Vilnius University
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Experiment No. 10

ATTENUATION OF GAMMA RAYS

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Contents

The aim of the experiment

Measure attenuation curves of gamma radiation with various photon energies, using various absorbers. Observe regularities of interaction of gamma radiation with matter: 1) the exponential attenuation law, 2) the decrease of the attenuation coefficient with increasing energy of gamma quanta, 3) the increase of the attenuation coefficient with increasing atomic number of the absorber, 4) the proportionality of the attenuation coefficient corresponding to Compton scattering to the atomic number of the absorber.

1. Tasks

1. Measure attenuation curves of gamma radiation emitted by nuclides ^{137}Cs (cesium-137) and ^{60}Co (cobalt-60), using aluminum, iron and lead as absorbers.
2. Plot the attenuation curves.
3. Determine the attenuation coefficients and atomic interaction cross sections.
4. Test the validity of the exponential attenuation law.
5. Determine the direction of change of the attenuation coefficient when the photon energy or the atomic number of the absorber is increased.
6. By comparing the mass attenuation coefficients, determine, if possible, the dominant interaction mechanism.

2. Control questions

1. Define the main processes of interaction between gamma radiation and matter: Compton scattering, photoelectric absorption and pair creation.
2. Explain the concept of interaction cross section.
3. Derive the exponential attenuation law. Define the concept of the attenuation coefficient. Define the components of the attenuation coefficient corresponding to different interactions.
4. What is the general shape of the dependence of cross sections of Compton scattering and photoelectric absorption on the gamma quanta energy and on the atomic number of the absorber?

Recommended reading:

1. Krane K. S. Introductory Nuclear Physics. New York: John Wiley & Sons, 1988. p. 198 – 204, 217 – 220, 392 – 394.
2. Lilley J. Nuclear Physics: Principles and Applications. New York: John Wiley & Sons, 2001. p. 24 – 25, 136 – 142.
3. Knoll G. F. Radiation Detection and Measurement. 3rd Edition. New York: John Wiley & Sons, 2000. p. 48 – 55.

3. The types of ionizing radiation

Ionizing radiation is a flux of subatomic particles (e. g. photons, electrons, positrons, protons, neutrons, nuclei, etc.) that cause ionization of atoms of the medium through which the particles pass. ***Ionization*** means the removal of electrons from atoms of the medium. In order to remove an electron from an atom, a certain amount of energy must be transferred to the atom. According to the law of conservation of energy, this amount of energy is equal to the decrease of kinetic energy of the particle that causes ionization. Therefore, ionization becomes possible only when the energy of incident particles (or of the secondary particles that may appear as a result of interactions of incident particles with matter) exceeds a certain threshold value – the ***ionization energy*** of the atom. The ionization energy is usually of the order of 10 eV (1 eV = $1{,}6022 \cdot 10^{-19}$ J).

Ionizing radiation may be of various nature. The ***directly ionizing radiation*** is composed of high-energy charged particles, which ionize atoms of the material due to Coulomb interaction with their electrons. Such particles are, e. g., high-energy electrons and positrons (beta radiation), high-energy ^{4}He nuclei (alpha radiation), various other nuclei. ***Indirectly ionizing radiation*** is composed of neutral particles which do not directly ionize atoms or do that very infrequently, but due to interactions of those particles with matter high-energy free charged particles are occasionally emitted. The latter particles directly ionize atoms of the medium. Examples of indirectly ionizing radiation are high-energy photons (ultraviolet, X-ray and gamma radiation) and neutrons of any energy. Particle energies of various types of ionizing radiation are given in the two tables below.

Table 1. The scale of wavelengths of electromagnetic radiation

Spectral region	Approximate wavelength range	Approximate range of photon energies
Radio waves	100000 km – 1 mm	$1 \cdot 10^{-14}$ eV – 0,001 eV
Infrared rays	1 mm – 0,75 μm	0,001 eV – 1,7 eV
Visible light	0,75 μm – 0,4 μm	1,7 eV – 3,1 eV
Ionizing electromagnetic radiation:		
Ultraviolet light	0,4 μm – 10 nm	3,1 eV – 100 eV
X-ray radiation	10 nm – 0,001 nm	100 eV – 1 MeV
Gamma radiation	< 0,1 nm	> 10 keV

Table 2. Particle energies corresponding to ionizing radiation composed of particles of matter

Radiation type	Approximate range of particle energies
Alpha (α) particles (^{4}He nuclei)	4 MeV – 9 MeV
Beta (β) particles (electrons and positrons)	10 keV – 10 MeV
Thermal neutrons	< 0,4 eV
Intermediate neutrons	0,4 eV – 200 keV
Fast neutrons	> 200 keV
Nuclear fragments and recoil nuclei	1 MeV – 100 MeV

The mechanism of interaction of particles with matter depends on the nature of the particles (especially on their mass and electric charge). According to the manner by which particles interact with matter, four distinct groups of particles can be defined:

1) heavy charged particles (such as alpha particles and nuclei),

2) light charged particles (such as electrons and positrons),

3) photons (neutral particles with zero rest mass),

4) neutrons (neutral heavy particles).

This experiment concerns only the third mentioned type of particles (gamma radiation).

4. Interaction of gamma radiation with matter

As in the case of charged particles (e. g., electrons, protons, alpha particles), interaction of photons of gamma radiation with matter is of electromagnetic nature. However, the exact physical mechanism of that interaction is quite different than in the case of charged particles, because:

1) Photons do not have electric charge, therefore they do not participate in Coulomb interaction. Photon interaction cross section is much smaller than interaction cross sections of charged particles.
2) The photon rest mass is zero, therefore their velocity is always equal to the velocity of light. I. e., photons can not be slowed down in matter (unlike charged particles). Photons can be only scattered or absorbed.

Photon ***absorption*** is an interaction process when the photon disappears and all its energy is transferred to atoms of the material or to secondary particles. Photon ***scattering*** is an interaction process when the photon does not disappear, but changes direction of its propagation. In addition, the scattered photon may transfer a part of its energy to an atom or an electron of the material. There are two interaction processes whereby a photon is absorbed and several types of scattering (of which one type is much more important than the others). Those interaction processes are defined below.

4.1. *The concept of interaction cross section*

In nuclear and particle physics, probabilities of various interactions between particles are frequently expressed in terms of their "cross sections". A "cross section" of a given type of interaction (e.g., photon scattering) can be visualized as a flat round region centered on the target particle (in the case discussed – on the target atom) and having an area (σ) such that the geometrical probability of an incident particle (e.g., a photon) hitting that area is equal to the probability of that interaction event. If concentration of target atoms is n_a, then their number inside a layer with surface area S and infinitesimal thickness dx is equal to $n_a \cdot S \cdot \mathrm{d}x$. If incident particles can hit any point on the surface S with equal probability, then the probability of hitting any one of the mentioned circular regions is equal to their combined area ($\mathrm{d}S' = \sigma \cdot n_a \cdot S \cdot \mathrm{d}x$) divided by S (see Fig. 1):

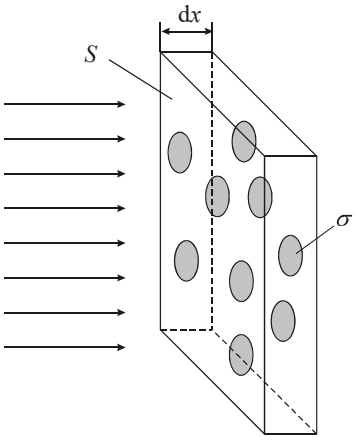

Fig. 1. For explanation of interaction cross section σ

$$\mathrm{d}P = \frac{\mathrm{d}S'}{S} = \sigma\, n_a\, \mathrm{d}x\,. \qquad (4.1)$$

Thus, the probability dP of a given type of interaction can be expressed in terms of its cross section σ, which acts as an "effective" cross-sectional area of the target atom for a given type of interaction (the cross section of a particular type of interaction may be much less or much greater than the actual geometric cross-sectional area of the atom).

4.2. *Compton scattering*

From the quantum mechanical point of view, a scattering event is a collision of two particles – a photon and an electron or a photon and an atom. From the laws of conservation of energy and momentum it follows that due to scattering by electrons of the material photon energy must decrease (because a part of that energy must be transferred to the electrons). This effect, which was first described in 1922 by American physicist A. Compton, became one of the cornerstones of quantum mechanics, because it proved that electromagnetic radiation under certain circumstances behaves like particles. Such type of scattering, when photon energy decreases, is called ***Compton scattering***. When photon energy is large (of the order of 10 keV or more), Compton scattering is the dominant scattering mechanism.

Since a single Compton scattering event is a result of photon's interaction with a single electron, the *atomic* Compton scattering cross section σ_C is equal to the *electronic* Compton scattering cross section σ_{Ce} times the number of electrons in an atom (the latter number is equal to the atomic number Z):

$$\sigma_C = Z\sigma_{Ce}\,. \qquad (4.2)$$

By definition, σ_{Ce} does not depend on Z. Thus, the atomic Compton scattering cross section is directly proportional to the atomic number of the material. When the photon energy is sufficiently large (of the order of 100 keV or larger), σ_{Ce} decreases with increasing photon energy.

4.3. *Photoelectric absorption*

Photoelectric absorption is a type of interaction of a photon with an atom when the atom absorbs all energy of the photon (i. e. the photon disappears) and one of atomic electrons is removed from the atom. This electron is called the photoelectron. Photoelectric absorption is also called "photoionization" or "internal photoelectric effect" (the word "internal" is used in order to avoid confusion with the "external" photoelectric effect, which is emission of electrons when electromagnetic radiation hits a material). The atomic cross section of photoelectric absorption is characterized by an especially strong dependence on the atomic number Z of the material and on photon energy. When photon energy is of the order of 100 keV, the mentioned cross section is approximately equal to

$$\sigma_f \approx 10^{-37} Z^5 / (h\nu)^{7/2}, \tag{4.3}$$

where the cross section σ_f is expressed in m^2, and $h\nu$ is the photon energy in MeV (h is the Planck constant, and ν is the photon frequency, which is the same as frequency of the incident electromagnetic wave). From Eq. (4.3) it follows that photoelectric absorption cross section rapidly increases with increasing atomic number Z and decreasing photon energy $h\nu$.

4.4. *Electron-positron pair production*

In the electric field of an atomic nucleus, a photon may stop existing by transforming all its energy into relativistic energy of two new particles – a free electron and a positron (electron's antiparticle). Since the recoil energy of the nucleus is relatively small, the law of conservation of energy during such an event can be written as follows:

$$h\nu = m_+c^2 + m_-c^2, \tag{4.4}$$

where m_+c^2 are m_-c^2 the total relativistic energies of the positron and the electron (m_+ and m_- are the total relativistic masses of the positron and the electron). Since m_+ and $m_-$are always larger than the electron's rest mass m_0, from (4.4) it follows that pair production is only possible when photon energy is larger than two rest energies of an electron: $2m_0c^2 \approx 1.02$ MeV. This is the so-called "threshold energy" of pair production. Although pair production becomes possible when photon energy exceeds the mentioned threshold value, the pair production cross section σ_p exceeds the Compton scattering cross section σ_C only when the photon energy approaches and exceeds 10 MeV. At photon energies less than 3 MeV, the frequency of pair production events is much smaller than the frequency of Compton scattering events.

Since the pair production becomes possible only at energies greater than $2m_0c^2$, the pair production cross section is exactly zero when $h\nu < 2m_0c^2$. When photon energy exceeds the mentioned "threshold" value of 1.02 MeV, the pair production cross section starts growing rapidly, but eventually this growth slows down and the cross section stabilizes ("saturates"). This saturation corresponds to photon energies satisfying the condition $h\nu >> 137m_0c^2Z^{-1/3}$. The maximum value of the pair production cross section is equal to

$$(\sigma_p)_{max} \approx 1{,}9 \cdot 10^{-31} Z^2 \ln(183Z^{-1/3}) \text{ [m}^2\text{]}. \tag{4.5}$$

4.5. *The attenuation coefficient*

The total cross section of interaction of a gamma radiation photon with an atom is equal to the sum of all three mentioned partial cross sections:

$$\sigma = \sigma_C + \sigma_f + \sigma_p. \tag{4.6}$$

Depending on the photon energy and the absorber material, one of the three partial cross sections may become much larger than the other two. Then the corresponding interaction process is the dominant one. Fig. 2 shows the intervals of photon energy $h\nu$ and atomic number Z corresponding to the case when one of the three interaction processes dominates. Obviously, the photoelectric absorption dominates at small values of photon energy, Compton scattering dominates at intermediate energies, and pair production dominates at high energies. The width of the energy interval corresponding to the Compton effect increases with decreasing atomic number of the material.

Using the definition of interaction cross section (see Section 4.1), it is easy to derive the dependence of gamma radiation intensity on thickness of absorber material. If a narrow parallel beam of

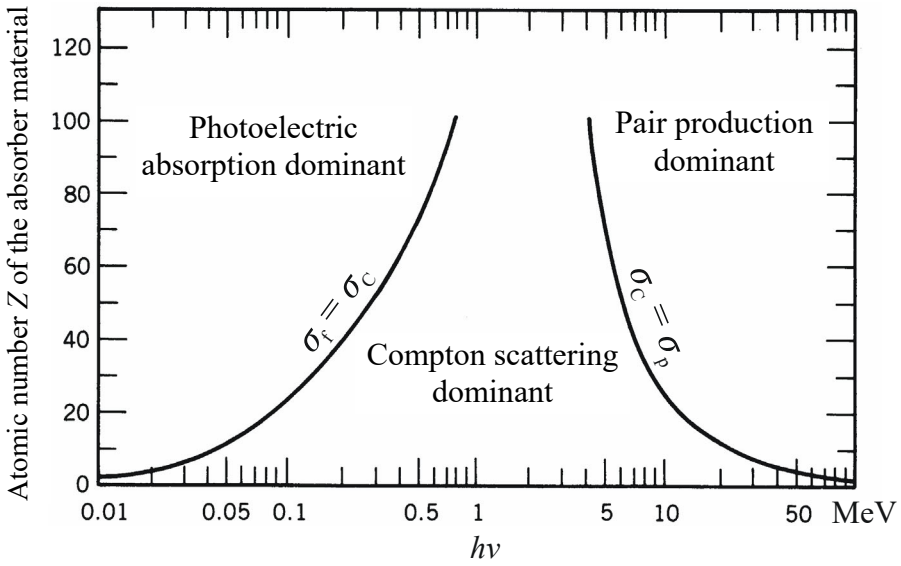

Fig. 2. The relative importance of various processes of gamma radiation interaction with matter (from [1])

gamma radiation falls normally upon a layer of absorber material, and if the detector detects only the photons that passed through this layer without any kind of interaction with its material (see Fig. 3), then the interaction probability dP on the left-hand side of Eq. (4.1) can be interpreted as the relative decrease of the photon count rate I (also called "intensity") after passing a layer with thickness dx:

$$\mathrm{d}P = -\frac{\mathrm{d}I}{I}, \tag{4.7}$$

where $-\mathrm{d}I$ is the absolute decrease of radiation intensity. After substituting this expression for dP in Eq. (4.1) and integrating, the dependence of radiation intensity I on the layer thickness x is obtained. This dependence is exponential:

$$I(x) = I_0 \mathrm{e}^{-\sigma \cdot n_\mathrm{a} \cdot x}, \tag{4.8}$$

where I_0 is the intensity of the incident beam at the layer surface ($x = 0$), and n_a is the atomic concentration in the material of the layer. This equality can be written as follows:

$$I(x) = I_0 \mathrm{e}^{-\mu x}, \tag{4.9}$$

where μ is the ***attenuation coefficient***:

$$\mu = \sigma \cdot n_\mathrm{a}. \tag{4.10}$$

The exponential function (4.9) is shown graphically in Fig. 4. Fig. 4b suggests the method of measuring the attenuation coefficient μ: it is obtained by linear fitting of the dependence of intensity logarithm on absorber thickness.

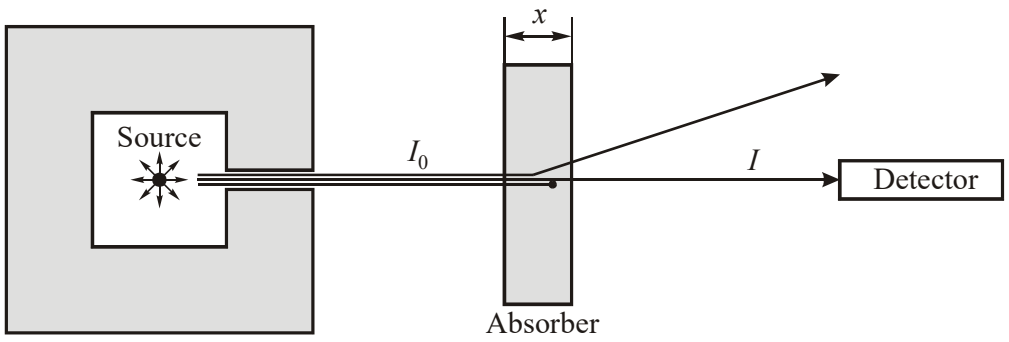

Fig. 3. Diagram of an experiment for measuring absorption of gamma radiation in a material. A narrow parallel beam of radiation is incident upon a layer of material ("absorber") with thickness x. In this layer, some photons are absorbed or scattered. The detector counts only the photons that did not interact with the material.

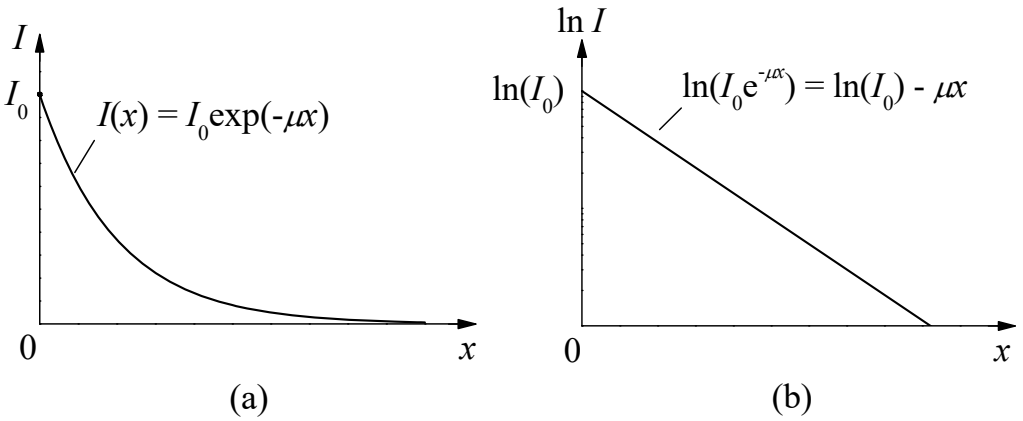

Fig. 4. Dependence of radiation intensity (a) and its logarithm (b) on absorber thickness

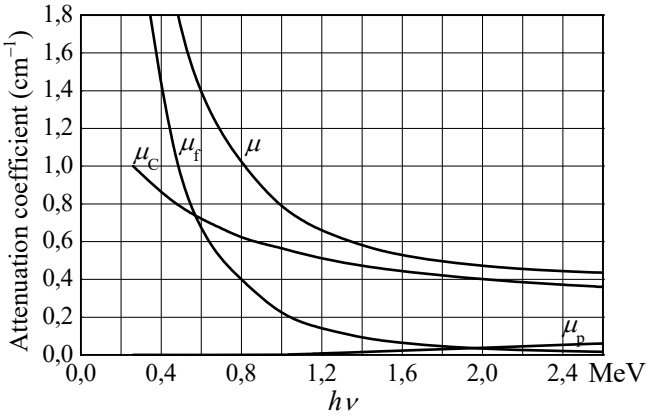

Fig. 5. Dependence of the "partial" attenuation coefficients of lead on photon energy

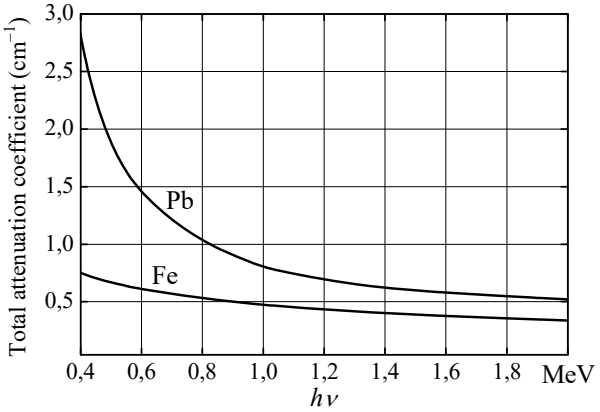

Fig. 6. Dependence of the total attenuation coefficient of lead and iron on photon energy

Since the interaction cross section is a sum of cross sections of three types of interaction (see (4.6)), the attenuation coefficient μ can be expressed as a sum of three "partial" attenuation coefficients corresponding to each of the three interaction processes:

$$\mu = \mu_C + \mu_f + \mu_p . \tag{4.11}$$

Expressions of the coefficients μ_C, μ_f and μ_p are obtained by substituting the corresponding partial cross section for σ in Eq. (4.10).

Since the attenuation coefficient is proportional to the interaction cross section, the energy dependence of the attenuation coefficient has the same shape as the energy dependence of the interaction cross section. The dependence of attenuation coefficients μ_C, μ_f and μ_p on energy of gamma photons, when the absorbing material is lead, is shown in Fig. 5. It can be seen that photoelectric absorption is the dominant type of interaction at low photon energies ($h\nu < 0.5$ MeV). The attenuation coefficient μ depends not only on photon energy, but also on the absorber material. This dependence is mainly a result of the strong dependence of interaction cross section σ on the atomic number Z of the target material (see Eqs. (4.2), (4.3), and (4.5)). Since cross sections of all three mentioned types of interaction increase with Z (albeit with different rate), the attenuation coefficient μ also increases with Z. For example, the attenuation coefficient of lead ($Z = 82$) is greater than the attenuation coefficient of iron ($Z = 26$). This is illustrated in Fig. 6. It should also be noted that the fraction of photoionization events in the total number of interaction events is much greater in lead than in iron, because the increase of the photoelectric absorption cross section with Z ($\sim Z^5$) is much faster than the increase of the Compton scattering and pair production cross sections ($\sim Z$ and $\sim Z^2$, respectively).

Since the attenuation coefficient μ depends on photon energy $h\nu$, the exponential attenuation law (4.9) is observed only in the case of monochromatic (single-energy) radiation. In such a case, the measured value of the attenuation coefficient can be used to determine the photon energy, if the mentioned dependence of μ on $h\nu$ known.

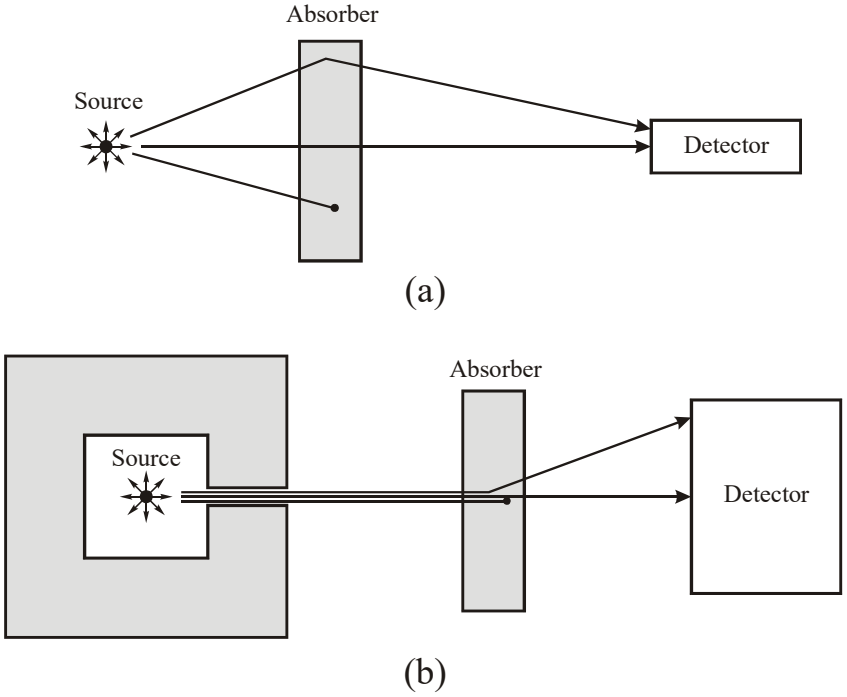

Fig. 7. Examples of "bad geometry" during investigation of absorption of gamma radiation in a material. (a) The incident radiation is not collimated, or the absorber is too close to the source. (b) The absorber is too close to the detector. In both cases, the detector counts not only the photons that crossed the absorber without any kind of interaction, but also some of the scattered photons.

The experiment for measuring absorption of gamma radiation, whose diagram is shown in Fig. 3, is sometimes called the "parallel-beam" or "good-geometry" measurement. The process of forming a parallel beam of radiation is called ***collimation***. It is achieved by allowing the radiation to pass through a narrow channel in a sufficiently thick layer of a strongly absorbing material. Such a device is called a ***collimator***. The geometry shown in Fig. 3 is "good" in the sense that the detector counts only the photons that did not interact with the absorber. This is an ideal case, which is never achieved in practice. A real detector can always detect some of the photons that were scattered in the absorber (if the scattering angle is small enough). Fig. 7 illustrates two examples of "bad geometry". In both cases, the geometry is "bad" in the sense that the probability to detect scattered photons is relatively large. In the case of Fig. 7a, this is because the beam or radiation is not collimated, or the absorber is too close to the source. In the case of Fig. 7b, the beam is well collimated, but the absorber is too close to the detector. In both cases, the decrease of the photon count rate with increasing thickness of the absorber is slower than it should be according to Eq. (4.9). Consequently, if the value of the attenuation coefficient μ is determined by fitting the experimental dependence of the count rate on thickness, then the experimental value of μ will be less than the true value of μ (which corresponds to the case of "good geometry").

When characterizing an absorber material, it is sometimes more convenient to use the so-called "mass attenuation coefficient" instead of the "linear" attenuation coefficient used above. The ***mass attenuation coefficient*** is defined as the ratio of the linear attenuation coefficient and absorber density:

$$\mu_m = \frac{\mu}{\rho}. \tag{4.12}$$

If Compton scattering is the dominant interaction process in two materials with different atomic numbers, then those two materials have approximately equal mass attenuation coefficients. This is because the atomic cross section of Compton scattering is proportional to the atomic number Z (see Eq. (4.2)). Then the attenuation coefficient (4.10) is proportional to the electron concentration Zn_a, which, in turn, is approximately proportional to density ρ of the material. If the attenuation coefficient is strongly influenced by other interaction processes (photoelectric absorption and pair production), then the expression of the attenuation coefficient includes the terms proportional to higher powers of the atomic number Z (the photoelectric cross section is proportional to Z^5 and the pair production cross section is proportional to Z^2). In this case, the attenuation coefficient μ is no longer proportional to ρ (that is to say, the mass attenuation coefficient μ_m is no longer constant).

5. Experimental setup

The experimental equipment is shown in Fig. 8.

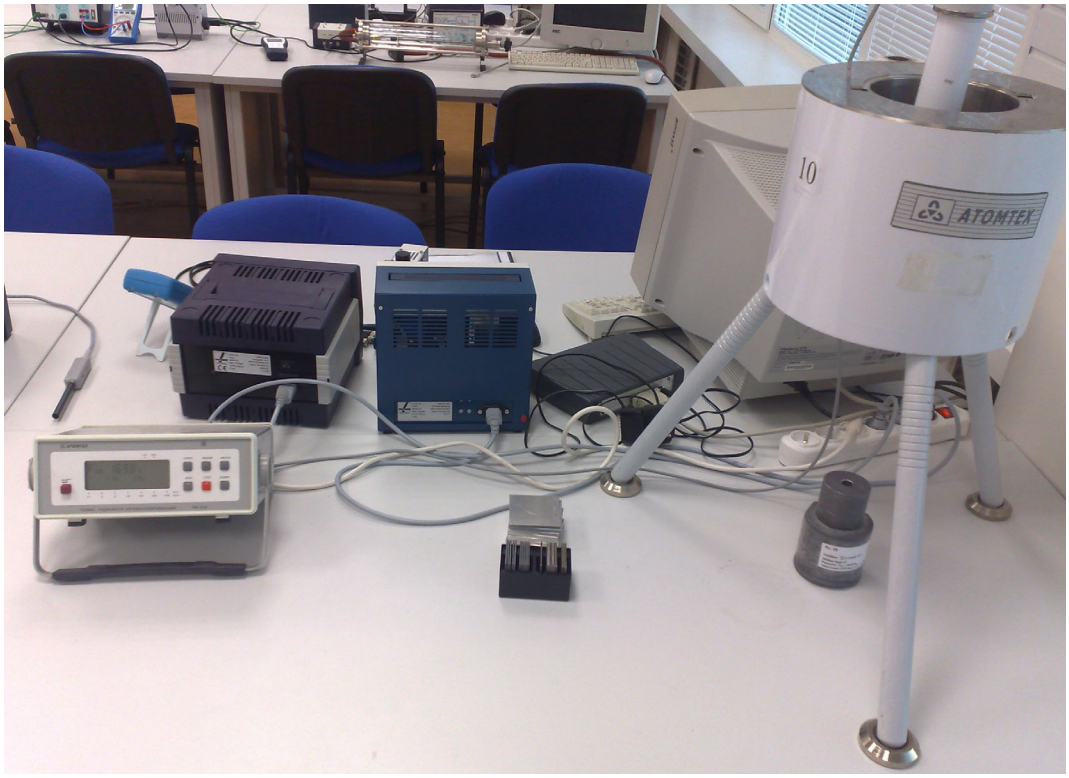

Fig. 8. The experimental equipment. The control unit of the radiometer RKG-01A is seen on the left. The housing of the scintillation detector is on the right (the detector is inserted into it from the top). On the table under the detector, there is a lead container with one of the two investigated sources (^{137}Cs). A collimator (a lead block with a channel for obtaining a narrow beam of radiation) is placed upon the source. In the middle, there is a box with lead and iron absorbers. A stack of aluminum absorbers is behind it.

The equipment consists of the following devices:

1. The gamma radiometer RKG-01A with a scintillation detector (see Fig. 8). Detector's dead time is very small (less than 10^{-6} s), therefore it has no effect on measurement results.

2. The ^{137}Cs radioactive source (see Fig. 9).

(a) (b)

Fig. 9. The lead container with the ^{137}Cs sample: (a) container with a lid; (b) container without the lid. In the latter photo, a hole from which the gamma radiation is emitted can be seen

3. The ^{60}Co radioactive source (see Fig. 10). The radioactive material is at one end of the source package. That end is marked by a groove around the perimeter of the package.

4. A set of lead, iron and aluminum absorbers.

Fig. 10. The ^{60}Co source. The radioactive material is at the top

The radiometer used in this experiment is optimized for detection of two radioactive nuclides – ^{137}Cs and ^{40}K (potassium-40). ^{137}Cs emits photons with energy 0,662 MeV (see Fig. 11), and ^{40}K emits photons with energy 1,46 MeV. Therefore, the radiometer has two "channels" – the "Cs channel", which records the count rate of photons with energy close to 0,7 MeV, and the "K channel", which records the count rate of photons with energy around 1,4 MeV. Photons with other energies are also counted (those counts are usually placed into the nearest "channel"), but their detection efficiency is less than detection efficiency of photons with the mentioned two energies. If the energy of an incident photon is between the mentioned two values, then it can be counted in any one of the channels (but not in both channels simultaneously). This is the case with ^{60}Co, which emits photons with two energies – 1,17 MeV and 1,33 MeV (see Fig. 12). Therefore, when measuring radiation of the ^{60}Co source, the sum of both channels must be recorded. When measuring radiation of the ^{137}Cs source, only the contents of the "Cs channel" must be recorded.

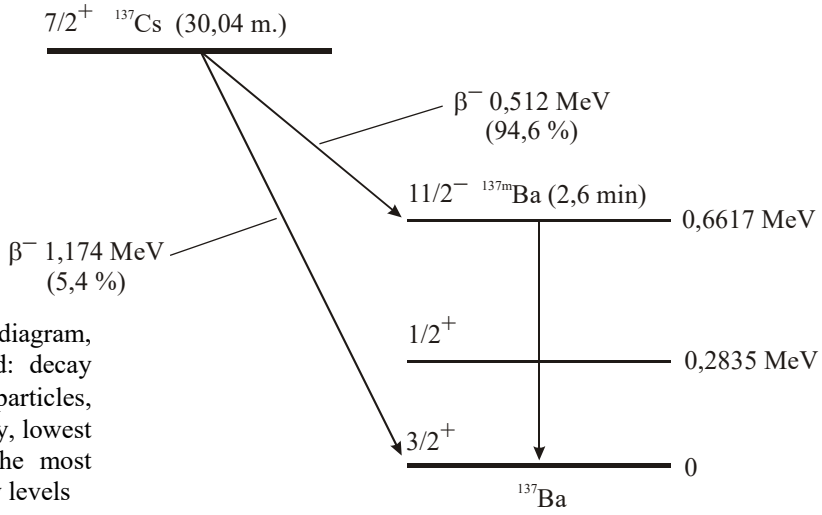

Fig. 11. Decay diagram of ^{137}Cs. In this diagram, the following information is presented: decay half-lives, the largest energies of β particles, probabilities of various modes of β decay, lowest energy levels of ^{137}Ba nucleus and the most probable transition between ^{137}Ba energy levels

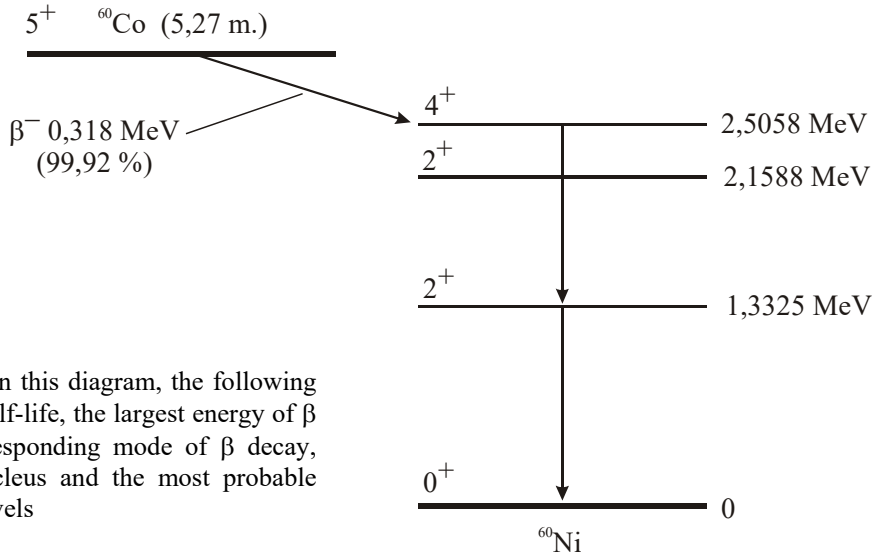

Fig. 12. Decay diagram of ^{60}Co. In this diagram, the following information is presented: decay half-life, the largest energy of β particles, probability of the corresponding mode of β decay, lowest energy levels of ^{60}Ni nucleus and the most probable transitions between ^{60}Ni energy levels

6. Measurement procedure

1. Switch on the control unit of the radiometer RKG-01A (press the button **ВКЛ** in the bottom left corner of the front panel).
2. Place the radioactive sources away from the detector (a distance of 0,5 m is sufficiently large). Press the button **ФОН** (Russian for "Background") on the control unit (see Fig. 13). Hold the button pressed until you hear a beep. Then the LCD display of the control unit shows the results of previous measurements (first, the result of the Cs channel is shown for 3 s, then the result of the K channel is shown for 3 s), and after that the new measurement starts. The display shows two numbers: the mean counting rate (in s^{-1}) and the relative standard error (i. e. the relative uncertainty) of the mean counting rate (see Fig. 13). The mean counting rate and its relative uncertainty are re-calculated each second (the random fluctuations of the counting rate decrease, which is reflected by the decrease of the relative standard deviation). The radiometer always measures two counting rates (one for each channel) simultaneously, although only one of them is displayed. It is possible to switch the display between the two channels without stopping the measurement process: in order to do that, press the button **ФОН** again.
3. Wait until the relative error in the Cs channel becomes 5 % or less (the relative error in the K channel can be ignored). When it happens, stop the measurements by pressing the red button **СТОП** (Russian for "Stop"). Write down the measured values of background counting rates in both channels (in order to switch the display between channels when the radiometer is stopped, press the red button **СТОП** repeatedly). ***Note***: during this entire experiment, only the mentioned two buttons will be needed to operate the control unit: button **ФОН** for starting a measurement, and button **СТОП** for ending it.
4. Place the ^{137}Cs source on the table under the detector. The source container must be approximately in the center of the tripod holding the detector (see Fig. 8). Uncover the container. Put the collimator (a cylindrical lead brick with a narrow channel) upon the container (see Fig. 14).
5. Measure the lead absorption curve by adding the lead plates upon collimator (i. e. gradually increasing total thickness of the layer of lead) and doing one measurement at each thickness. Each measurement must be stopped when the relative error of the Cs channel becomes 1 % or 0 %. Thickness must be changed from 0 (when there is no absorber, as shown in Fig. 14) to 20 mm in increments of 2 mm. The measurement results must be recorded in a table with two columns: the first column for values of absorber thickness, and the second column for the counting rate. ***Notes***: 1) Each measurement is started by pressing the button **ФОН** and ended by pressing the red button **СТОП**. 2) Since thickness of one lead plate is either 1 mm or 2 mm, the total thickness must be incremented by placing either one 2 mm-tick plate or two 1 mm-thick plates. 3) Since the activity of the ^{137}Cs sample is relatively large, at smallest values of absorber thickness there is no need to wait – already

Fig. 13. The front panel of the control unit of the radiometer RKG-01A (the symbol „0 %" means that the relative standard error is less than 0,5 %)

Fig. 14. The ^{137}Cs container with the collimator placed upon it

Fig. 15. The ^{137}Cs container with the collimator and absorbers placed upon it

after 1 s the displayed relative error can be as small as 1 % or even 0 % (as in Fig. 13). 4) the number, dimensions and thicknesses of iron plates are the same as those of lead plates. The easiest way to distinguish between lead and iron absorbers is by presence or absence of a hole in one corner. That hole is present in lead plates (see Fig. 15), but absent in iron plates.

6. Repeat Step 5 with iron absorbers (again, the total thickness must be varied from 0 to 20 mm in increments of 2 mm).
7. Repeat the measurements using aluminum plates. Thickness of all aluminum plates is 5 mm. The total aluminum thickness must be varied from 0 to 40 mm in increments of 5 mm.
8. Remove the collimator from the ^{137}Cs container, put the lid upon the container and put the container away from the detector.
9. Put a platform for the ^{60}Co source under the detector (in the same place where the ^{137}Cs source was). This will ensure a sufficiently small distance between the source and the detector (at the time of this writing, the activity of the ^{60}Co sample is very small, and this source must therefore be placed as close to the detector as possible). For example, a spare power supply may be used as the mentioned platform (see Fig. 16).

Fig. 16. The ^{60}Co source must be placed on a raised platform in order to reduce the distance between the source and the detector, because the activity of this source is relatively small. In this photo, a power supply for Experiment No. 3 is used as the mentioned platform.

Fig. 17. The holder of the ^{60}Co source

Fig. 18. The ^{60}Co source inserted into the holder

Fig. 19. The ^{60}Co holder with the collimator placed on top of it

10. Put the ^{60}Co source holder under the detector (in the same place where the ^{137}Cs source was). The ^{60}Co source holder is a steel cylinder with a hole in it (see Fig. 17). Then insert the ^{60}Co source into that hole as shown in Fig. 18 (take notice of the correct orientation of the source). Place the lead collimator upon the source (see Fig. 19).
11. Repeat Steps 5, 6, 7 with those three changes:
    (a) In the case of lead and iron absorbers, change the thickness in increments of 4 mm (not 2 mm as with the ^{137}Cs source). This means that the total number of thickness values is six (not eleven as with the ^{137}Cs source), and each increase of the thickness should be achieved by placing either four additional 1 mm plates, or two additional 2 mm plates, or two 1 mm plates and one 2 mm plate. In the case of aluminum, the sequence of thickness values should be the same as with the ^{137}Cs source.
    (b) Stop each measurement when the relative error in the Cs channel decreases to 10 % (not 1 % as with the ^{137}Cs source).
    (c) Record count rates in both channels (not only in the Cs channel, but in the K channel, too). Then calculate their sum for each thickness.
12. Extract the ^{60}Co source from the steel holder and place it away from the detector. Measure the background again (Steps 2 and 3).
13. Switch off the control unit. Show the tables with the results to the laboratory supervisor for signing.

7. Analysis of experimental results

1. Subtract the background count rate (n_b) from the values of the count rate with the source present (n). Plot dependences of $\ln(n - n_b)$ on thickness x of lead, iron and aluminum. Plot dependences of $\ln(n - n_b)$ on mass thickness $x\rho$, where ρ is density of the absorber material (densities of lead, iron and aluminum are equal to 11.29 g/cm^3, 7.86 g/cm^3 and 2.70 g/cm^3, respectively). Thus, four graphs must be plotted, with three curves in each of them (two graphs with dependences on thickness x and two graphs with dependences on mass thickness $x\rho$).
2. Using the method of linear fitting, estimate the attenuation coefficients (μ) and mass attenuation coefficients (μ / ρ) and their standard errors. The fitted functions should be the same ones that were plotted previously (see above), i.e., $\ln(n - n_b)$. Linear fitting can be done using various computer programs, or it can be done "by hand", using a calculator (see Section 8). The fitted lines must be shown in the same graphs where the measurement data are plotted.
3. Using Eq. (4.10), calculate the interaction cross sections (σ) and their standard errors for each of the six pairs of source and absorber. In order to calculate the atomic concentration n_a, the densities and standard atomic weights A of the absorbers must be used. The standard atomic weights of lead, iron and aluminum are equal to 207.2 g/mol, 55.85 g/mol and 26.98 g/mol, respectively. Since the number

of atoms in one mole of a chemical element is equal to the Avogadro constant $N_A = 6.02 \times 10^{23}$ mol^{-1}, the mass of an atom (in grams) is equal to A / N_A.

4. Determine the true mass attenuation coefficients from Fig. 20 and compare them with the measured values. In order to use Fig. 20, the photon energy must be known. ^{137}Cs emits photons with energy 0,662 MeV (see Fig. 11). ^{60}Co emits photons with two similar energies 1,33 MeV and 1,17 MeV (see Fig. 12; the second energy corresponds to transition of the daughter nucleus ^{60}Ni from the third excited energy level to the first excited energy level, and the photon energy is equal to the difference of those two energy levels). Therefore, in the case of ^{60}Co, the true value of μ must be determined using the mean photon energy 1,25 MeV.
5. Discuss the observed regularities and compare them with theory (see tasks No. 4 – 6). If mass attenuation coefficients in any two absorbers (with different atomic numbers Z, but using the same radioactive source) are not significantly different from each other, then one may conclude that in both those materials the dominant interaction type is Compton scattering (see the last paragraph of Section 4.5).

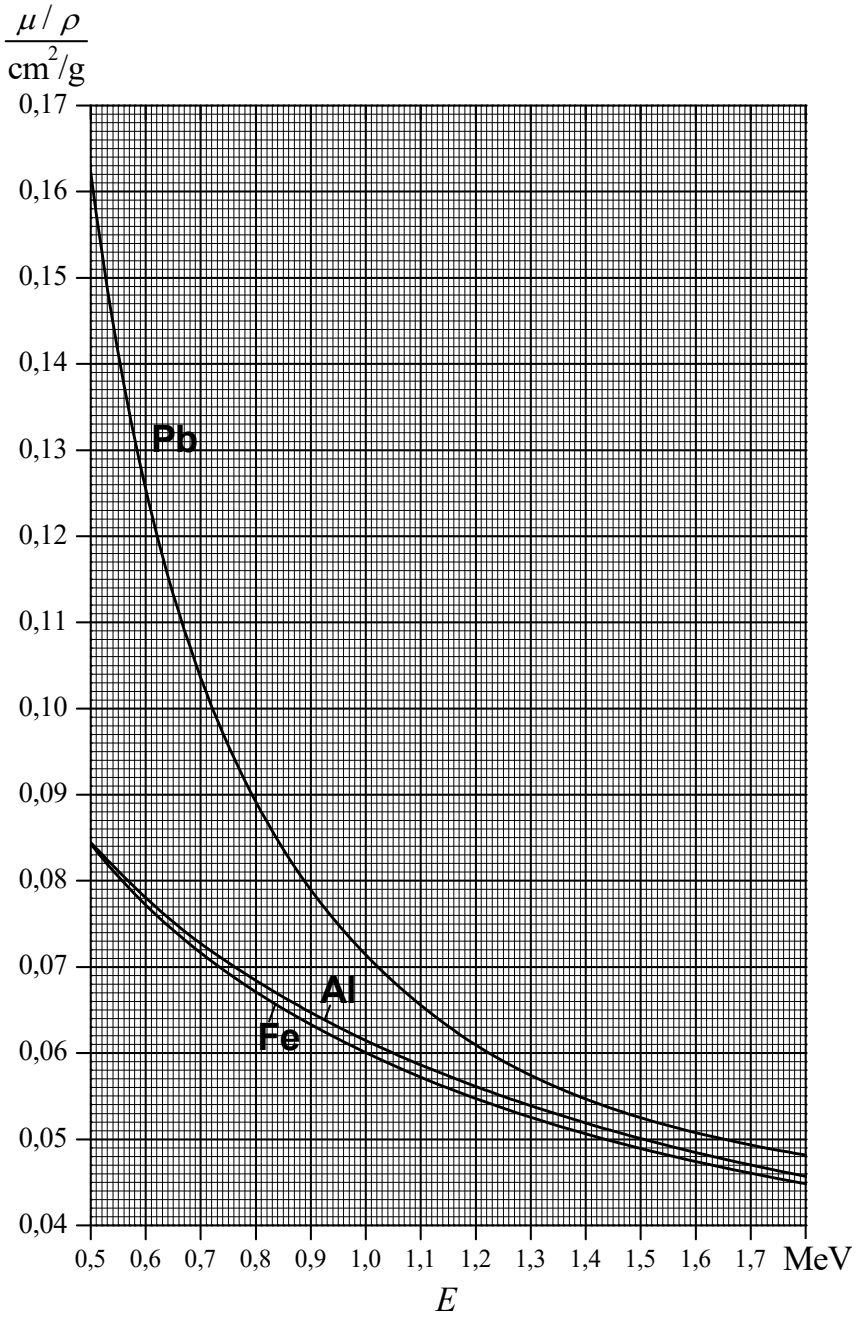

Fig. 20. Dependences of the mass attenuation coefficient of gamma radiation in lead, iron and aluminum on the photon energy (from http://atom.kaeri.re.kr/cgi-bin/w3xcom)

8. Linear fitting

The aim of linear fitting is determination of the least squares estimates of the coefficients A and B of the linear equation

$$y = A + B \cdot x, \tag{8.1}$$

The essence of the method of least squares is the following. Let us assume that a data set consists of the argument values $x_1, x_2, \ldots, x_{n-1}, x_n$ and corresponding values of the function $y(x)$. A typical example is a set of measurement data. In such a case, n is the number of measurements. The measured function values will be denoted $y_1, y_2, \ldots, y_n$. The "theoretical" value of y at a given argument value x_k is a function of the unknown coefficients A and B (see (8.1)), hence we can write $y(x_k) = y(x_k; A,B)$ ($k = 1, 2, \ldots, n$). The problem of estimating the coefficients A and B is formulated as follows. The most likely values of A and B are the values that minimize the expression

$$F(A,B) \equiv \sum_{k=1}^{n} \left[y\left(x_k; A, B\right) - y_k \right]^2 . \tag{8.2}$$

Expression (8.2) is the sum of squared deviations of theoretical values from the measured ones (hence the term "least squares"). That sum is also called "the sum of squared errors" (SSE). This expression always has a minimum at certain values of A and B. However, even if the form of the theoretical function $y(x)$ correctly reflects the true relationship between the measured quantities y and x, those "optimal" values of A and B, which correspond to the minimum SSE, do not necessarily coincide with the true values of A and B (for example, because of measurement errors). The method of least squares only allows estimation of the most likely values of A and B.

Everything that was stated above about the method of least squares also applies to the case when the theoretical function is nonlinear. Regardless of the form of that function and of the number of unknown coefficients, a SSE expression of the type (8.2) must be minimized. However, when $y(x)$ is the linear function (8.1), this problem can be solved analytically (i.e., A and B can be expressed using elementary functions), but when $y(x)$ is nonlinear, this problem can only be solved numerically (applying an iterative procedure).

If $y(x)$ is the linear function (8.1), then the SSE expression (8.2) can be written as follows:

$$F(A,B) \equiv \sum_{k=1}^{n} (A + Bx_k - y_k)^2 = nA^2 + B^2 \sum_{k=1}^{n} x_k^2 + \sum_{k=1}^{n} y_k^2 + 2AB \sum_{k=1}^{n} x_k - 2A \sum_{k=1}^{n} y_k - 2B \sum_{k=1}^{n} x_k y_k . \tag{8.3}$$

It is known that partial derivatives of a function with respect to all arguments at a minimum point are zero. After equating to zero the partial derivatives of the expression (8.3) with respect to A and B, we obtain a system of two linear algebraic equations with unknowns A and B. Its solution is

$$B = \frac{n \sum_{k=1}^{n} x_k y_k - \left(\sum_{k=1}^{n} x_k \right) \left(\sum_{k=1}^{n} y_k \right)}{n \sum_{k=1}^{n} x_k^2 - \left(\sum_{k=1}^{n} x_k \right)^2}, \tag{8.4}$$

$$A = \frac{1}{n} \sum_{k=1}^{n} y_k - \frac{B}{n} \sum_{k=1}^{n} x_k . \tag{8.5}$$

The B coefficient is called the "slope" of the straight line, and the A coefficient is called the "intercept". The standard deviations (or "errors") of those two coefficients are calculated according to formulas

$$\Delta A = \sqrt{\frac{F_{\min}}{n(n-2)} \left(1 + \frac{\bar{x}^2}{D_x} \right)}, \tag{8.6}$$

$$\Delta B = \sqrt{\frac{F_{\min}}{n(n-2)D_x}}, \tag{8.7}$$

where $F_{\min}$ is the minimum value of the SSE (8.3), i.e., the value of SSE when A and B are equal to their optimal values (8.4) and (8.5), $\bar{x}$ is the average argument value:

$$\bar{x} = \frac{1}{n} \sum_{k=1}^{n} x_k , \tag{8.8}$$

and D_x is the variance of the argument values:

$$D_x = \frac{1}{n} \sum_{k=1}^{n} (x_k - \bar{x})^2 = \frac{1}{n} \sum_{k=1}^{n} x_k^2 - \bar{x}^2 . \tag{8.9}$$