John Wiley & Sons, 2001. 393 p.
2. Krane K. S. Introductory Nuclear Physics. New York: John Wiley & Sons, 1988. 845 p.
3. Knoll G. F. Radiation Detection and Measurement. 3rd Edition. New York: John Wiley & Sons, 2000. 802 p.