A sample laboratory report

Experiment No. 8

ARTIFICIAL RADIOACTIVITY

Name of the student:

Date the experiment was performed:

Objective of the experiment

Test the exponential law of radioactive decay, measure half-lives of radioactive nuclides from their decay curves, investigate the phenomenon of activity saturation.

Tasks

1. Measure the decay curve of a silver sample, initially consisting of 51,35 % $^{107}_{47}\mathrm{Ag}$ (silver-107) and 48,65 % $^{109}_{47}\mathrm{Ag}$ (silver-109) after its bombardment by neutrons (activation). Repeat those measurements 4 times, each one corresponding to a different activation time.
2. Plot the decay curves of the mixture of radioactive nuclides $^{108}\mathrm{Ag}$ (silver-108) and $^{110}\mathrm{Ag}$ (silver-110) and use those curves to calculate the half-life of $^{108}\mathrm{Ag}$.
3. Plot the activation curve of $^{108}\mathrm{Ag}$.
4. Using the calculated half-life of $^{108}\mathrm{Ag}$, isolate and plot the contribution of $^{110}\mathrm{Ag}$ to the overall decay curve and calculate the half-life of $^{110}\mathrm{Ag}$.
5. Discuss the results.

Theory

A "nucleus" is the central part of an atom, where most of the atom's mass is concentrated. The nucleus consists of positively charged protons (charge $+e$) and neutrons, which have zero electric charge. Protons and neutrons are collectively called "nucleons". Since the mass of the proton is very close to the mass of the neutron, the total number of nucleons in a nucleus is frequently called the "mass number" of the nucleus. If the net charge of an atom is zero, then the number of electrons in the atom is equal to the number of protons in the nucleus. This number is the same as the atomic number of the chemical element. The mass number (A) and the atomic number (Z) are usually specified to the left of the element symbol as indices, using the format $^{A}_{Z}\mathrm{X}$, where "X" is the element symbol, e.g., $^{12}_{6}\mathrm{C}$. Atomic species that have the same number of protons in the nucleus, but different numbers of neutrons, are called "isotopes" of that element. Atomic species that have the same mass number, but different number of protons (and neutrons), are called "isobars" (by this definition, all isobars are different chemical elements). Two atomic species that differ either in the composition or in energy of their nuclei are called different "nuclides".

Nuclear binding energy (B) of a given nucleus is defined as the total work that must be done by an external force to break the nucleus into its constituent nucleons. The relation of B to the mass of the nucleus follows from the well-known equivalence of mass (m) and energy (E):

$$E = mc^2, \tag{1}$$

where c is the speed of light. In this relation, E is the so-called "relativistic energy" of the nucleus, including both the rest energy and the kinetic energy, and m is the relativistic mass. If the nucleus is at rest, then m is equal to the rest mass (m_0), and E is equal to the rest energy (E_0). The rest energy is equal to the sum of rest energies of all nucleons, minus the binding energy:

$$E_0 = [Zm_\mathrm{p} + (A-Z)m_\mathrm{n}]c^2 - B. \tag{2}$$

Bearing in mind the mass-energy equivalence, equality (2) implies that the rest mass of a nucleus (m_0) is less than the sum of rest masses of all its nucleons:

$$m_0 = Zm_\mathrm{p} + (A-Z)m_\mathrm{n} - \Delta m, \tag{3}$$

where Δm is the so-called "mass defect" of the nucleus, which is equal to

$$\Delta m = \frac{B}{c^2}. \tag{4}$$

Radioactivity is a property of some nuclei to spontaneously transform into a different nuclide, emitting one or more particles. Such a transformation event (or a multitude of such events in a radioactive sample) is called "radioactive decay". The types of radioactive decay are classified according to the type of particles emitted during it. Most of the naturally occurring radioactive nuclei decay by emitting an α particle (a nucleus of helium-4, i.e., $^{4}_{2}\mathrm{He}$), or a β particle (i.e., either a free electron, which in this context is called a "β^- particle", or a positron, which in this context is called a "β^+ particle"). Those types of decay are called "alpha decay" or "beta decay", respectively. If a nucleus has an excess internal energy (in other words, if it is excited), then it may lose that energy by emitting it as a photon. Those photons are

called “gamma quanta”, and such radiation is called “gamma radiation”. Such transformation of a nucleus is sometimes called “gamma decay”, although it only involves a decrease of internal energy a nucleus, and does not involve a change of its composition. There are also other types of decays, which are only infrequently observed in nature (such as spontaneous fission), or which can be produced only in very short-lived artificial nuclides (such as emission of neutrons or protons). Any spontaneous transmutation of a nucleus is possible only when that transmutation causes a decrease of the total rest energy of the system. I.e., in a radioactive decay, the total rest mass of the decay products is always less than the rest mass of the parent nucleus. The decrease of the rest energy is equal to the kinetic energy of the decay products (i.e., the daughter nucleus and the particles emitted during the decay).

Due to radioactive decay, the number of atoms of a radioactive nuclide decreases exponentially with time:

$$N(t) = N_0 \mathrm{e}^{-\lambda t}, \tag{5}$$

where $N(t)$ is the number of atoms (nuclei) at time t, N_0 is the initial number of nuclei (at time $t = 0$), and λ is the decay constant. The decay constant determines the rate at which the number of radioactive nuclei decreases: the larger the decay constant, the faster the radioactive decay. Another way to specify the rate of decrease of the number of radioactive nuclei is by specifying the half-life, which is related to the decay constant as follows:

$$T_{1/2} = \frac{\ln 2}{\lambda}. \tag{6}$$

The half-life is the time needed for the number of radioactive nuclei to decrease by half.

The rate of decay (i.e., the number of decays per unit time) is called “activity” of the sample. Its unit of measurement is called “becquerel” ($1\ \mathrm{Bq} = 1\ \mathrm{s}^{-1}$). From Eq. (5) it follows that activity also decreases exponentially with time (if the radioactive decay is the only factor causing the change of N):

$$\Phi(t) \equiv -\frac{\mathrm{d}N}{\mathrm{d}t} = \lambda N_0 \mathrm{e}^{-\lambda t} = \lambda N(t). \tag{7}$$

Transmutation of a nucleus can also be induced artificially, by bombarding it with appropriate particles. In such case, this transmutation is called a “nuclear reaction”. Thus, the main difference between radioactive decay and a nuclear reaction is that radioactive decay is spontaneous, i.e., it involves only one primary particle (a single nucleus), whereas a nuclear reaction involves at least two primary particles (a nucleus and another particle that collides with it).

A nuclear reaction that is applied in this experiment is the so called “radiative neutron capture”, whose generalized equation is the following:

$${}^{A}_{Z}\mathrm{X} + {}^{1}_{0}\mathrm{n} \rightarrow {}^{A+1}_{Z}\mathrm{X}^{*} \rightarrow {}^{A+1}_{Z}\mathrm{X} + \gamma. \tag{8a}$$

This equation should be interpreted as follows. Due to interaction of the initial nucleus (${}^{A}_{Z}\mathrm{X}$) with the incident neutron (${}^{1}_{0}\mathrm{n}$), the neutron is captured into the nucleus, increasing its mass number by 1. The resulting nucleus is the so-called “compound nucleus”, which is a short-lived intermediate stage of the reaction. The compound nucleus is excited (this is denoted by the asterisk in its notation ${}^{A+1}_{Z}\mathrm{X}^{*}$). It emits the excess energy as gamma radiation (γ). Thus, the radiative neutron capture is a two-stage process: 1) formation of an excited compound nucleus; 2) gamma decay of the compound nucleus. When writing the equation of the radiative neutron capture, the initial stage of the reaction (formation of the compound nucleus) is usually omitted:

$${}^{A}_{Z}\mathrm{X} + {}^{1}_{0}\mathrm{n} \rightarrow {}^{A+1}_{Z}\mathrm{X} + \gamma. \tag{8b}$$

Another example of a nuclear reaction is the following reaction that is frequently used to produce neutrons (which subsequently can be used, e.g., in the reaction (8b)):

$${}^{9}_{4}\mathrm{Be} + {}^{4}_{2}\mathrm{He} \rightarrow {}^{12}_{6}\mathrm{C} + {}^{1}_{0}\mathrm{n}. \tag{9}$$

A typical neutron source, where the latter reaction takes place, is a mixture of metallic beryllium powder and a small amount of an alpha-radioactive material (e.g., plutonium isotope ${}^{239}\mathrm{Pu}$).

In nuclear and particle physics, probabilities of various nuclear reactions are frequently expressed in terms of their “cross sections”. A “cross section” of a given type of interaction (e.g., radiative neutron capture) can be visualized as a flat round region centered around the target nucleus and having an area (σ) such that the geometrical probability of an incident particle (e.g., a neutron) hitting this area is equal to the probability of this interaction event. If concentration of target nuclei is n, then their number inside a layer with surface area S and thickness $\mathrm{d}x$ is equal to $n \cdot S \cdot \mathrm{d}x$. If incident particles can hit any point on the

surface S with equal probability, then the probability of hitting any one of the mentioned circular regions is equal to their combined area ($\mathrm{d}S' = \sigma \cdot n \cdot S \cdot \mathrm{d}x$) divided by S (see Fig. 1):

$$\mathrm{d}P = \frac{\mathrm{d}S'}{S} = \sigma\, n\, \mathrm{d}x\,. \qquad (10)$$

Thus, the probability $\mathrm{d}P$ of a given type of interaction can be expressed in terms of its cross section σ.

In general, reaction cross section depends on energies of primary particles. In particular, cross sections of reactions caused by neutrons have the property that is usually called "$1/\upsilon$ law". This law states that cross section of a reaction caused by incident neutrons (e.g., radiative neutron capture) is inversely proportional to velocity (υ) of incident neutrons:

$$\sigma \sim \frac{1}{\upsilon}\,. \qquad (11)$$

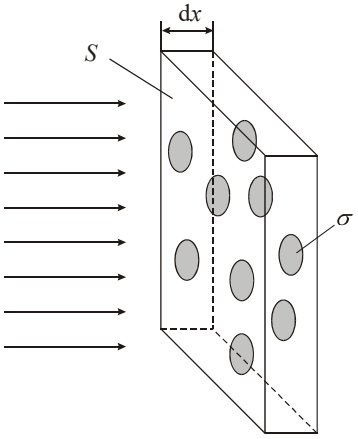

Fig. 1. For explanation of interaction cross-section σ

Neutrons that are produced in the reaction (9) have an average energy of the order of a few megaelectronvolts. In order to maximize the frequency of reaction events ("reaction rate"), neutrons have to be slowed down ("moderated"), so that their average kinetic energy becomes of the order of kT, where k is the Boltzmann constant, and T is the absolute temperature (at room temperature $kT \approx 0.025$ eV). This slowing down is achieved by surrounding the volume where the reaction takes place by a material containing a large quantity of hydrogen atoms (e.g., water or organic compounds such as paraffin). Since the mass of the proton is approximately the same as the mass of the neutron, it follows from the laws of conservation of energy and momentum that the neutron should transfer most of its initial kinetic energy to protons (the nuclei of hydrogen atoms) in just a few elastic collisions. [E.g., in the case of a single central collision the neutron would almost stop to a halt, whereas the proton would acquire almost all kinetic energy of the incident neutron.]

The secondary (or "daughter") nucleus ${}^{A+1}_{Z}\mathrm{X}$, which is formed as a result of the radiative neutron capture, is frequently radioactive and subsequently decays by emitting an electron. This is the so-called "β^- decay":

$${}^{A+1}_{Z}\mathrm{X} \rightarrow {}^{A+1}_{Z+1}\mathrm{Y} + e^- + {}^{0}_{0}\tilde{\nu}\,. \qquad (12)$$

Here, "e^-" denotes an electron, and "${}^{0}_{0}\tilde{\nu}$" denotes an antineutrino – a neutral particle with zero (or extremely small) rest mass, which is also emitted during β^- decay. We see that in β^- decay one neutron in the nucleus transforms into the proton. The reason why the most likely type of radioactive decay of a substance created by neutron capture is the β^- decay is the requirement that the proton-to-neutron ratio $Z / (A - Z)$ should change during the decay in the direction opposite to its change due to neutron capture. Since the capture of the neutron causes a decrease of this ratio, the decay must be such that $Z / (A - Z)$ increases. This means that Z must increase. The only type of decay that satisfies the latter requirement is the β^- decay.

In general, a process whereby a sample is made radioactive due to its bombardment by particles is called "activation". Although many types of particles can be used as projectiles during activation, neutrons are used much more often than other types of particles. During activation, the sample's activity increases because of constant production of the radioactive nuclide in it. When activation is terminated, the sample activity begins to decrease exponentially according to Eq. (7). A plot of the time dependence of the sample activity during activation is called "activation curve", and a plot of its decrease during radioactive decay (after terminating activation) is called "decay curve". A typical activation curve is shown in Fig. 2. This dependence can be derived from the differential equation

$$\frac{\mathrm{d}N}{\mathrm{d}t} = \sigma N_{\mathrm{p}} j - \lambda N\,, \qquad (13)$$

where N is the number of atoms of the daughter nuclide, N_{p} is the number of atoms of the parent nuclide, σ is the neutron capture cross section and j is the density of neutron flux (i. e. the number of neutrons per unit area per unit time). If we compare Eq. (13) with the law of radioactive decay (7), we can see that there is an additional term ($\sigma N_{\mathrm{p}} j$) on the right-hand side of the equation. This term reflects the fact that during the activation, the nuclei not only decay; they are also created due to neutron capture. Thus, there are two competing processes: an increase of the number of daughter nuclei due to neutron capture (this increase is reflected by the positive term $\sigma N_{\mathrm{p}} j$ in Eq. (13)), and the decrease of that number due to

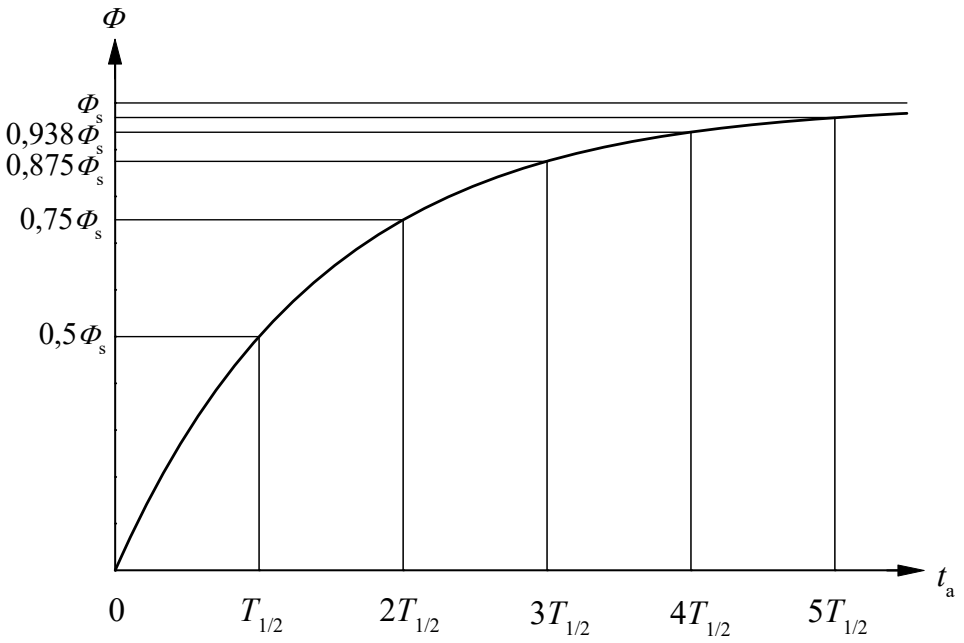

Fig. 2. Activation curve $\Phi = \Phi_s(1-e^{-\lambda t_a})$. t_a is activation time, Φ is activity of the sample, Φ_s is the saturation activity, λ is the decay constant

decay of the daughter nuclei (this decrease is reflected by the negative term $-\lambda N$). Since the number of nuclei that capture a neutron is many orders of magnitudes smaller than the number of parent nuclei N_p, the latter number can be assumed to be constant. Thus, the first term on the right-hand side of Eq. (13) is practically constant, whereas the absolute value of the second term increases with time (due to the increase of the number of daughter nuclei N). This means that the increase of N during activation slows down continuously and eventually the value of N stabilizes. This happens when the rate of nuclide production ($\sigma N_p j$) becomes equal to the rate of its decay (λN). This stable value of N is called the „saturation value“. The mathematical expression of dependence of N on t_a is obtained by solving the differential equation (13) with initial condition $N(0) = 0$. This solution is

$$N(t_a) = N_{sat}(1-e^{-\lambda t_a}), \tag{14}$$

where N_{sat} is the mentioned saturation value of the number of radioactive nuclei. The corresponding dependence of activity on activation time has the same shape:

$$\Phi(t_a) = \lambda N(t_a) = \lambda N_{sat}(1-e^{-\lambda t_a}). \tag{15}$$

This function is shown in Fig. 2. Obviously, the number of daughter nuclei (and activity) saturates after a time which is roughly equal to 4 – 5 half-lives.

Although, as mentioned, the decrease of the number of parent nuclei (N_p) due to activation is relatively small, it should be taken into account after a prolonged exposure to an intense neutron flux. Although the process of deactivation restores the initial zero activity of the sample, it does not restore the nuclei that participated in the neutron capture. As clear from the above, the two transformations of the nucleus, which are expressed by Eq. (8b) and Eq. (12), produce a different chemical element ($^{A+1}_{Z+1}Y$).

During this experiment, decay curves of a mixture of radioactive silver isotopes ^{108}Ag and ^{110}Ag are measured. Those isotopes are produced in the following two nuclear reactions:

$$^{107}_{47}Ag + {}^{1}_{0}n \rightarrow {}^{108}_{47}Ag + \gamma, \tag{16a}$$

$$^{109}_{47}Ag + {}^{1}_{0}n \rightarrow {}^{110}_{47}Ag + \gamma, \tag{16b}$$

which occur when a sample of natural silver (51.35 % $^{107}_{47}$Ag and 48.65 % $^{109}_{47}$Ag) is exposed to neutrons. The reaction products $^{108}_{47}$Ag and $^{110}_{47}$Ag are β^- radioactive. I. e., they subsequently decay as follows:

$$^{108}_{47}Ag \underset{142s}{\rightarrow} {}^{108}_{48}Cd + e^- + \bar{\nu}_e, \tag{17a}$$

$$^{110}_{47}Ag \underset{24,6s}{\rightarrow} {}^{110}_{48}Cd + e^- + \bar{\nu}_e \tag{17b}$$

(the values of the half-life of both decays are indicated below the arrows). Due to the significant difference of the two decay half-lives, each decay curve consists of two clearly defined regions – the initial region of fast decrease of ^{110}Ag activity (the "fast component"), followed by a slower decrease of ^{108}Ag activity (the "slow component").

List of Equipment Used

The equipment consists of the following components:

1) Geiger-Müller counter,
2) **Isotrak** ratemeter,
3) personal computer,
4) Pu-Be neutron source. The source is at the bottom of a steel container, which is filled with neutron moderator – paraffin,
5) a sample of natural silver.

Procedure

1. The silver sample was inserted into the container with the neutron source.
2. During the activation (while the sample was being bombarded with neutrons), background radiation was measured.
3. After the predefined activation time (10 min, 5 min, 2 min or 1 min), the sample was extracted from the neutron source and placed upon the counter, whereupon its decay curve was measured for 10 minutes. During the first 2 minutes, the counts were recorded every 10 seconds. During the last 8 minutes, the counts were recorded every 60 seconds.
4. The sample was placed away from the counter for complete deactivation (additional 10 minutes). This time was again used to measure the background.

The above procedure was repeated four times with different activation times. At the end of the measurements, the table with data of four decay curves and the average background count rate was printed.

Measurement Data

The "raw" decay curve data is given in the table below. Duration of activation is denoted "t_a". The four columns with title "Time (s)" gives the time elapsed since the end of activation.

Table 1. Decay curve data

t_a = 10 min		t_a = 5 min		t_a = 2 min		t_a = 1 min	
Time (s)	Count	Time (s)	Count	Time (s)	Count	Time (s)	Count
19.265	209	14.806	192	15.617	198	13.597	195
29.201	165	24.740	177	25.762	167	23.733	134
39.338	128	34.883	134	35.702	124	33.674	105
49.272	106	44.815	101	45.842	99	43.601	79
59.211	88	54.754	92	55.776	91	53.741	63
69.358	74	64.895	75	65.715	66	63.679	45
79.291	73	74.833	72	75.856	57	73.825	50
89.431	55	84.769	61	85.787	51	83.766	26
99.371	50	94.908	56	95.927	42	93.696	28
109.303	61	104.840	35	105.867	34	103.840	25
119.450	35	114.981	34	115.806	26	113.780	17
129.383	39	124.920	16	125.947	29	123.707	11
189.405	180	184.945	191	185.967	141	183.950	107
249.647	148	244.974	125	245.996	106	243.968	103
309.666	140	305.004	104	306.025	96	303.997	78
369.695	126	365.245	97	366.267	80	364.027	80
429.938	94	425.264	100	426.286	61	424.268	70
489.956	90	485.294	88	486.315	70	484.288	77
549.985	98	545.536	68	546.345	77	544.317	72
610.227	82	605.555	72	606.374	70	604.347	84

The average background radiation count was 1.117 ± 0.022 particles per second, or 67.0 particles per minute.

Analysis of Data

The decay curves were analyzed using the method of nonlinear least squares fitting. Fitting was done using the following sequence of steps:

I. The second part of each decay curve, corresponding to 60 s-long measurement, was fitted by a single exponential function $C_1 e^{-\lambda_1 t}$ with *two* varied parameters (C_1 and λ_1).

II. An average $\langle\lambda_1\rangle$ was calculated.

III. The second part of each decay curve, corresponding to 60 s-long measurement, was again fitted by a single exponential function $C_1 e^{-\langle\lambda_1\rangle t}$ with *one* varied parameter C_1.

IV. For each decay curve, the exponential function $C_1 e^{-\langle\lambda_1\rangle t}$ was subtracted from the initial part, which corresponds to 10 s-long measurements. Before this operation, the pre-exponential factor C_1 was recalculated using this equation:

$$C_1' = \frac{e^{\langle\lambda_1\rangle\cdot 10\,\mathrm{s}} - 1}{e^{\langle\lambda_1\rangle\cdot 60\,\mathrm{s}} - 1} C_1 . \tag{18}$$

V. Since only the fast component remains after the mentioned subtraction, it was fitted by a single exponential function $C_2 e^{-\lambda_2 t}$ (with two varied parameters) for each decay curve.

Those calculations were done using the graphing and data analysis program **Origin 6.1** and the customized Origin template file provided at the laboratory. The graphs with measurement data (after subtracting the average background count and after subtracting the contribution of the slow component at $t < 120$ s) and the theoretical fitting curves are presented in Fig. 3 – Fig. 7. The least squares estimate of the decay constant of ^{108}Ag (calculated by the fitting program) is

$$\langle\lambda_1\rangle = (0.00560 \pm 0.00055)\ \mathrm{s}^{-1},$$

where the number after the symbol "±" is the estimate of the standard error of $\langle\lambda_1\rangle$.

The weighted average value $\langle\lambda_2\rangle$ of the decay constant of ^{110}Ag was calculated as follows:

$$\langle\lambda_2\rangle = \frac{1}{D_2}\left[\frac{\lambda_2^{(1)}}{(\Delta\lambda_2^{(1)})^2} + \frac{\lambda_2^{(2)}}{(\Delta\lambda_2^{(2)})^2} + \frac{\lambda_2^{(3)}}{(\Delta\lambda_2^{(3)})^2} + \frac{\lambda_2^{(4)}}{(\Delta\lambda_2^{(4)})^2}\right], \tag{19}$$

where $\Delta\lambda_2^{(i)}$ is the standard error of the parameter λ_2 obtained by fitting the fast initial part of the i-th decay curve (i = 1, 2, 3, 4), and the coefficient D_2 is defined as follows:

$$D_2 \equiv \frac{1}{(\Delta\lambda_2^{(1)})^2} + \frac{1}{(\Delta\lambda_2^{(2)})^2} + \frac{1}{(\Delta\lambda_2^{(3)})^2} + \frac{1}{(\Delta\lambda_2^{(4)})^2} . \tag{20}$$

The values of $\lambda_2^{(i)}$ and $\Delta\lambda_2^{(i)}$ are given in the legend of graphs presented in Fig. 3 – Fig. 6. The estimate of the standard error of the weighted average $\langle\lambda_2\rangle$ is

$$\Delta\langle\lambda_2\rangle = 1/\sqrt{D_2} . \tag{21}$$

By using equations (19) – (21), the following values were obtained:

$$D_2 = 2.678 \times 10^5\ \mathrm{s}^2, \quad \langle\lambda_2\rangle = (0.0327 \pm 0.0019)\ \mathrm{s}^{-1} . \tag{22}$$

The half-lives of ^{108}Ag and ^{110}Ag (denoted T_1 and T_2, respectively) were calculated using Eq. (6). The estimate of the standard error of the half-life was calculated as follows:

$$\Delta T_{1,2} = \frac{\ln 2}{\langle\lambda_{1,2}\rangle^2}\Delta\langle\lambda_{1,2}\rangle . \tag{23}$$

The resulting estimates of T_1 and T_2 and their standard errors are given below:

T_1 = (123.8 ± 12.2) s,
T_2 = (21.21 ± 1.25) s.

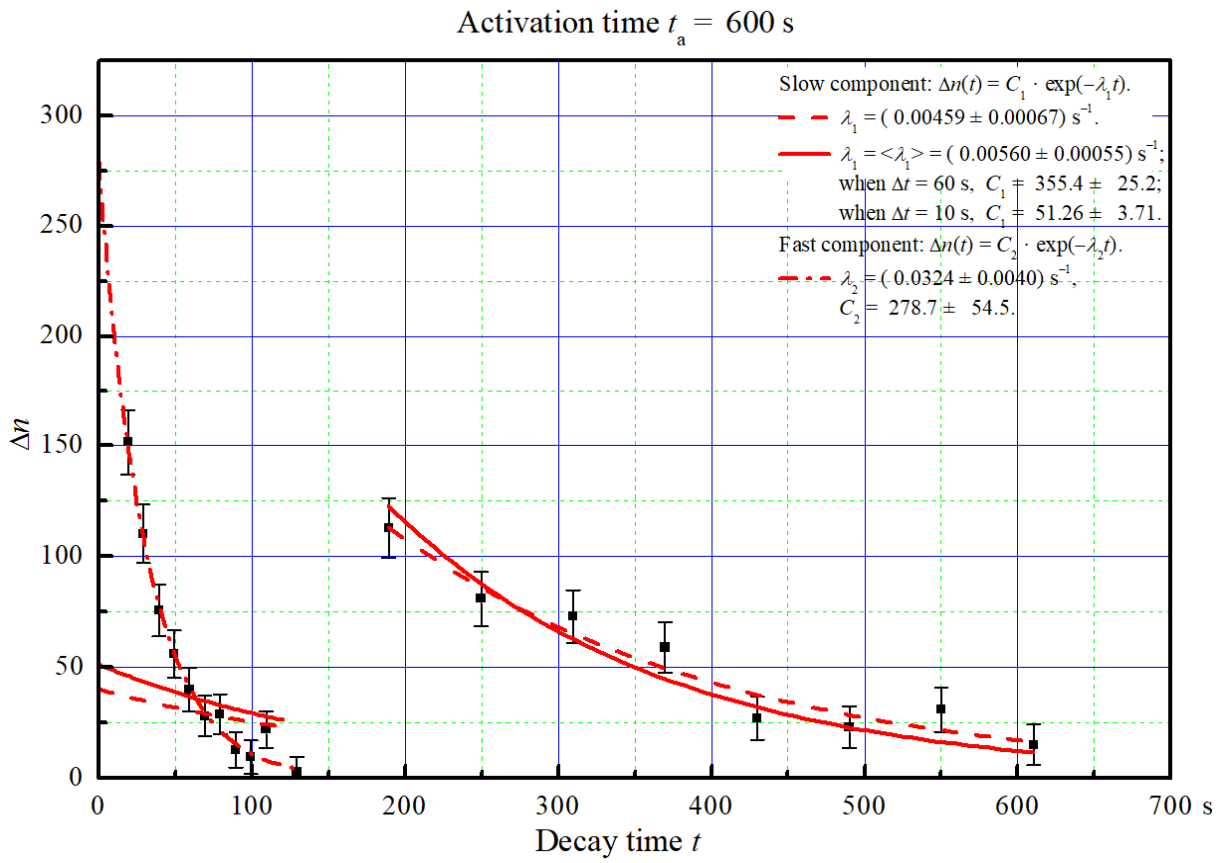

Fig. 3. The decay curve corresponding to the 10-minute long activation and the results of nonlinear fitting

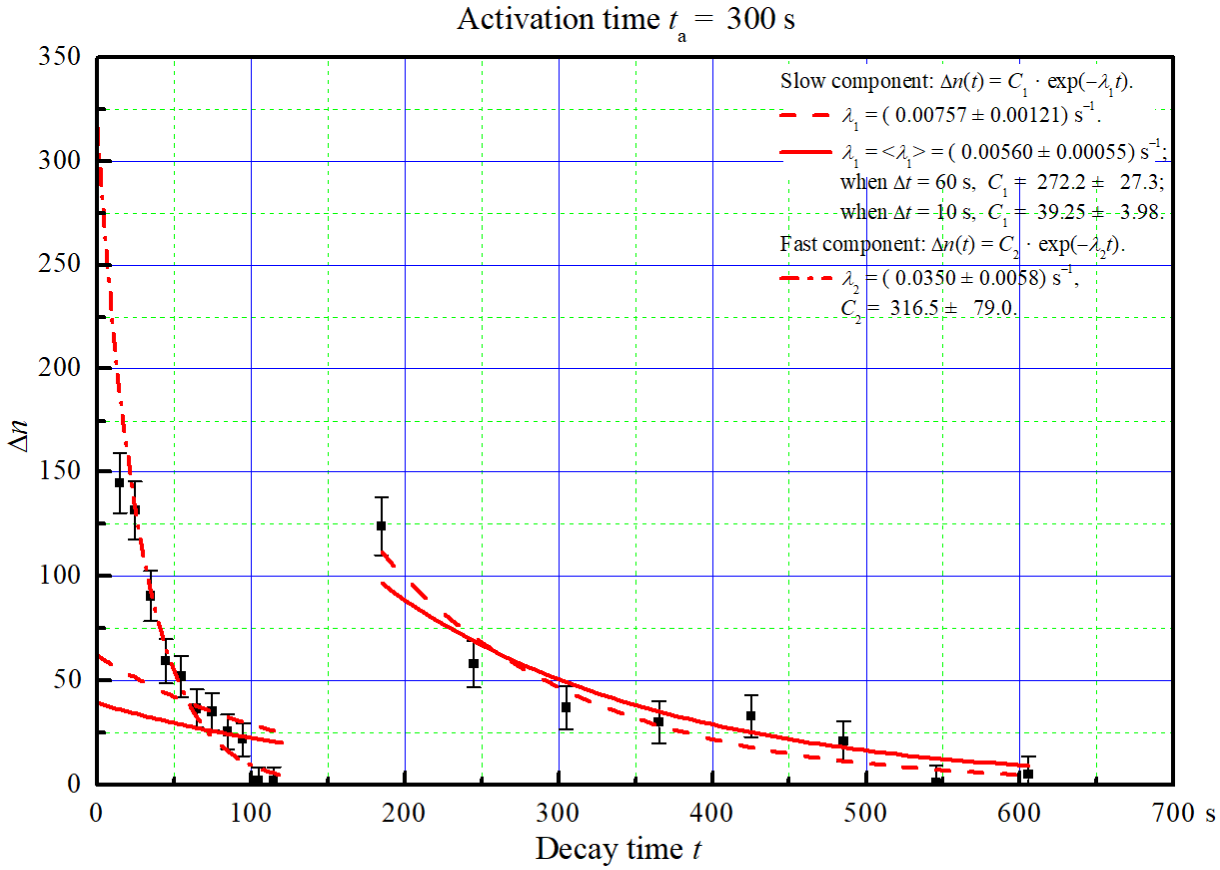

Fig. 4. The decay curve corresponding to the 5-minute long activation and the results of nonlinear fitting

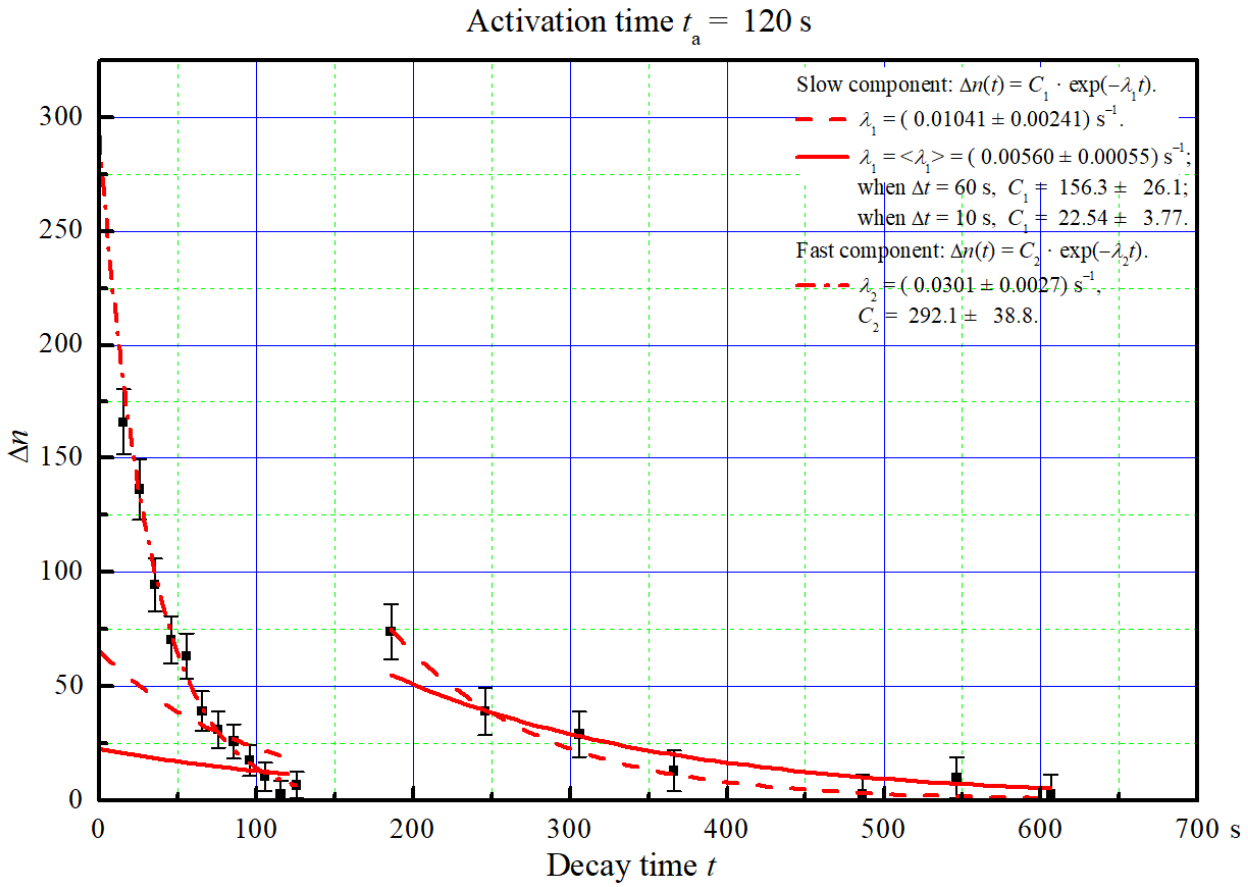

Fig. 5. The decay curve corresponding to the 2-minute long activation and the results of nonlinear fitting

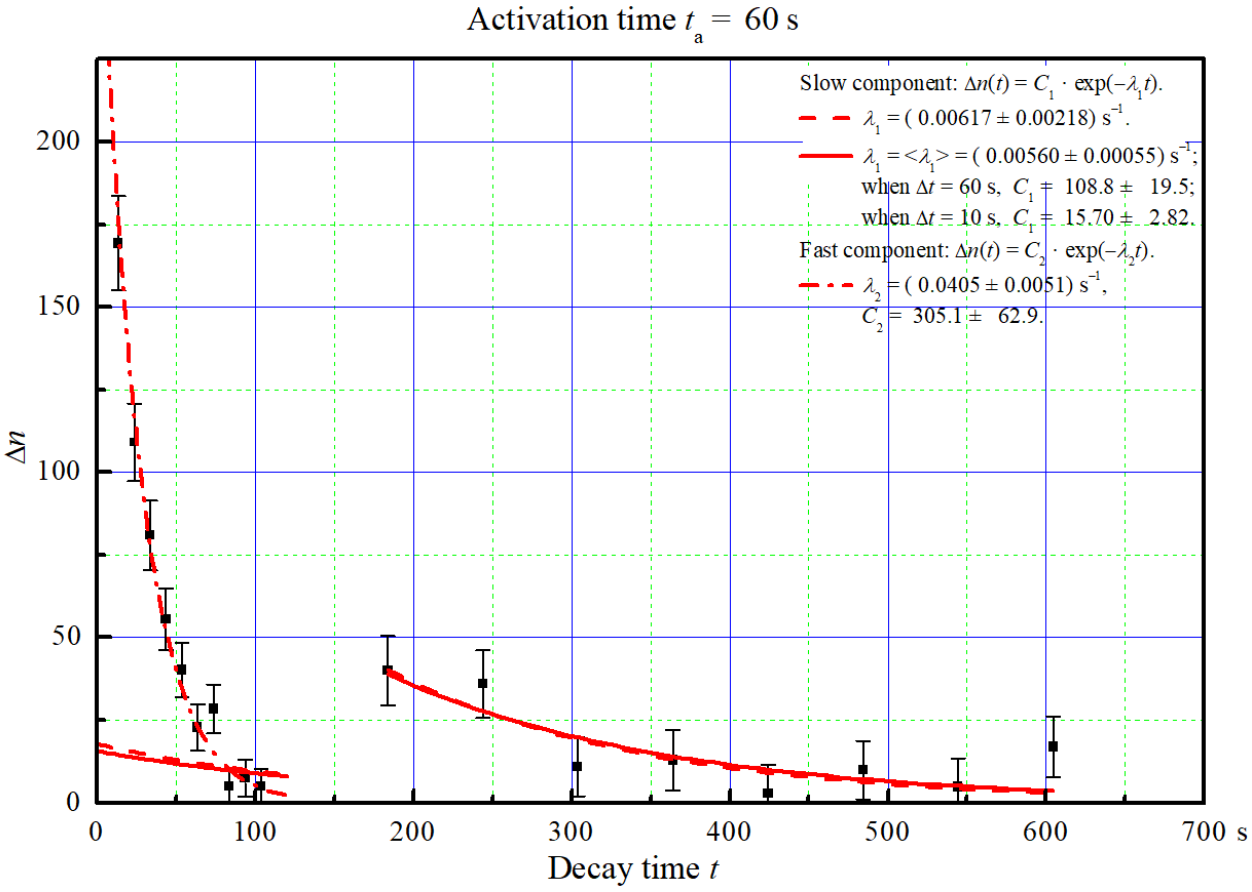

Fig. 6. The decay curve corresponding to the 1-minute long activation and the results of nonlinear fitting

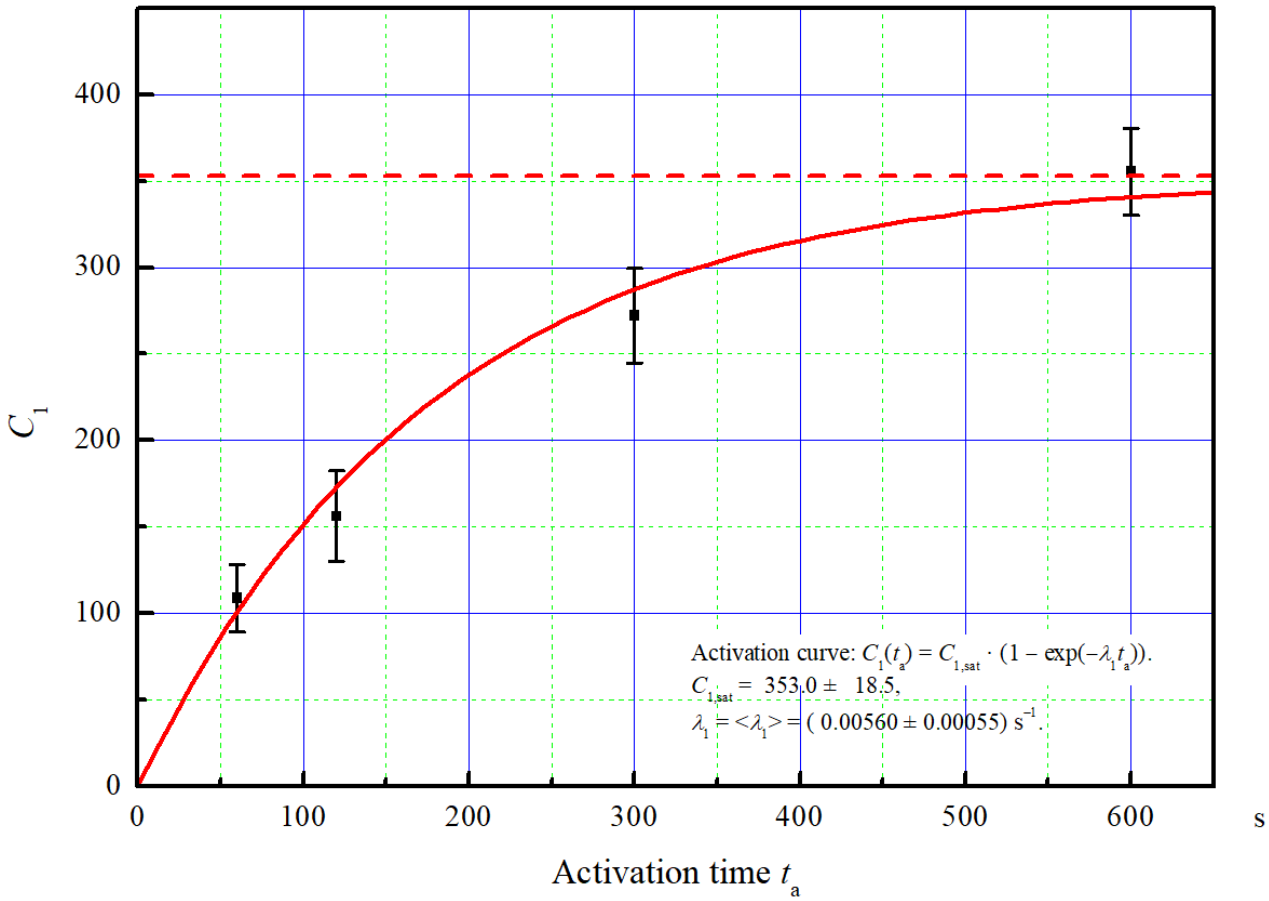

Fig. 7. Activation curve corresponding to the isotope ^{108}Ag (scatter plot), the fitted line (solid red line) and the fitted saturation value of the pre-exponential factor C_1 (horizontal dashed line).

Discussion of Results

The measured decay curves, which are presented in Fig. 3 – Fig. 6, are approximately exponential. This conclusion follows from comparison of experimental data (scatter plots) with theoretical curves (lines), which are known to be exponential. Thus, in this respect the experiment is in accord with the exponential decay law given by Eq. (7). The deviations of measurement data from the fitted curves increase with decreasing activation time, which was expected, because a decrease of activation time causes a decrease of the sample activity at the end of activation and a corresponding increase of random relative errors of activity measurement. According to Poisson statistics, the standard relative error of activity measurement is approximately equal to the inverse square root of the total number of particles detected during the activity measurement. However, in this experiment a major cause of errors is background radiation. The data presented in Table 1 and the value of the background count rate (which is given below Table 1) indicate that the background radiation is the main contributor to the measured particle count when the time elapsed since the end of activation exceeds approximately 5 minutes, and consequently the background radiation is the main source of random fluctuations of measurement data in the mentioned time range. A decrease of the sample activity causes an increase of the effect of background radiation on measurement accuracy, i.e., it becomes more difficult to separate the decay curve from the background "noise".

By comparing the decay curves corresponding to different activation times, it can be seen that variation of the initial part (the fast component) of the decay curves with activation time is much less pronounced than the variation of the slow component. The same conclusion follows from comparison of the values of pre-exponential factors C_1 and C_2, which correspond to different activation times: the factor C_1, which is proportional to activity of ^{108}Ag at the end of activation, varies much more significantly than the factor C_2, which is proportional to activity of ^{110}Ag at the end of activation (see the numbers in the legend in Fig. 3 – Fig. 6). C_2 seems to be approximately constant (apart from random variations). This indicates that activity of ^{110}Ag is practically equal to the maximum ("saturation") value in all four cases, which is in accord with the fact that the half-life of ^{110}Ag (24.6 s) is much less than the shortest activation

time (1 min). The variation of C_1 with activation time is shown in Fig. 7. Again, we can see good agreement with the theoretical activation curve, which is given by Eq. (15).

The estimates of the two half-live are close to the true values, which are given in Eq. (17a) and (17b). In particular, the true half-life of ^{108}Ag (142 s) belongs to the 95 % confidence interval, whose endpoints are (123.8 ± 24.4) s, although it does not belong to the 68 % confidence interval, whose endpoints are (123.8 ± 12.2) s. One can conclude that the observed difference of the experimental and true half-life of ^{108}Ag is probably mostly accidental, and it is caused by random nature of radioactive decay. The true half-life of ^{110}Ag (24.6 s) does not belong to the 95 % confidence interval, whose endpoints are (21.2 ± 2.5) s. It follows that the experimental value of the half-life of ^{110}Ag is probably distorted by systematic errors (also called "bias"). One possible reason of these errors is the fact that the fitted decay curve for ^{110}Ag is obtained after subtraction of the fitted decay curve for ^{108}Ag from the initial data points (in the range $t < 120$ s). The latter decay curve is slightly overestimated at the smallest times due to the overestimation of the decay constant λ_1 of ^{108}Ag. This causes an overestimation of the relative rate of decrease of the curve that is obtained as a result of the mentioned subtraction, i.e., an overestimation of the decay constant λ_2 of ^{110}Ag and a consequent underestimation of T_2. Since the same (average) fitted value of λ_1 is used in all four cases, the same effect of underestimating T_2 is observed for each of the four activations, causing the overall bias towards smaller values of T_2. However, as follows from this explanation, the main reason of this "bias" in the value of T_2 is ultimately the same one that causes the error of T_1. I.e., it is probably caused by the random nature of radioactive decay.

The mentioned errors of the values of T_1 and T_2 could be decreased by increasing the beta activity of the silver sample, or by increasing the absolute efficiency of the detector, and by decreasing the background. The sample activity could be increased by using a neutron source with larger neutron flux, or by using a larger sample, or by optimizing the sample position near the neutron source during activation. The absolute efficiency of the detector could be increased by optimizing the sample position near the detector. The background count rate could be decreased by shielding the detector and sample from external radiation sources during measurement of decay curves.

Conclusions

1. It is possible to perform accurate measurements of decay half-life by weighted nonlinear least squares fitting of decay curves consisting of more than one exponential segments, when the measured half-lives are in the range (10 – 200) s. Presumably, this range could be extended to shorter times (down to several milliseconds) and to longer times (up to a year). The errors are mostly a result of Poisson statistics and they can be decreased by optimizing the measurement geometry and using detector shielding, or by using a neutron source with larger neutron flux.

2. The induced activity of the sample that is being bombarded with neutrons saturates during the time that exceeds the value of the half-life by a factor of approximately 4.

References

Lilley J. Nuclear Physics: Principles and Applications. New York: John Wiley & Sons, 2001.