

Vilnius University
Faculty of Physics
Laboratory of Atomic and Nuclear Physics

Experiment No. 8

ARTIFICIAL RADIOACTIVITY

by Andrius Poškus
(e-mail: andrius.poskus@ff.vu.lt)

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The aim of the experiment

Test the exponential law of radioactive decay, measure half-lives of radioactive nuclides from their decay curves, investigate the phenomenon of activity saturation.

1. Tasks

1. Measure the decay curve of a silver sample, initially consisting of 51,35 % $^{107}_{47}\text{Ag}$ (silver-107) and 48,65 % $^{109}_{47}\text{Ag}$ (silver-109) after its bombardment by neutrons (activation). Repeat those measurements 4 times, each one corresponding to a different activation time.
2. Plot the decay curves of the mixture of radioactive nuclides ^{108}Ag (silver-108) and ^{110}Ag (silver-110) and use those curves to calculate the half-life of ^{108}Ag .
3. Plot the activation curve of ^{108}Ag .
4. Using the calculated half-life of ^{108}Ag , isolate and plot the contribution of ^{110}Ag to the overall decay curve and calculate the half-life of ^{110}Ag .
5. Discuss the results.

2. Control questions

1. What is the nucleus and what are its constituents? Explain the concepts of isotopes and isobars.
2. What is nuclear binding energy? How is it related to the mass of the nucleus?
3. What is radioactivity? What are the types of radioactive decay? How is the kinetic energy of decay products related to masses of the parent nucleus and decay products (use the principle of mass-energy equivalence)?
4. What is the time dependence of the number of radioactive nuclei? What is the decay constant and the half-life?
5. What is the activity of a radioactive substance? How does the activity depend on time?
6. Explain the concept of nuclear reaction. What is the difference between nuclear reaction and radioactive decay?
7. Explain the concept of radiative neutron capture.
8. Explain the concept of a nuclear reaction cross section. Why neutrons must be slowed down ("moderated") when they are used to create new nuclides by the capture reaction?
9. Why the best neutron moderators are the materials containing large quantities of hydrogen atoms?
10. The concepts of activation and activation curve. Why the activity of the sample that is being bombarded with neutrons stops increasing after a certain time?
11. Why the most likely type of radioactive decay of a substance created by neutron capture is the β^- ("beta minus") decay?
12. Does the number of silver atoms change after the activation of the silver sample and its complete deactivation?

Recommended reading:

1. Krane K. S. *Introductory Nuclear Physics*. New York: John Wiley & Sons, 1988. p. 160 – 165, 169 – 170, 173 – 175, 217 – 220, 230 – 231, 378 – 380, 392 – 394, 416 – 421, 447 – 451, 456 – 459.
2. Lilley J. *Nuclear Physics: Principles and Applications*. New York: John Wiley & Sons, 2001. p. 3 – 42, 93 – 104, 108 – 110, 113 – 116, 142 – 148, 212 – 213.
3. Knoll G. F. *Radiation Detection and Measurement*. 3rd Edition. New York: John Wiley & Sons, 2000. p. 21 – 23, 55 – 57.

3. The nucleus and its constituents

Atomic nucleus consists of Z *protons* and N *neutrons* (they are collectively called *nucleons*). Z is called the *atomic number*, and the total number of nucleons ($A = Z + N$) is called the *mass number*.

The size of the nucleus is of the order of 10^{-15} m.

The basic properties of the atomic constituents:

	charge	mass (u)	spin (\hbar)	magnetic moment (J T^{-1})
proton	e	1.007276	1/2	1.411×10^{-26}
neutron	0	1.008665	1/2	-9.66×10^{-27}
electron	$-e$	0.000549	1/2	9.28×10^{-24}

In the table above, “u” denotes the atomic unit of mass, which is equal to $1.6605391 \times 10^{-27}$ kg, \hbar is the reduced Planck constant (1.054572×10^{-34} J·s). and e is the elementary charge ($1.602176634 \times 10^{-19}$ C).

4. Isotopes, isotones and isobars

Common notation of a nucleus: ${}^A_Z X$, where X is a symbol of a chemical element (for example, ${}^{12}_6\text{C}$). *Isotopes* are atoms whose nuclei have the same Z , but different N . For example, carbon has three naturally occurring isotopes: ${}^{12}_6\text{C}$, ${}^{13}_6\text{C}$ and ${}^{14}_6\text{C}$. *Isotones* are atoms whose nuclei have the same N , but different Z . *Isobars* have the same mass A , but different Z and N .

5. Nuclear mass and energy

The very existence of a nucleus means that nucleons are bound together by a force that is strong enough to counteract the Coulomb repulsion of protons. The *binding energy* (B) of a nucleus is the energy (i.e., the work by external forces) required to separate it into its constituent nucleons.

The binding energy is the opposite of the total potential energy of interaction between the nucleons. Since the nucleons are bound together, the potential energy is negative. Hence, $B > 0$.

Relationship between mass and energy:

$$E = mc^2$$

Hence, the negative term $-B$ in the total energy of a nucleus means that the mass of the nucleus is less than the sum of masses of the constituent nucleons by the amount equal to B/c^2 :

$$M(A, Z) = Zm_p + (A - Z)m_n - B/c^2$$

The shape of this curve is a result of the combined effect of the nuclear and electrostatic (Coulomb) forces. The nuclear force is short-range ($\sim 10^{-15}$ m), hence it binds a nucleon only to its nearest neighbours. The Coulomb force is long-range (each proton interacts with all other protons of the nucleus).

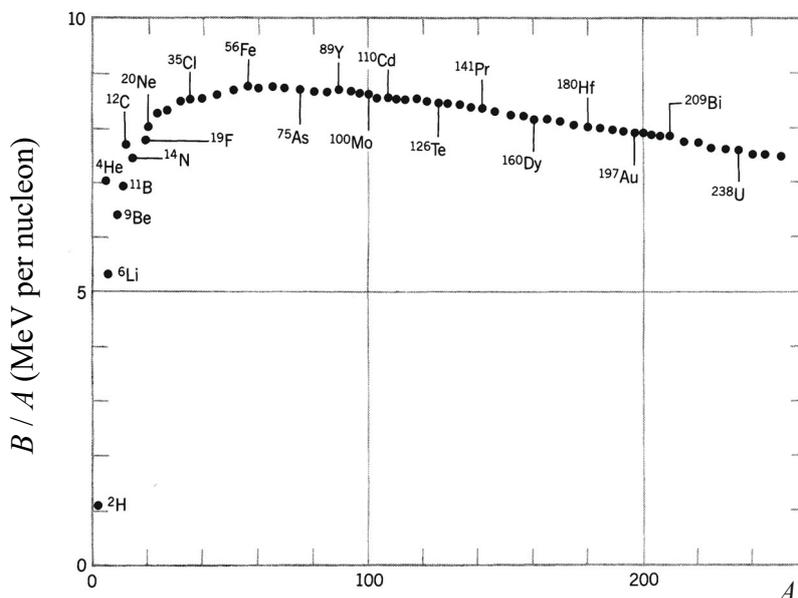


Fig. 1. Binding energy per nucleon as a function of mass number A

Decrease of B/A with increasing A at large values of A reflects an increase of the relative weight of the Coulomb interaction in larger nuclei. This decrease of B/A means that energy could be released by breaking a heavy nucleus into two, roughly equal fragments (*fission*).

Decrease of B/A with decreasing A at small values of A reflects the fact that in smaller nuclei a larger fraction of nucleons are on the surface of the nucleus. This decrease of B/A means that energy could be released by combining two light nuclei into a single nucleus (*fusion*).

6. Occurrence and stability of nuclei

For each value of the mass number A , there is a particular value of Z corresponding to the smallest mass of the nucleus. This value corresponds to the stable nucleus. If Z is larger (a “proton-rich” nucleus) or smaller (a “neutron-rich” nucleus), then the nucleus is unstable (*radioactive*). Such a nucleus eventually transforms into a stable nucleus by series of *decays*. There are various types of decays. For example, a proton may be converted into a neutron, or vice versa (such a type of radioactivity is called *beta radioactivity*). Alternatively, the nucleus may emit an α particle (*alpha radioactivity*).

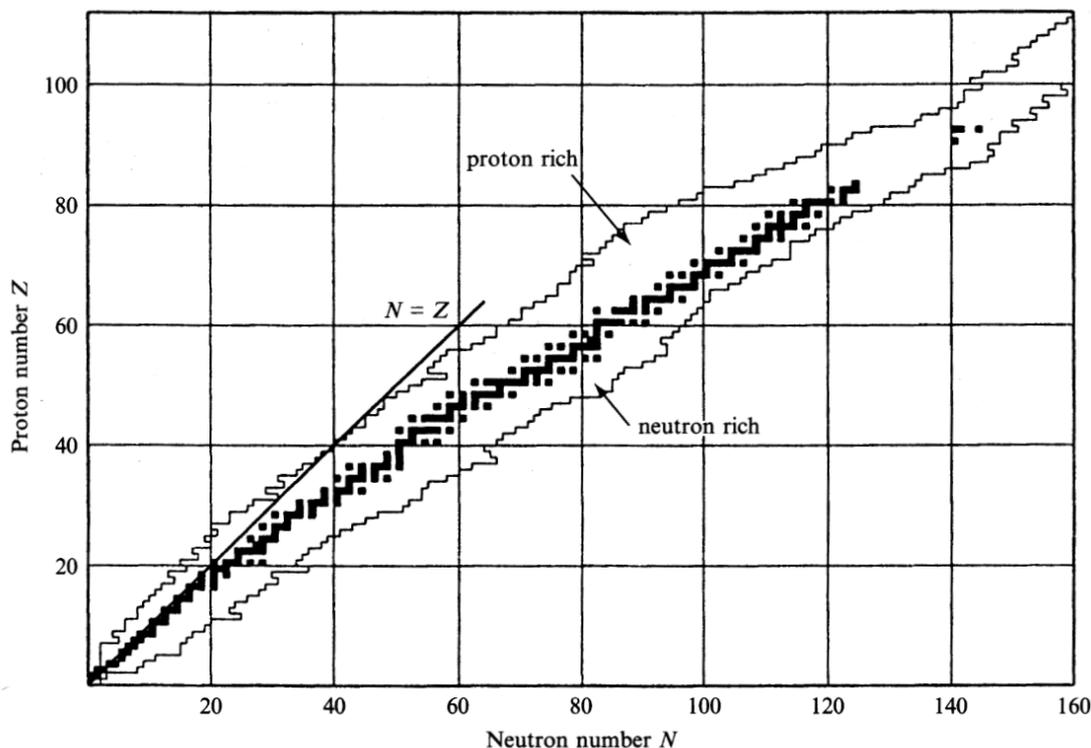


Fig. 2. Stable and unstable nuclei plotted according to proton number Z and neutron number N . Regions of known proton-rich and neutron-rich nuclei are indicated on either side of the band of stable nuclei (and very long-lived unstable nuclei), which are represented as black squares.

7. The law of radioactive decay

In radioactive decay, an unstable nucleus (called “parent”) is transformed into a more stable nuclide (called the “daughter”). If the daughter nuclide is also radioactive, the process continues in a *decay chain* until a stable nuclide is reached.

Radioactivity is a random process. We cannot know exactly when a given unstable nucleus will decay and can only specify a probability per unit time that it will do so. That probability is called the *decay constant*. It is frequently denoted by the Greek letter λ . Another quantity, which is related to the decay constant, is the decay *half-life* ($t_{1/2}$), which is the time needed for half the nuclei in a sample to decay:

$$t_{1/2} = \frac{\ln 2}{\lambda}. \quad (7.1a)$$

The mean time until the decay of a nucleus is called its mean *lifetime*:

$$\tau = \frac{1}{\lambda}. \quad (7.1b)$$

If a given radioactive nuclide is not created (i.e., it is not a daughter of another nuclide, and it is not created in any nuclear reaction), then its amount decreases exponentially with time:

$$N(t) = N(0)\exp(-\lambda t) \equiv N(0)2^{-t/t_{1/2}}. \quad (7.2)$$

Decay rate $-dN/dt$ is called **activity**. Unit of activity is becquerel (Bq): $1 \text{ Bq} = 1 \text{ s}^{-1}$.

All naturally occurring, and the majority of artificially produced, radioactive nuclei are either α active, β active, or both, and emit a combination of α , β and γ radiation. Artificially produced unstable nuclei may also decay by emitting protons, neutrons or even heavy ions.

Equation (7.2) is the formulation of the so-called “law of radioactive decay”. An equivalent way to formulate it is to take time derivatives of both sides of the equation:

$$\frac{dN}{dt} = -\lambda N. \quad (7.3)$$

This is the first-order differential equation whose solution corresponding to the initial condition $N(0) = N_0$ is (7.2). The left side of this equation is opposite to activity. Since N decreases with time, from Eq. (7.3) it follows that activity also decreases exponentially with time:

$$\Phi(t) \equiv -\frac{dN}{dt} = \lambda N(t) = \lambda N_0 e^{-\lambda t}. \quad (7.4)$$

The logarithm of this function is a straight line (see Fig. 3). Its intercept gives the logarithm of the initial activity $\ln(\lambda N_0)$, and its slope is the opposite of the decay constant $(-\lambda)$. Thus, analysis of the decay curve is a simple method of determining the decay constant λ and the half-life $T_{1/2}$.

If a radioactive source is composed of several radioactive nuclides, then the total activity of the source is equal to the sum of activities of constituent nuclides. If those nuclides are independent (i.e. if neither of them is a product of decay of another nuclide present in the same sample), then the overall decay curve is a sum of several exponential terms. For example, in the case of two independent radioactive nuclides,

$$\Phi = \Phi_1 + \Phi_2 = \Phi_{01}e^{-\lambda_1 t} + \Phi_{02}e^{-\lambda_2 t} = \lambda_1 N_{01}e^{-\lambda_1 t} + \lambda_2 N_{02}e^{-\lambda_2 t}, \quad (7.5)$$

where Φ_1 and Φ_2 activities of the two nuclides, Φ_{01} and Φ_{02} are the corresponding initial activities, λ_1 and λ_2 are the decay constants, N_{01} and N_{02} are the initial quantities of both nuclides. In this case, the time dependence of the logarithm of activity consists of two linear regions and a transition region between them (see Fig. 4).

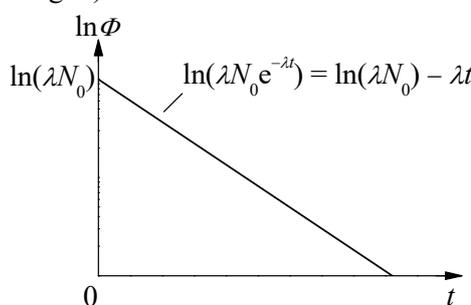


Fig. 3. Time dependence of the natural logarithm of activity

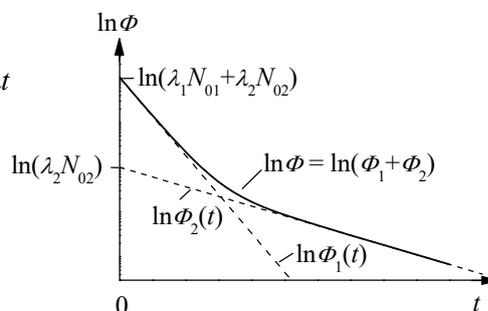
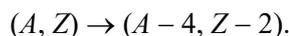


Fig. 4. Time dependence of the natural logarithm of activity of a mixture of two independent radioactive nuclides

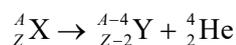
8. Types of radioactive decay

8.1. Alpha decay

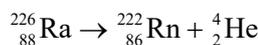
During α emission, the parent nucleus loses both mass and charge:



A generalized equation of α decay:



An example:



Main properties of α decay:

1. $Z > 82$.
2. Discrete energies of α particles emitted by a particular nuclide.

3. Narrow energy range of emitted α particles: $E = (4 - 8,7)$ MeV.
4. Very strong dependence of the decay half-life on the particle energy E . It is given by the **Geiger and Nuttall law** of alpha decay:

$$\lg t_{1/2} = C + \frac{D}{\sqrt{E}}$$

Property No. 1 is related to the fact that α decay is caused by Coulomb repulsion of protons.

The emitted particle is an α particle (and not, e.g., a proton), because, when an α particle is emitted from a nucleus, the total rest mass of the system decreases. The decay energy (Q_α) is released in the form of kinetic energies of the daughter nucleus and the α particle:

$$Q_\alpha = (m_p - m_D - m_\alpha)c^2 = E_D + E_\alpha$$

As evident from the table below, the decay energy is only positive for the case when the emitted particle is an α particle, hence this is the only possible type of spontaneous decay.

Table 3.1 Decay energy for various types of decay of the ^{232}U nucleus

Emitted particle	Decay energy (MeV)	Emitted particle	Decay energy (MeV)
n	-7,26	^4He	+5,41
^1H	-6,12	^5He	-2,59
^2H	-10,70	^6He	-6,19
^3H	-10,24	^6Li	-3,79
^3He	-9,92	^7Li	-1,94

8.2. Beta decay and electron capture

During β decay, the atomic number changes by ± 1 , and the mass number does not change:

$$(A, Z) \rightarrow (A, Z \pm 1).$$

There are three types of β decay:

β^- decay: an electron is emitted from the nucleus, Z increases by 1,

β^+ decay: a positron is emitted from the nucleus, Z decreases by 1,

electron capture: an atomic electron is captured into the nucleus, Z decreases by 1.

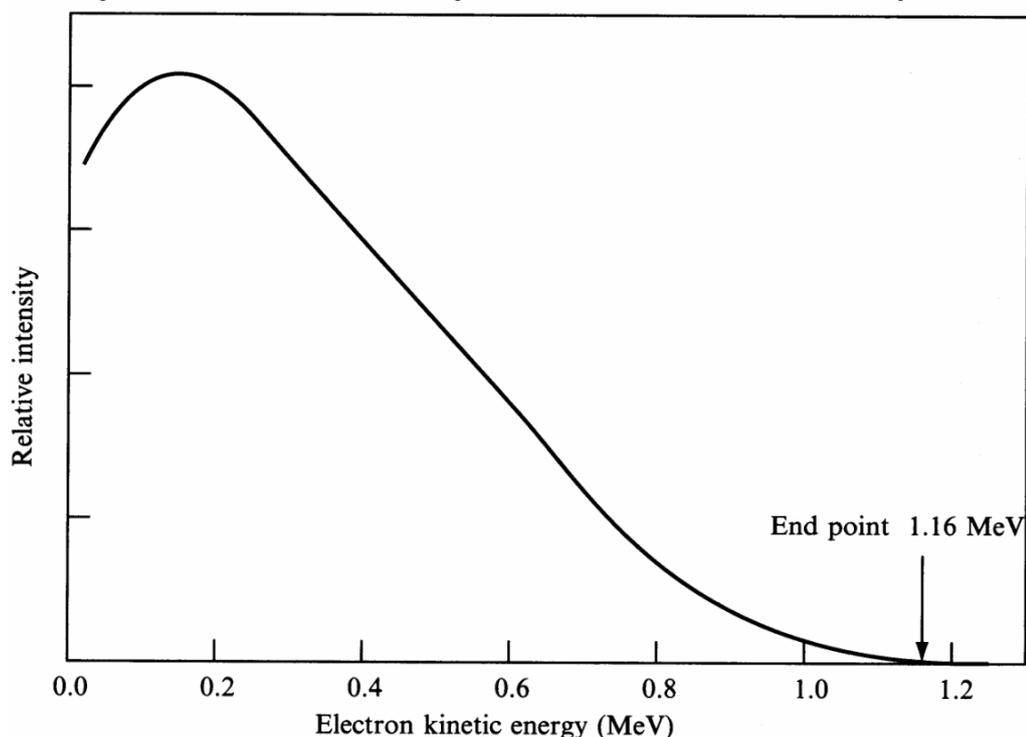


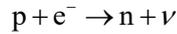
Fig. 5. Energy spectrum of β particles emitted by ^{210}Bi

Unlike the energy spectrum of α particles or gamma photons, the β particle energy spectrum is continuous. However, it is known that the total decay energy is uniquely defined (it is equal to the difference of the rest energies of the parent nucleus and daughter nuclei). This means that a part of the decay energy is carried away by another particle. That particle is neutrino or antineutrino.

During β^- or β^+ decay, one neutron inside the nucleus transforms into a proton or vice versa:



During electron capture, one proton inside the nucleus transforms into a neutron (as in β^+ decay):



8.3. Gamma decay and internal conversion

The **gamma (γ) radiation** is electromagnetic radiation whose wavelength is much less than the distance between atoms of a solid material, i.e., much less than 10^{-10} m (1 Å).

The most common physical mechanism of γ radiation (also called “ γ decay”) is a quantum transition of a nucleus from an energy level E_a to a lower energy level E_b in the same nucleus. When this occurs, the transition energy ($E_a - E_b$) may appear in the form of a γ -ray photon:

$$E = h\nu = E_a - E_b,$$

where ν is the radiation frequency. Typical energies of γ -ray photons: (0.01 – 5) MeV.

Two other physical mechanisms by which a nucleus may lose excitation energy:

In the case of **internal conversion**, an electron is ejected from one of the atomic orbits. The energy of the ejected electron is equal to the difference between the excitation energy E and the binding energy ε_r of the electron:

$$E_e = E - \varepsilon_r.$$

In the case of **internal pair formation**, an electron-positron pair is created. In this case, the excitation energy is converted into the rest energy of two new particles – an electron and a positron – and to their kinetic energy. This process is usually weak, except for very high transition energies, and can only occur if the available energy exceeds that needed to create the pair, which is $2m_0c^2 = 1.022$ MeV, where m_0 is the rest mass of an electron.

Typical lifetimes of excited nuclei: (10^{-14} – 10^{-6}) s. However, long-lived excited states with lifetimes in excess of 1 min are also possible. They are called **metastable states**.

Gamma radiation can also be emitted during annihilation reactions. The most common example of such a reaction is annihilation of an electron and a positron:

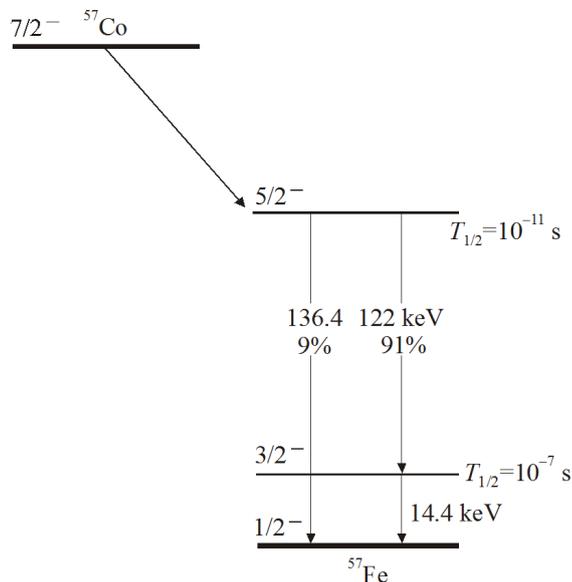
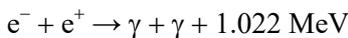


Fig. 6. The decay diagram of ^{57}Co (decay type – electron capture). During this decay, excited nuclei of ^{57}Fe are formed, which emit γ -ray photons of three energies

9. Nuclear reactions

9.1. Introduction. The concept of a nuclear reaction

A **nuclear reaction** is any transformation of a nucleus caused by interaction (“collision”) with an incident particle. Thus, the main difference between radioactive decay and a nuclear reaction is that radioactive decay is spontaneous, i.e., it involves only one primary particle (a single nucleus), whereas a nuclear reaction involves at least two primary particles (a nucleus and another particle that collides with it). If a nucleus X is bombarded with a particle “a” and transforms into a nucleus Y, emitting a particle “b”, then such a reaction is written as follows:



A group of reactions which have the same incident and emitted particles (“a” and “b”, respectively) is called “(a,b) reactions”. Reaction energy, or Q value:

$$Q = (m_a + m_x - m_b - m_y)c^2$$

If $Q > 0$, then the reaction is **exothermic**, and if $Q < 0$, then it is **endothermic**.

9.2. The concept of the reaction cross section

Reaction cross section σ is an area whose value is chosen on the basis of this requirement: *the geometric probability for an incident particle to strike that area must be equal to the probability that the incident particle will induce the nuclear reaction.*

$$dP = \frac{dS'}{S} = \sigma n dx$$

When several types of collision outcomes are possible,

$$\sigma = \sum_i \sigma_i$$

When several types of targets are present,

$$\sigma = \sum_i p_i \sigma_i$$

(p_i is the fraction of targets of i -th type). Cross sections are usually measured in barns. $1 \text{ b} = 10^{-28} \text{ m}^2$.

Mean free path is the average distance travelled by the incident particle until the reaction event:

$$l = \frac{1}{\sigma n}$$

(n is the target concentration). Derivation of the mean free path expression:

$$N = \sigma x n, \quad l = \frac{x}{N} = \frac{x}{\sigma x n} = \frac{1}{\sigma n}$$

If several types of collisions are possible,

$$\frac{1}{l} = \sum_i \frac{1}{l_i}$$

Macroscopic cross section is the reaction cross section corresponding to unit volume of the target material:

$$\Sigma = \frac{dP}{dx} = \sigma n = \frac{1}{l}$$

Reaction rate R is the number of reaction events per unit time in a given volume V . Derivation of the expression for R :

$$\frac{dP}{dt} = \sigma n v$$

$$R = \frac{dP}{dt} n_{\text{inc}} V = \sigma n v n_{\text{inc}} V = \sigma N v n_{\text{inc}} = \sigma N j = \Sigma V j,$$

where n_{inc} is concentration of incident particles.

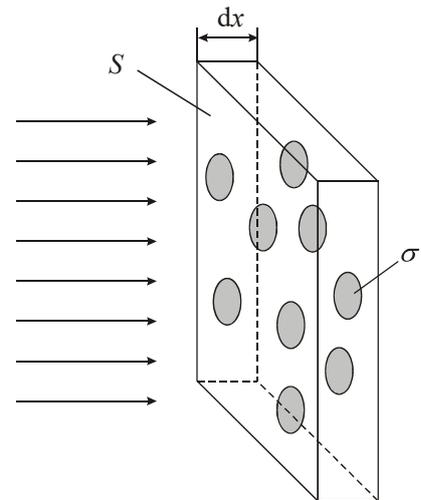


Fig. 7. Explanation of the reaction cross section σ

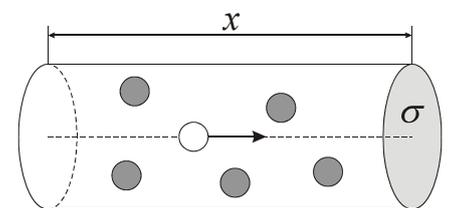


Fig. 8. For derivation of the mean free path

A cross section can be defined for any type of interaction. Usually, the interaction is defined by:

- (1) types and energies of primary particles (e.g., incident and target particles);
- (2) the reaction outcome, i.e., types and energies of secondary particles (“reaction products”).

However, when defining the reaction outcome, we can be even more specific and analyze only the reactions when secondary particles move in specific directions. This is when the concept of “differential cross section” is useful. Let us define $d\sigma(\theta, \phi)$ as cross section of such interaction when the secondary particle moves into an infinitesimal solid angle $d\Omega$ in direction defined by polar and azimuthal angles (θ, ϕ) . Then the **differential cross section** is defined by

$$\sigma_{\Omega} = \frac{d\sigma}{d\Omega}$$

Unit of measurement of the differential cross section is b / sr (barn per steradian).

Total cross section:

$$\sigma = \int \sigma_{\Omega} d\Omega \equiv \int_0^{2\pi} d\phi \int_0^{\pi} \sigma_{\Omega} \sin \theta d\theta$$

Usually, σ_{Ω} does not depend on ϕ . Then

$$\sigma = 2\pi \int_0^{\pi} \sigma_{\Omega} \sin \theta d\theta$$

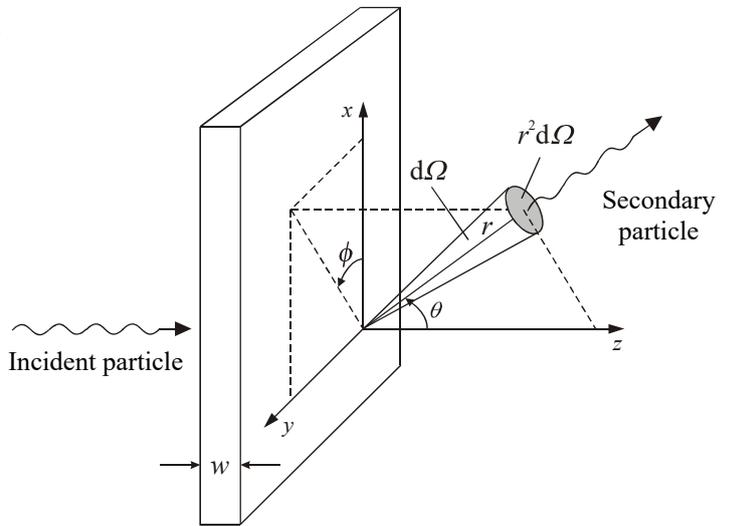
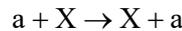


Fig. 9. For explanation of differential cross section

9.3. Examples of nuclear reactions

9.3.1. Elastic scattering



A special case of elastic scattering is elastic Coulomb scattering, or **Rutherford scattering**.

$$F = \frac{1}{4\pi\epsilon_0} \cdot \frac{zZe^2}{r^2}$$

If we assume that the target nucleus is infinitely massive, then the Rutherford scattering differential cross section is given by

$$\sigma_{\Omega} = \frac{d^2}{16 \sin^4(\theta/2)} = \left(\frac{zZe^2}{4\pi\epsilon_0} \right)^2 \left(\frac{1}{4E} \right)^2 \frac{1}{\sin^4(\theta/2)},$$

where d is the minimum distance of approach in the case of a central collision between a nucleus with charge $+Ze$ and an incident particle with charge $+ze$ and kinetic energy E :

$$d = \frac{zZe^2}{4\pi\epsilon_0 E}$$

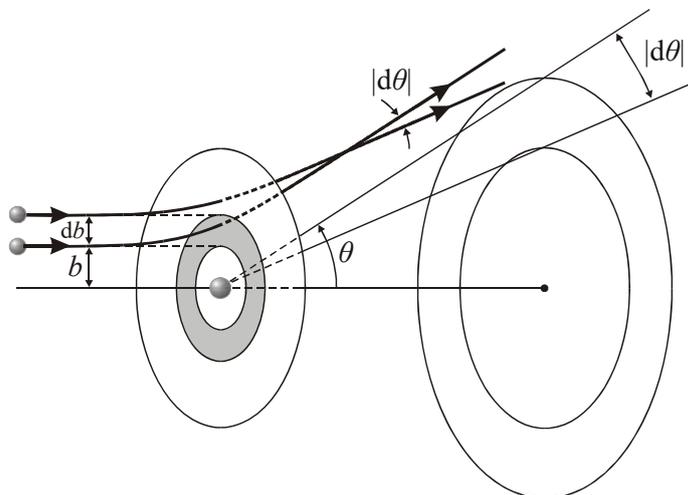


Fig. 10. Rutherford scattering: when the impact parameter of an incident particle is between b and $b + db$, then the scattering angle is between θ and $\theta + d\theta$ (db and $d\theta$ are of opposite sign)

9.3.2. Direct reactions

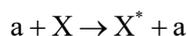
A **direct nuclear reaction** is a reaction when the incident particle only interacts with a small number of nucleons, which are near the surface of the target nucleus.

Properties of direct reactions:

- relatively small exchange of energy, momentum and mass,
- extremely short duration of interaction ($\sim 10^{-22}$ s).

Examples of direct reactions:

Inelastic scattering:



Transfer reactions:

stripping reaction (e.g., (d,p), (α ,d) and (^{16}O , ^{12}C)),
pickup reaction (e.g., (p,d), (p,t) and (^{16}O , ^{17}O)).

9.3.3. Compound nucleus reactions

In the case of a central or near-central collision between an incident particle and the target nucleus, a so-called “compound nucleus reaction” is possible. It proceeds in two stages:

- Both interacting particles merge into an intermediate (“compound”) nucleus, which is in an excited state.
- After $10^{-16} - 10^{-18}$ s the compound nucleus loses the excess energy, emitting one or more particles.

Properties of the emitted particles depend only on properties of the compound nucleus and do not depend on the method of its formation. In this respect, the second stage is similar to radioactive decay. Sometimes, several decay types (“decay channels”, or “decay branches”) are possible. Likewise, several ways to form a given compound nucleus may exist.

The emitted particles may be:

- nucleons,
- nuclei,
- gamma photons,
- internal conversion electrons.

Emission of nucleons is similar to evaporation of a hot liquid. However, if excitation energy is less than nucleon binding energy or only slightly larger than the latter, then gamma emission or internal conversion is more likely. In such a case, the incident particle is “captured” into the target nucleus, hence such a reaction is called a **capture reaction**.

If the incident particle is a nucleus (e.g., a proton), then formation of the compound nucleus is inhibited by the Coulomb barrier. However, if the incident particle is a neutron, then there is no Coulomb barrier, hence formation of the compound nucleus becomes possible even at low energies of the incident neutron. In such a case, reaction probability is inversely proportional to the neutron velocity (this is the so-called “ $1/v$ law”).

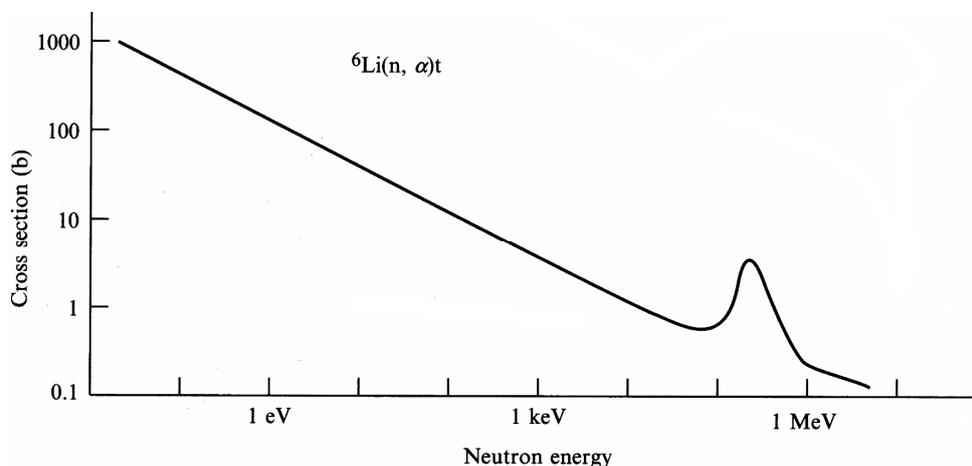


Fig. 11. Dependence of the cross section of the nuclear reaction ${}^6\text{Li}(n, \alpha)t$ on neutron energy. A resonance is evident at 250 keV

9.3.4. Resonance in compound nuclear reactions

Frequently, energy dependence of a compound-nucleus reaction cross section has several maxima. They correspond to an increased probability of formation of the compound nucleus. Such an increase is observed when the excess energy supplied to the compound nucleus becomes exactly equal to the energy that is needed to excite the nucleus into one of its excited energy levels (“natural oscillation frequencies”). This phenomenon is called *resonance*. Near the resonance energy, the reaction cross section is approximately given by

$$\sigma(E) \sim \frac{1}{(E - E_r)^2 + (\Gamma/2)^2}$$

(this is the so-called *Lorentz function*, or *Breit-Wigner function*).

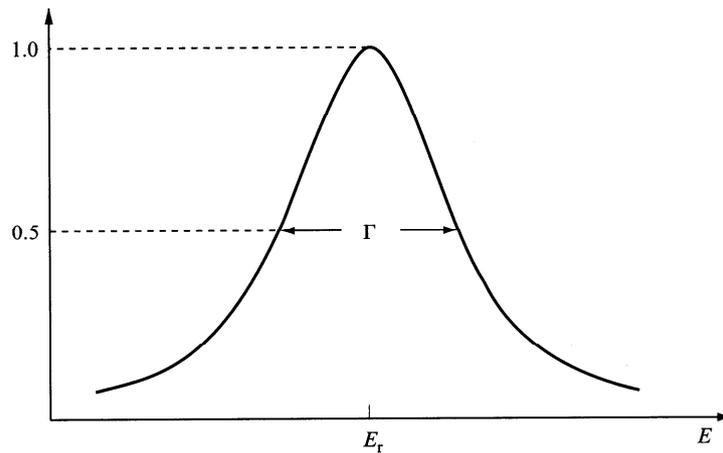


Fig. 12. The Breit-Wigner resonance curve

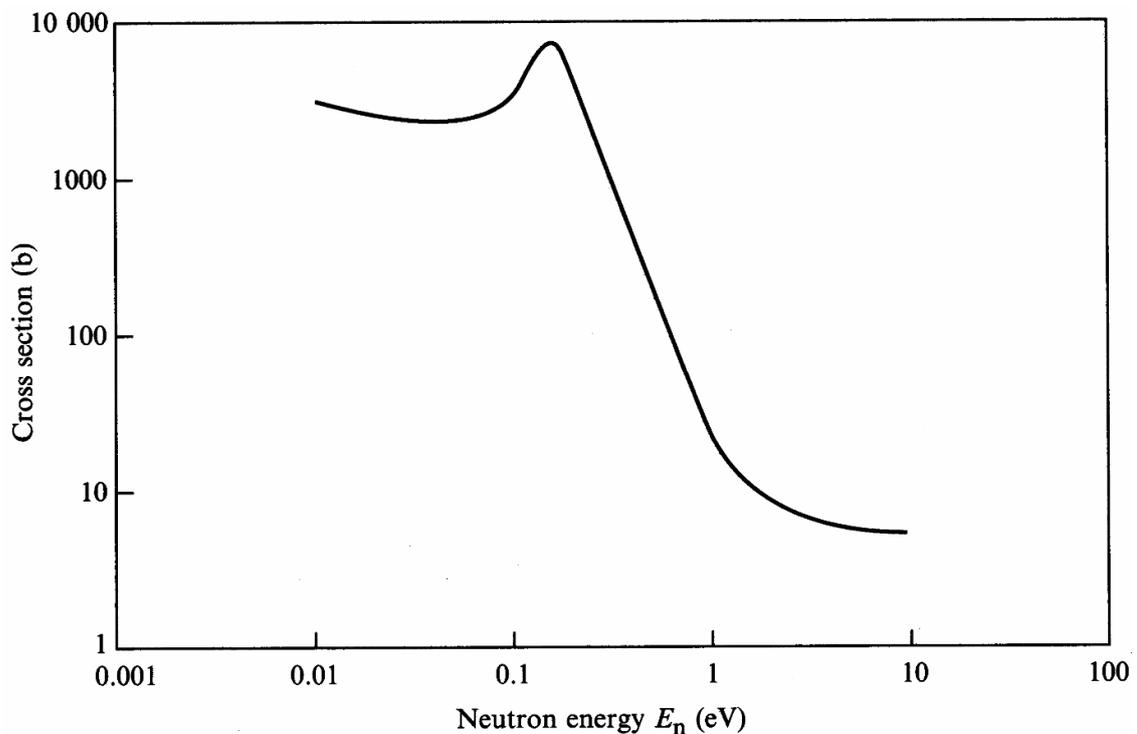


Fig. 13. The total cross section of neutron interaction with an isotope of cadmium ^{113}Cd . A pronounced resonance at 0.17 eV is evident. The asymmetry of the peak is mainly a result of superposition of two curves: the “ $1/v$ law” and the Breit-Wigner resonance curve.

9.4. Activation by neutron capture

A nuclear reaction that is applied in this experiment is the so called “radiative neutron capture”, whose generalized equation is the following:



This equation should be interpreted as follows. Due to interaction of the initial nucleus (${}^A_Z\text{X}$) with the incident neutron (${}^1_0\text{n}$), the neutron is captured into the nucleus, increasing its mass number by 1. The resulting nucleus is the so called “compound nucleus”, which is a short-lived intermediate stage of the reaction. The compound nucleus is excited (this is denoted by the asterisk in its notation ${}^{A+1}_Z\text{X}^*$). It emits the excess energy as gamma radiation (γ). Thus, the radiative neutron capture is a “compound nucleus reaction” (see also Section 9.3.3), i.e., a two-stage process: 1) formation of an excited compound nucleus; 2) gamma decay of the compound nucleus. When writing the equation of the radiative neutron capture, the initial stage of the reaction (formation of the compound nucleus) is usually omitted:



The following reaction is frequently used to produce neutrons (which subsequently can be used, e.g., in the reaction (9.1b)):



A typical neutron source, where the latter reaction takes place, is a mixture of metallic beryllium powder and a small amount of an alpha-radioactive material (e.g., plutonium isotope ${}^{239}\text{Pu}$).

In order to increase the probability that a nucleus will capture a neutron (i.e., to increase the cross section of reaction (9.1b)), the neutrons are usually slowed down (“moderated”). According to the “1 / v law” mentioned in Section 9.3.3, slower neutrons are more likely to be captured. Neutrons are moderated by elastic collisions with light nuclei. According to the laws of conservation of energy and momentum, an elastic collision between an incident particle (a neutron in the case discussed) and a target particle (a light nucleus in the case discussed) causes transfer of a part of kinetic energy of the incident particle to target particle. This transfer is largest when the masses of the two particles are equal to each other. Consequently, the best moderators of neutrons are those that contain the largest quantity of hydrogen: the mass of the proton (nucleus of the hydrogen atom) is very similar to the mass of the neutron.

In this experiment, the investigated substance is silver. Natural silver consists of two stable isotopes – ${}^{107}_{47}\text{Ag}$ and ${}^{109}_{47}\text{Ag}$ (51,35 % ${}^{107}_{47}\text{Ag}$ and 48,65 % ${}^{109}_{47}\text{Ag}$). When those isotopes are bombarded by neutrons, the following nuclear reactions take place:



The reaction products ${}^{108}_{47}\text{Ag}$ and ${}^{110}_{47}\text{Ag}$ are β^- radioactive. I.e., they subsequently decay as follows:



(half-lives are indicated under the arrows). The process of creating radioactive nuclides by neutron bombardment is called **activation**. The nuclides created by neutron capture are typically β^- radioactive, because they are in the “neutron-rich zone”, i.e., below the black squares in Fig. 2. The type of decay is such as to return the nucleus back to the zone of stability (black squares in Fig. 2). Obviously, if the nucleus is neutron-rich, then the β^- decay is the only type of decay satisfying the mentioned requirement: it causes a decrease of the neutron number (denoted “ N ” in Fig. 2) by 1 and an increase of the proton number Z by 1, i.e., the corresponding point moves left and upwards in Fig. 2.

After activation of a silver sample, its total activity decays with time as a sum of two exponentially decaying terms (7.5). However, in this experiment, the quantity that is measured directly is not the activity, but the number of beta particles detected by the Geiger-Müller counter over a fixed interval of time, $\Delta n(t)$. Fortunately, the number of detected particles is proportional to activity, therefore the overall shape of the function $\Delta n(t)$ is the same as that of the function $\Phi(t)$, i.e. it is a sum of two exponentials:

$$\Delta n(t) = C_1 e^{-\lambda_1 t} + C_2 e^{-\lambda_2 t}, \quad (9.5)$$

where $\Delta n(t)$ is the number of particles detected during the interval of time starting at $t - \Delta t$ and ending at t (here Δt denotes the measurement interval length). In Eq. (9.5), the first term corresponds to ^{108}Ag , and the second term corresponds to ^{110}Ag . Since decay of ^{108}Ag is slower than decay of ^{110}Ag , let us call the first term the “slow component”, and the second term the “fast component”. The coefficients C_1 and C_2 depend on detector parameters, measurement geometry (i.e. the shape of the sample and its position relative to the detector), activation time t_a and the duration of a single measurement Δt . In this experiment, only the latter two quantities are variable: decay curves are measured after four activations, each with a different activation time t_a , and there are two values of Δt used (during the first two minutes of each decay curve, the measurement interval is 10 s, and later on it is 60 s).

Since t_a is variable, it becomes possible to investigate not only the shape of individual decay curves, but also the dependence of that shape on t_a . Clearly, only the preexponential factors C_1 and C_2 depend on t_a (the decay constants λ_1 and λ_2 cannot depend on t_a , because those constants are internal parameters of the nuclides, and activation changes only the number of nuclei, not their internal properties). The pre-exponential factors are proportional to activities of corresponding nuclides at the end of activation. Let us discuss dependence of activity on activation time t_a . From Eq. (7.4) we know that activity is proportional to the number of radioactive nuclei. The dependence of the quantity of a particular nuclide on t_a during the activation can be determined from the differential equation

$$\frac{dN}{dt} = \sigma N_p j - \lambda N, \quad (9.6)$$

where N is the number of atoms of the daughter nuclide, N_p is the number of atoms of the parent nuclide, σ is the neutron capture cross section and j is the density of neutron flux (i.e. the number of neutrons per unit area per unit time). If we compare Eq. (9.6) with the law of radioactive decay (7.3), we can see that there is an additional term ($\sigma N_p j$) on the right-hand side of the equation. This term reflects the fact that during the activation, the nuclei not only decay; they are also created due to neutron capture. Thus, there are two competing processes: an increase of the number of daughter nuclei due to neutron capture (this increase is reflected by the positive term $\sigma N_p j$ in Eq. (9.6)), and the decrease of that number due to decay of the daughter nuclei (this decrease is reflected by the negative term $-\lambda N$). Since the number of nuclei that capture a neutron is many orders of magnitude smaller than the number of parent nuclei N_p , the latter number can be assumed to be constant. Thus, the first term on the right-hand side of Eq. (9.6) is practically constant, whereas the absolute value of the second term increases with time (due to the increase of the number of daughter nuclei N). This means that the increase of N during activation slows down continuously and eventually the value of N stabilizes. This happens when the rate of nuclide production ($\sigma N_p j$) becomes equal to the rate of its decay (λN). This stable value of N is called the „saturation value“. The mathematical expression of the dependence of N on t_a is obtained by solving the differential equation (9.6) with the initial condition $N(0) = 0$. This solution is

$$N(t_a) = N_{\text{sat}}(1 - e^{-\lambda t_a}), \quad (9.7)$$

where N_{sat} is the mentioned saturation value of the number of radioactive nuclei. The corresponding dependence of activity on activation time has the same shape:

$$\Phi(t_a) = \lambda N(t_a) = \lambda N_{\text{sat}}(1 - e^{-\lambda t_a}). \quad (9.8)$$

This function is shown in Fig. 14. Obviously, the number of daughter nuclei (and activity) saturates after a time which is roughly equal to 4 – 5 half-lives.

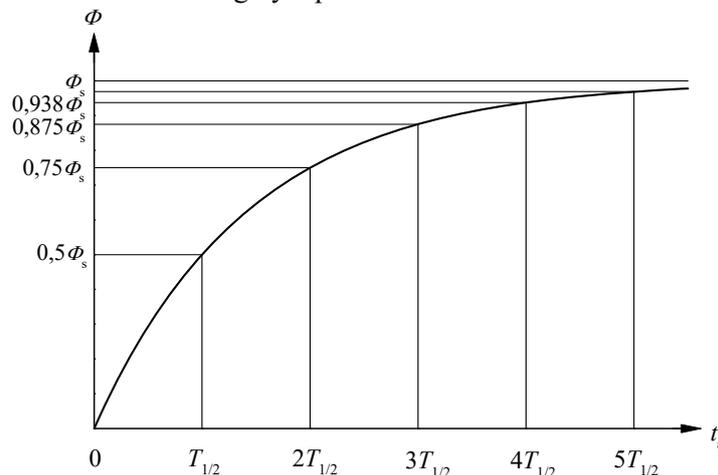


Fig. 14. Activation curve $\Phi = \Phi_s(1 - e^{-\lambda t_a})$. t_a is activation time, Φ is activity of the sample, Φ_s is the saturation activity, λ is the decay constant

As mentioned above, the pre-exponential factors in Eq. (9.5) are proportional to the corresponding activities at the end of activation. Therefore, their dependence on the activation time t_a is also of the shape defined by Eq. (9.8) and shown in Fig. 14. In particular, the dependence of the factor C_1 (which corresponds to the longer-lived nuclide ^{108}Ag) on t_a is the following:

$$C_1(t_a) = C_{1,\text{sat}}(1 - e^{-\lambda_1 t_a}), \quad (9.9)$$

where $C_{1,\text{sat}}$ is the saturation value of C_1 .

Now, let us discuss the dependence of C_1 on the duration of a single count Δt . Since the number of particles detected *per unit time* is proportional to activity of the sample, the number of particles detected *during the period from $t - \Delta t$ to t* is proportional to the integral of activity during the same period. From this it follows that C_1 is proportional to $\exp(\lambda_1 \Delta t) - 1$. Thus, if C_1 denotes the value of the pre-exponential factor corresponding to $\Delta t = 60$ s, and C'_1 denotes its value corresponding to $\Delta t = 10$ s, then those two values are related as follows:

$$C'_1 = \frac{e^{\lambda_1 \cdot 10\text{s}} - 1}{e^{\lambda_1 \cdot 60\text{s}} - 1} C_1. \quad (9.10)$$

Using the decay constant of ^{108}Ag ($\lambda_1 = \ln 2 / (142,2\text{s}) = 0,00487\text{ s}^{-1}$), the following expression of C'_1 is obtained from (9.10): $C'_1 = 0,147 C_1$.

Some of the detected particles may be emitted by other sources (e. g. cosmic radiation or natural radioactivity of the environment). Those particles show up as a constant term in the measured count. This term is referred to as the “background”. Eq. (9.5) corresponds to the ideal case when the background is zero. In general, one must take into account the background term Δn_b :

$$\Delta n(t) = C_1 e^{-\lambda_1 t} + C_2 e^{-\lambda_2 t} + \Delta n_b. \quad (9.11)$$

The background term Δn_b is measured when the investigated sample is sufficiently far away from the detector.

10. Experimental setup

The equipment consists of the following components:

- 1) cylindrical Geiger-Mueller counter (on the right of Fig. 15),
- 2) **Isotrak** ratemeter (on the left of Fig. 15),
- 3) personal computer,
- 4) Pu-Be neutron source. The source is at the bottom of a steel container, which is filled with neutron moderator – paraffin (see Fig. 16),
- 5) a sample of natural silver (see Fig. 17 and Fig. 18).



Fig. 15. The main components of the measuring equipment – **Isotrak** ratemeter (on the left) and Geiger-Müller counter (on the right). The counter is the metal tube. Near its center there is the sample holder, which ensures that the position of the sample relative to the detector is the same during all measurements (also see Fig. 18). The **Isotrak** ratemeter is connected with the detector and with the PC



Fig. 16. The container of the neutron source (the source is at the bottom of the container). The container is surrounded by lead bricks that serve to decrease intensity of gamma radiation in the environment



Fig. 17. The silver sample and its handle

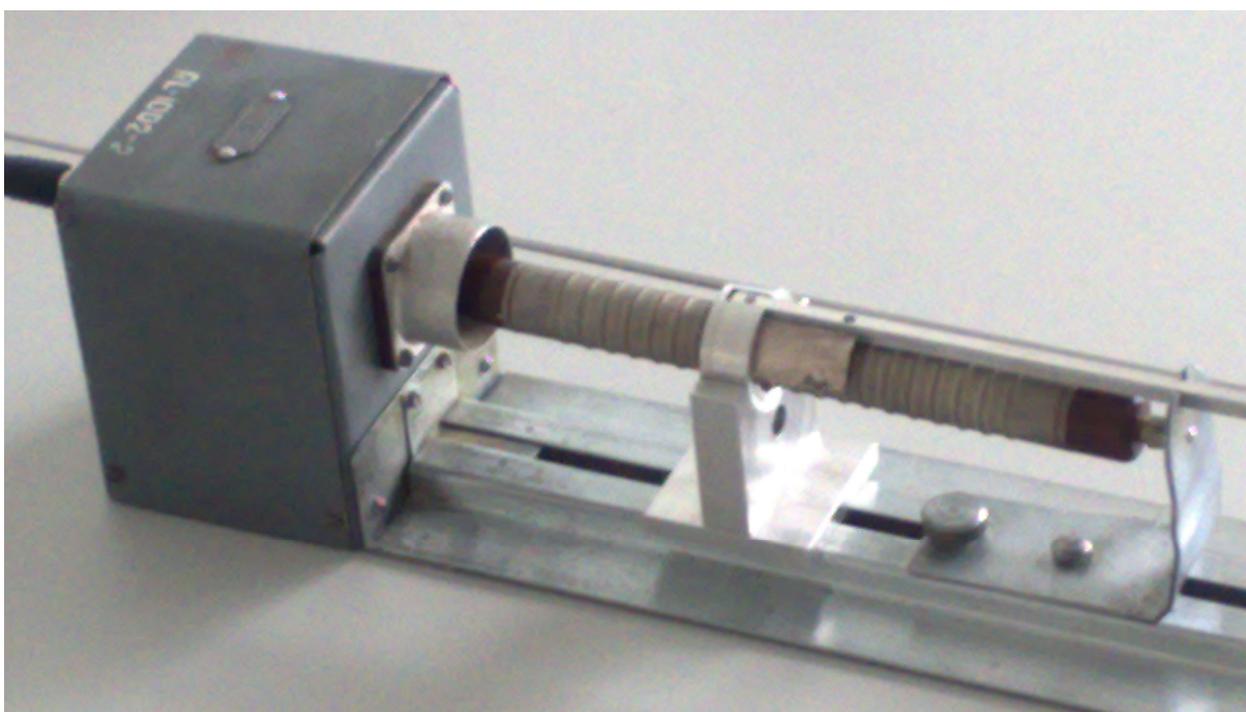


Fig. 18. The detector and the silver sample during the measurement of the decay curve

11. The measurement procedure

During this experiment, four decay curves (9.11) of the mixture of radioactive silver isotopes ^{108}Ag and ^{110}Ag are measured. Each of them corresponds to a different activation time t_a . Approximate values of the activation time are the following: 10 min, 5 min, 2 min and 1 min. After each activation, the decay curve is measured for 10 min, then the sample is placed away from the detector and the background is measured for additional 5 min (during this time, the silver sample completely deactivates and becomes ready for the next activation-deactivation cycle). Since each decay curve consists of two components – the slow one (it corresponds to decay of ^{108}Ag) and the fast one (it corresponds to decay of ^{110}Ag), two values of the counting time Δt are used: during the first 2 minutes it is $\Delta t = 10$ s, and during the last 8 minutes it is $\Delta t = 60$ s. I.e., in the initial part of the decay curve each point gives the number of particles detected during last 10 s, and in the final part of the decay curve each point gives the number of particles detected during last 60 s.

The computer program that receives data from the Isotrak ratemeter passes those data to the program “Origin 6” in real time. The program “Origin 6” displays the data in the form of worksheets and graphs. Fig. 19 shows an example of the Origin graph. The vertical bars centered on each point are the so-called “error bars”. The length of each error bar is twice the standard deviation of each measurement result (one standard deviation upwards from the point and one standard deviation downwards from the point). Those standard deviations are calculated under the assumption of Poisson distribution. I.e., the

standard deviation of each measurement result is equal to the square root of that result (the standard deviations will be subsequently used for calculating the weight factors during nonlinear fitting).

The following conventions regarding the times are used in the graphs with decay curves:

- 1) The time t , which is plotted on the X axis, is counted from the moment of time corresponding to the end of activation. Thus, the time $t = 0$ corresponds to extraction of the sample from the neutron source (and *not* to placement of the sample upon the detector!).
- 2) The time corresponding to each point is time of the *end* of the corresponding count. For example, if a point is at $t = 30$ s, this means that this point is the number of particles detected during the time interval $20 \text{ s} < t < 30 \text{ s}$.

As extraction of the sample from the neutron source and its placement upon the detector lasts for several seconds, the first point (at $t = 10$ s) is inaccurate. This is because the position of the sample relative to the detector during the time interval $0 < t < 10$ s is different from that position during the subsequent measurements. Hence, the first point is not used for fitting (if necessary, more points can be removed prior to fitting, too).

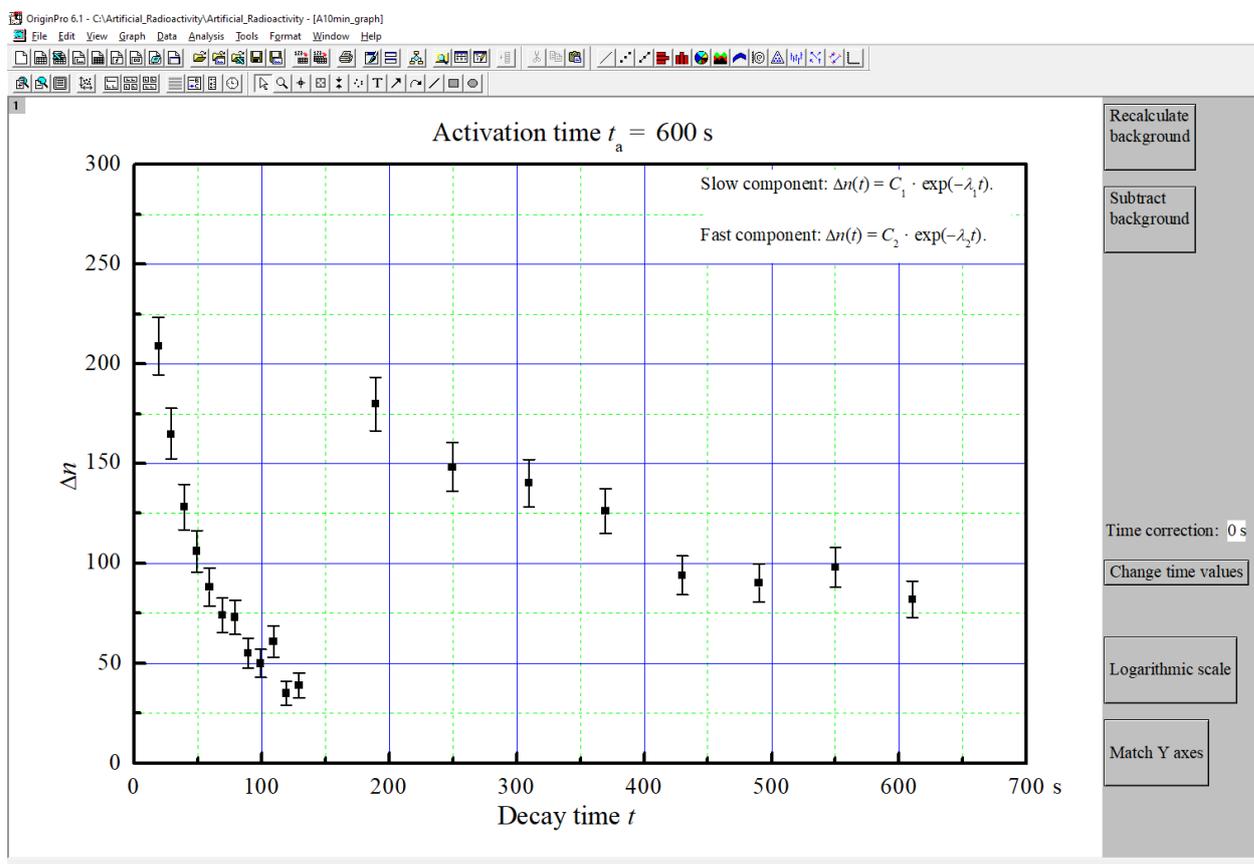


Fig. 19. An example of a decay curve

A detailed step-by-step procedure of the measurements is given below.

1. Switch on the PC and the **Isotrak** ratemeter. Set the detector voltage to (450 ± 20) V. In order to do that, it is necessary to press the button “Select” on the ratemeter several times until the control lamp “G-M Voltage” starts blinking, and press the button “Enter”. Then the display of the ratemeter shows the detector voltage (in volts). The voltage can be adjusted by turning a knob that is in the lower left corner of the ratemeter.

2. Start the program “Activ.exe”, which receives data from the ratemeter and passes it to the program “Origin 6.1”. The program “Activ.exe” is in the folder “C:\Artificial_Radioactivity”. It can also be started using a Start Menu shortcut. The initial window of the program “Activ.exe” is shown in Fig. 20a.

3. If the program “Origin” is open, close it. Click the button “Open Origin...”. In the “Open” dialog, create a new “Origin” project (or select an existing project, if you wish to add new data to an unfinished set of measurement results). In order to create a new project, navigate to a needed folder, enter the file name in the field “File name:” (that name must be different from the names of all other “Origin” projects located in that folder) and click the button “Open”. Then the program “Activ.exe” will create an empty project whose format is optimized for this experiment. **Important!** If the program “Origin” was already open when the button “Open Origin...” was clicked, then the data will be sent to the open “Origin” project, regardless of the file selected in the “Open” dialog. If several “Origin” projects are open, then the data will be sent to the project that was opened first.

4. If the program “Activ.exe” establishes connection with “Origin” successfully, then the button “Open Origin...” becomes disabled, and the button “Background” becomes enabled (see Fig. 20b). Then click the button “Prepare the ratemeter” (see Fig. 20b). This is needed in order to establish connection between the program “Activ.exe” and the ratemeter, as well as to set the measurement parameters (such as the duration of a single count). Sometimes the button “Prepare the ratemeter” has to be clicked several times. If the attempt to establish connection with the ratemeter is successful, then the following three control lamps (light-emitting diodes – “LED”) on the ratemeter should become lit (without blinking):

- „Gate“
- „10 s“
- „Continuous“

All other LEDs of the ratemeter should not be lit. Any other combination of LEDs that are lit, or existence of at least one blinking LED indicates that the required mode of operation has not been established yet, and the button “Prepare the ratemeter” has to be clicked again. Experience shows that this button may have to be clicked up to seven times before the attempt to set the required operation mode of the ratemeter is successful.

5. After that, click the button “Background”. Then the window of the program “Activ.exe” becomes as shown in Fig. 20c and it starts transferring the data to the “Origin” worksheet “Background” (this worksheet contains particle counts during 10 s-long measurements). **Note:** During the measurements, the LED “Gate” on the ratemeter must blink rapidly. If it does not blink, this means that the measurements have stopped. Then the measurement has to be restarted. In order to do that, click the button “Stop measuring the background”, repeat Step 4 and click “Background” again.

The “Origin” project has 10 windows:

background data worksheet window “Background“,

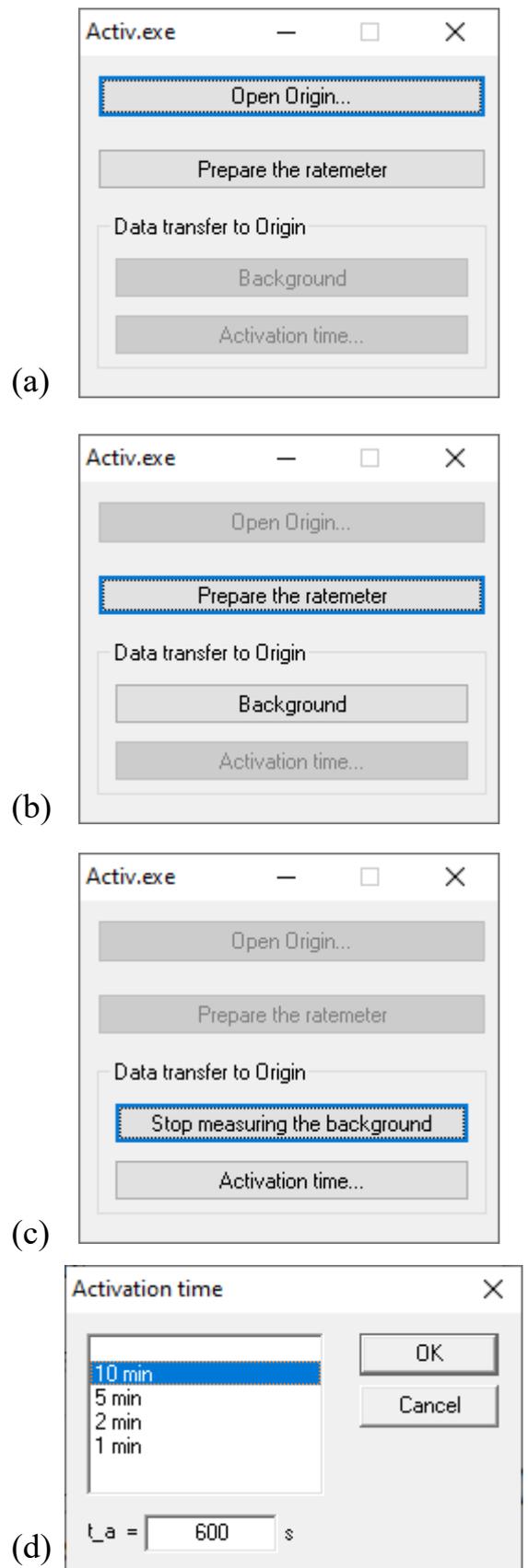


Fig. 20. The window of the program “Activ.exe“: (a) the initial window; (b) after establishing connection with “Origin”, before measuring the background, (c) during measurement of the background, when the activation time is not set yet; (d) the dialog window for selection of the activation time.

four windows with decay curve data (window titles are “A10min – worksheet”, “A5min – worksheet”, “A2min – worksheet” and “A1min – worksheet”),

four windows with graphs of decay curves (window titles are “A10min_graph”, “A5min_graph”, “A2min_graph” and “A1min_graph”),

activation curve graph window “Activation_curve”.

Any of those windows can be opened using the “Origin” menu “Window”. This can be done without stopping the measurements (the program “Activ.exe” will continue transferring the data to the correct worksheet even if its window is not open). A more convenient method of navigating between windows is based on the Origin’s internal “Project Explorer”, which can be opened using the menu command “View / Project Explorer”. In the latter case, it is recommended to use the Project Explorer view mode “List” (in order to change the view mode, right-click anywhere in the Project Explorer pane and then select “View / List” from the menu).

6. Take off the lid from the container of the neutron source (then the container looks as shown in Fig. 22a). After that, uncover the slit for insertion of the sample (i.e., pull the two narrow lead bricks to the sides as shown in Fig. 22b).

7. Set the activation time in the program “Activ.exe”. This is done by clicking the button “Activation time...” (see Fig. 20c). Note that it is not necessary to stop measuring the background, because the button “Activation time...” is enabled only during the measurement of the background. If the measurement of the background was stopped, then prepare the ratemeter again, i.e., repeat Step 4, and click the buttons “Background” and “Activation time...”.

8. In the activation time selection window (see Fig. 20d), select the needed activation time, i.e., click the corresponding item in the list (do not click “OK” yet).

9. Insert the sample into the neutron source container (see Fig. 23). When the sample is fully inserted (as shown in Fig. 23b), click the button “OK” in activation time selection window (see Fig. 20d) immediately. Then this window is closed and the program starts measuring the activation time. The program window becomes as shown in Fig. 21a. During the activation, the program continues measuring the background. During this stage, the time remaining until extraction of the sample is shown on the bottom button (see Fig. 21a).

10. When the time shown on the bottom button decreases to 0, the program stops measuring the background automatically and starts measuring the decay curve. In other words, the destination worksheet for the data transfer is changed: instead of the worksheet “Background”, the data are transferred to one of the four worksheets “A10min – worksheet”, “A5min – worksheet”, “A2min – worksheet”, “A1min – worksheet”, depending on the selected activation time (for example, if the 10 min activation was selected, then the data will be recorded in the worksheet “A10min – worksheet” and shown in the graph “A10min_graph” simultaneously). Then the text on the bottom button changes again: it becomes “Stop measuring the decay curve” (see Fig. 21b). When this happens, the sample must be extracted from the neutron source and placed upon the detector immediately, as shown in Fig. 18 and Fig. 24. Ideally, the total time of sample extraction from the neutron source and its placement upon the detector should be shorter than 10 s (then no additional data modifications will be needed in the future). If a longer time has passed, then take notice of the number of the initial points that will have to be excluded from fitting. For example, if the mentioned time is 25 s, then three initial counts will have to be removed (their approximate intervals are 0 – 10 s, 10 s – 20 s and 20 s – 30 s, counting since the end of activation).

11. Wait for 10 min until the measurement of the decay curve is stopped (the program “Activ.exe” stops the measurement automatically).

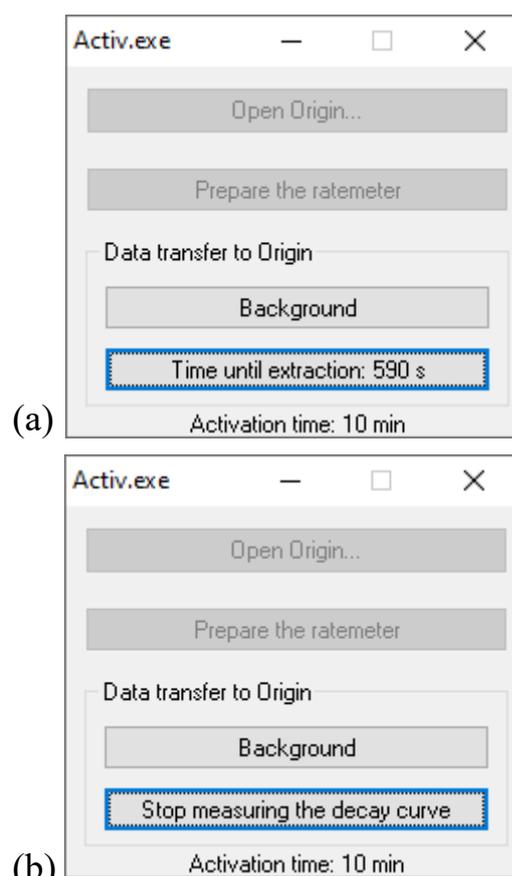


Fig. 21. The window of the program “Activ.exe”: (a) during activation, (b) while measuring the decay curve.



Fig. 22. The container of the neutron source: (a) without the lid; (b) with the sample slit uncovered

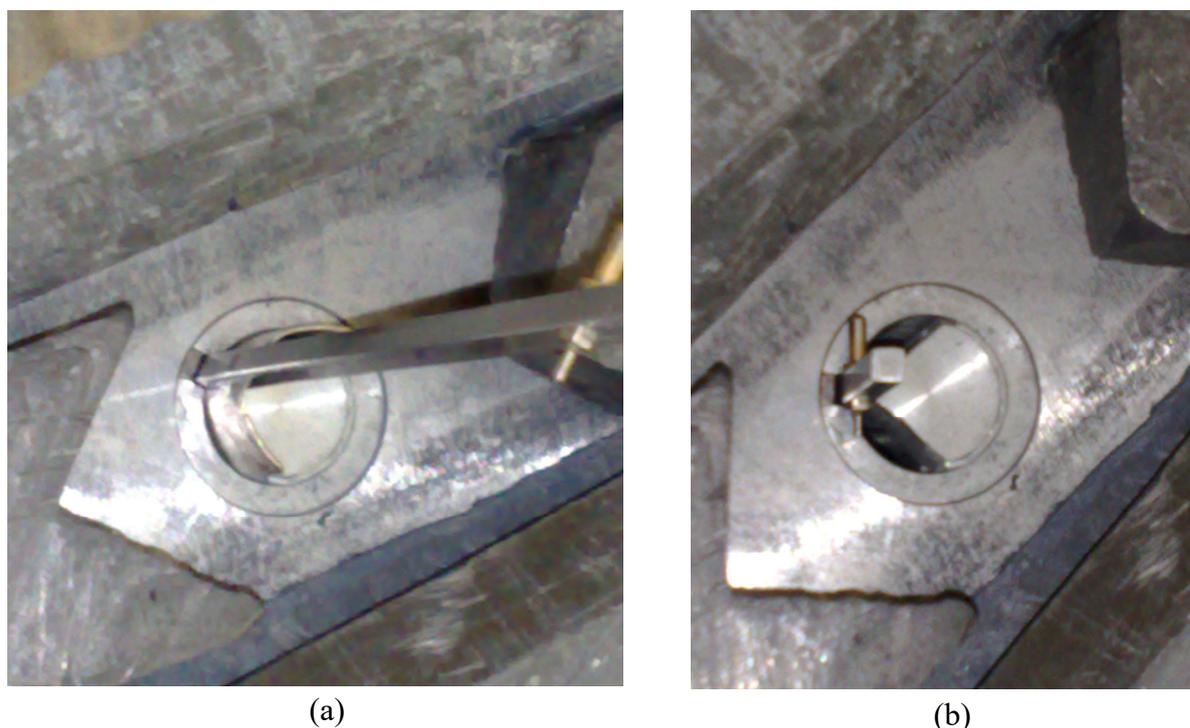
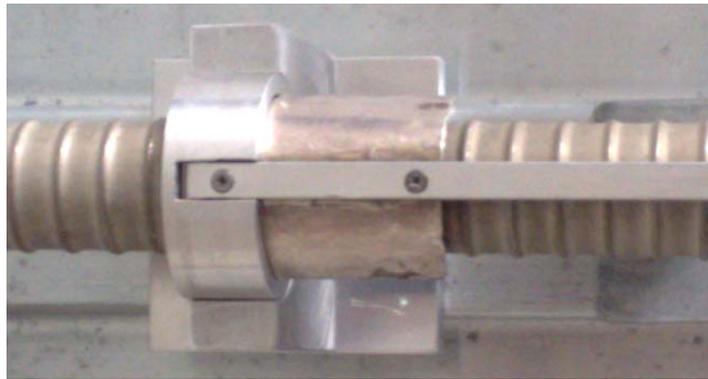


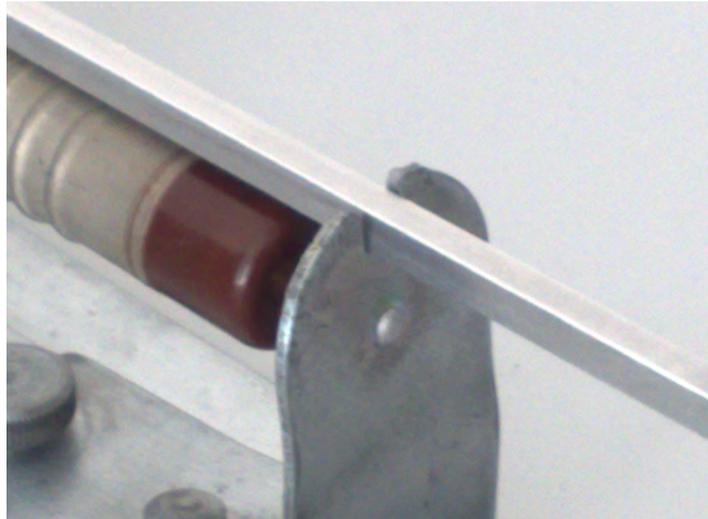
Fig. 23. Insertion of the sample into the neutron source container: (a) start; (b) end (the screw that is fastened upon the sample handle must touch the top of the container)

12. When the program “Activ.exe” stops the measurement, the program window becomes as shown in Fig. 20b again. Then place the sample sufficiently far away from the detector (a distance of 1 m is sufficient) and wait for additional 4–5 min until complete deactivation of the sample. This period of time should be used for measuring the background, i.e., adding more data to the worksheet “Background”. For this purpose, save the data file (“Origin” menu command “File / Save Project”), prepare the ratemeter (Step 4) and click the button “Background”. The time of this additional deactivation should be measured with any clock available. The lower limit of this time is about 4 min, and there is no upper limit (apart from the requirement to finish the experiment in time).

13. Repeat Steps 7 – 12 with a different activation time. In this way, four activation-deactivation cycles must be performed, which correspond to activation times 10 min, 5 min, 2 min and 1 min. Decay curve must be measured for 10 min (regardless of the activation time), and then additional 4–5 minutes must pass for complete deactivation of the sample. However, after measuring the third decay curve (after 2-minute activation), the additional deactivation time may be reduced to 3 min. After measuring the fourth (last) decay curve (after 1-minute activation), there is no need to wait for complete deactivation (or to measure the background). *Note:* The mentioned activation times are approximate. They have been chosen based on the requirement that one activation time is 2 – 5 times shorter than ^{108}Ag half-life, another one is close to the half-life, the third one is 2 – 3 times longer than the half-life and the fourth one



(a)



(b)

Fig. 24. Positioning of the silver sample on the detector while measuring the decay curve: (a) the end of the aluminum handle where sample is fastened must be pushed all the way into the holder; (b) the handle must be inside the cut of the detector stand

is 4 – 5 times longer than the half-life. Another set of activation times may be chosen (e. g., 12 min, 4 min, 1,5 min and 0,5 min). The true activation time (in seconds) must be specified in the text field that is at the top of the corresponding “Origin” graph (“Origin” reads the activation times from those fields). However, for correct functioning of the scripts that are associated with buttons visible on the right of the “Origin” project windows, the window titles must not be changed (for example, the titles of the windows corresponding to the longest activation time must be “A10min – worksheet” and “A10min_graph”, even if the corresponding true activation time is 12 min).

14. After ending the measurements, save the measurement data (“Origin” menu command „File / Save Project“ or „File / Save Project As...“), switch off the Isotrak ratemeter, cover the neutron source and print the worksheet data of all four decay curves. The printed numbers must not be processed (i.e., no background subtracted, etc.). All the printed data must be formatted as a single table and presented in a clear manner (this table can be created using a variety of programs, such as “Origin”, “Excel”, or “Word”). On the same sheet of paper, include the value of the average background, too. It is not necessary to include the standard deviations in the printed data. The list of printers in the “Print” dialog that pops up after selecting the menu command “File/Print” must contain the printer that is present in the laboratory. *Notes:* The printer that is currently used in the laboratory is not a network printer; instead it is connected to a computer that is connected to LAN. If the system cannot establish connection with the printer, this probably means that the mentioned computer or the printer is not switched on.

15. Write your name and surname on the printed sheets with measurement results. Show them to the laboratory supervisor for signing. Those sheets will have to be included in the final laboratory report.

The „Origin“ project file with measurement data and the scripts contained in that project may be used for analysis of the data and for creating the required graphs. That project file may be copied to a USB flash drive, or sent via Internet to another location for subsequent analysis.

12. Analysis of experimental results

One of the goals of this experiment is estimation of half-lives of nuclides ^{108}Ag and ^{110}Ag . This is done using the method of nonlinear least squares fitting. In order to minimize the fitting errors (the so-called “bias”), all the prior information about the investigated physical processes must be used. In the case discussed, there are two pieces of information that can be used:

- 1) When $t > 120$ s, the decay curve coincides with the slow component, because the short-lived nuclei of ^{110}Ag have practically completely decayed by this time. Thus, this part of the decay curve can be fitted by a single exponential function $C_1 e^{-\lambda_1 t}$.
- 2) The true values of the decay constants λ_1 and λ_2 do not depend on activation time (see the comment before Equation (9.6)).

Therefore, fitting is done using the following sequence of steps:

- I. The second part of each decay curve, corresponding to 60 s-long measurement, is fitted by a single exponential function $C_1 e^{-\lambda_1 t}$ with two varied parameters (C_1 and λ_1).
- II. An average $\langle \lambda_1 \rangle$ is calculated.
- III. The second part of each decay curve, corresponding to 60 s-long measurement, is fitted by a single exponential function $C_1 e^{-\langle \lambda_1 \rangle t}$ with one varied parameter C_1 .
- IV. The exponential function $C_1 e^{-\langle \lambda_1 \rangle t}$ is subtracted from the initial part, which corresponds to 10 s-long measurements. Before this operation, the pre-exponential factor C_1 must be re-calculated using Eq. (9.10).
- V. Since only the fast component remains after the mentioned subtraction, it is fitted by a single exponential function $C_2 e^{-\lambda_2 t}$.

The format of the “Origin” project with measurement results is designed to facilitate all steps of calculations. At the right of all windows of that project, there are push-buttons associated with scripts doing the various calculations and drawing the resulting graphs. Those scripts have been tested with two versions of “Origin”: “OriginPro 6.1” and “Origin 8”. In both versions, all calculations and plotting are done correctly (even though in “Origin 8” the error message “Error: Failed to execute script” appears).

Below is a step-by-step description of the analysis of measurement data.

1. Calculate the average background by clicking the button “Recalculate background” in any one of the windows (see Fig. 19). Then click the button “Subtract background” (see Fig. 19). This eliminates the background term Δn_b from the decay curves (see Eq. (9.11)). Then the decay curve graph window becomes as shown in Fig. 25. **Note:** After subtracting the background, some points in the region of large times may become negative, because nuclear decay is a random process, and because the background has been evaluated with an error. This is normal, and such points must not be excluded from analysis.
2. Click the button “Fit the slow component” (see Fig. 25). This script performs nonlinear fitting of the second part of the decay curve (corresponding to measurement time 60 s) by an exponential function $C_1 e^{-\lambda_1 t}$ with two varied parameters (C_1 and λ_1). Then a dashed line appears (see Fig. 26). This line is the exponential function corresponding to “optimum” values (in the least squares sense) of the varied parameters. The dashed line consists of two segments: the first one corresponds to $\Delta t = 10$ s, and the second one corresponds to $\Delta t = 60$ s. These two segments differ by the value of the pre-exponential factor (see Eq. (9.10)).
3. Perform steps 1 and 2 in the remaining three graph windows with the decay curves. If steps 1 and 2 are already done in any three of those windows, then after clicking the button “Fit the slow component” in the fourth window, the script associated with that button does the steps II and III that were mentioned earlier, i.e. it calculates the average $\langle \lambda_1 \rangle$ and its standard deviation, then repeatedly fits the slower part of each decay curve and displays the corresponding theoretical function (a solid curve consisting of two segments) in each decay curve window (see Fig. 27).

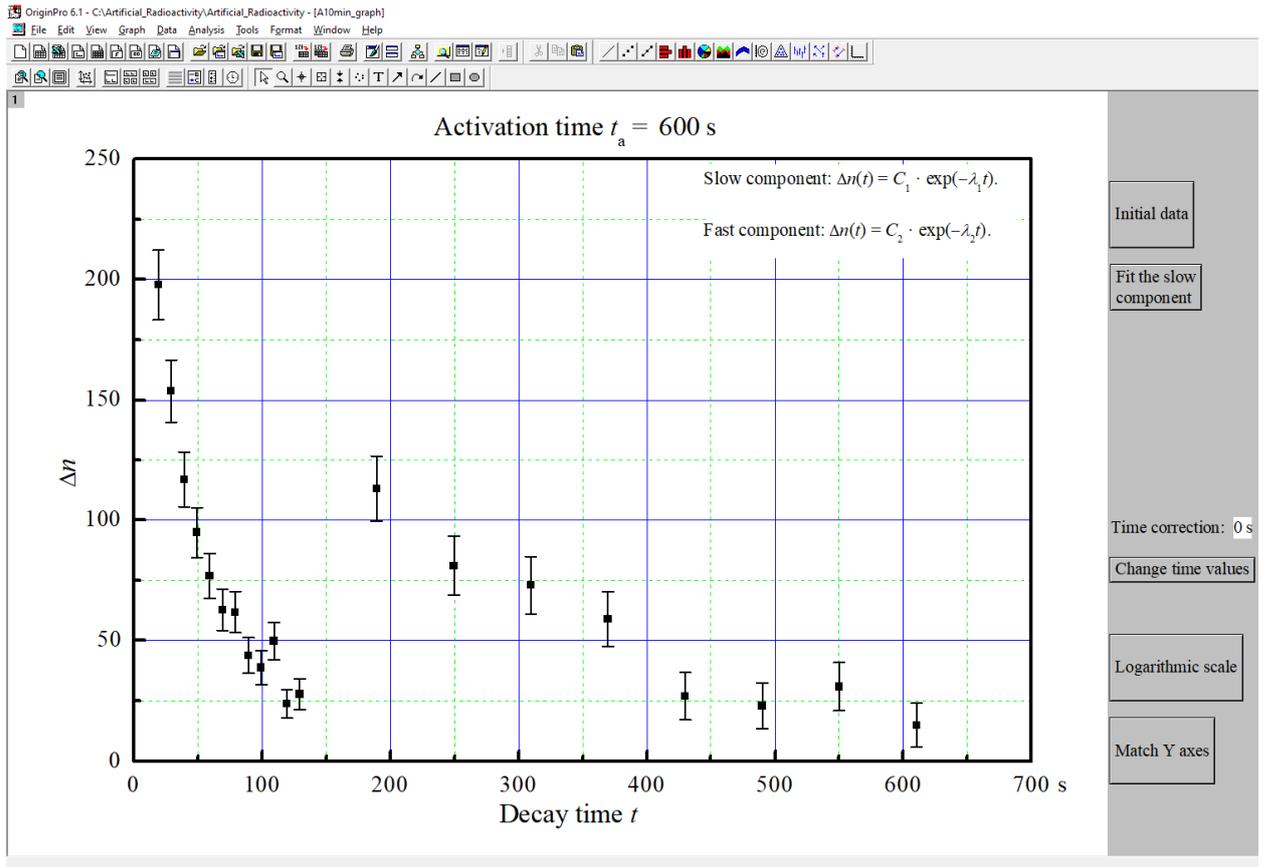


Fig. 25. The decay curve window after subtracting the background

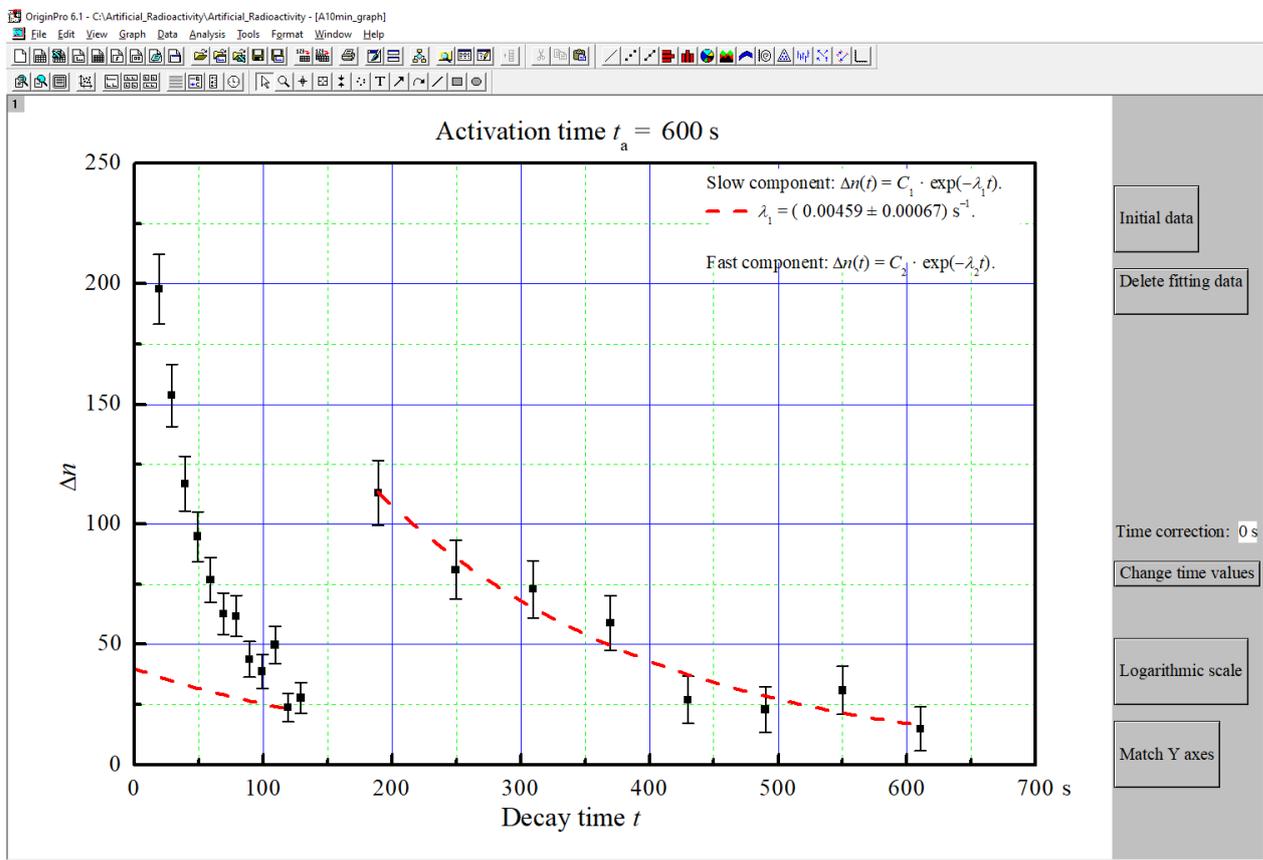


Fig. 26. The decay curve window after fitting the slow component

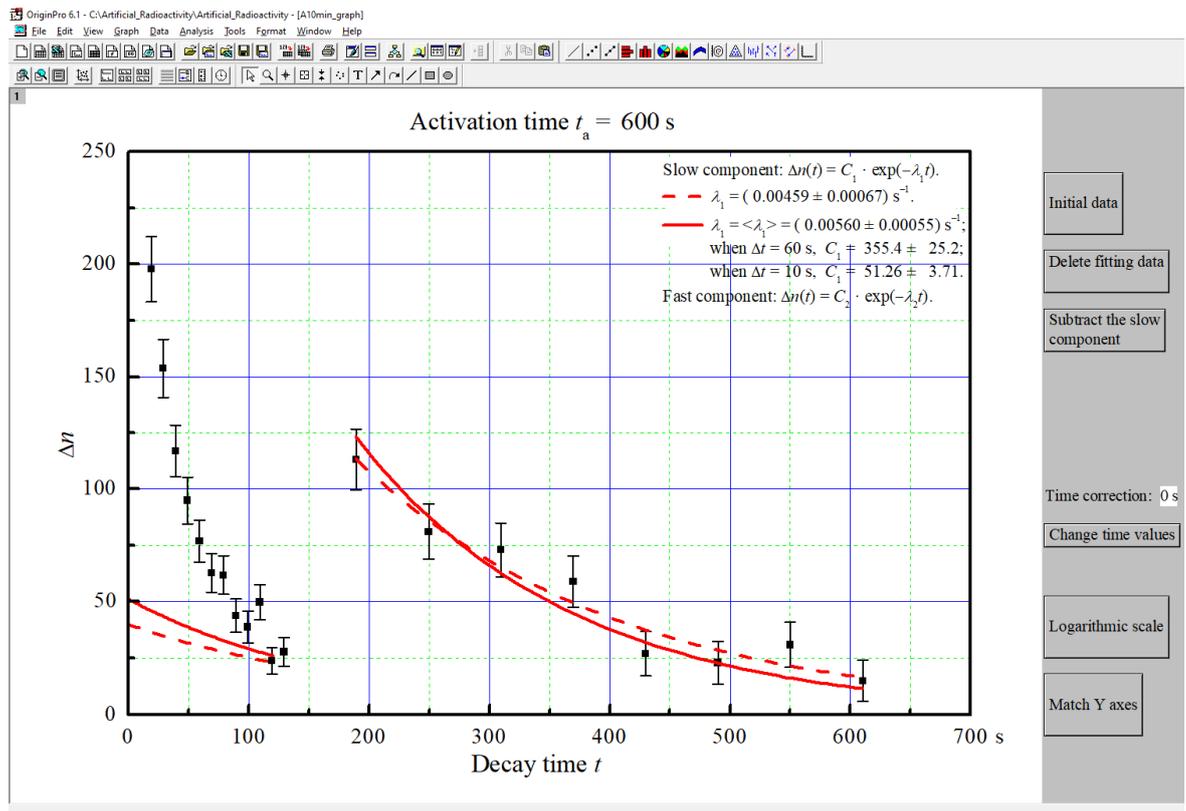


Fig. 27. The decay curve window after the second fitting of the slow component

In addition, after clicking the button “Fit the slow component” in the fourth window, the script draws the activation curve, i.e. the dependence of the coefficient C_1 on the activation time t_a , and fits that dependence by the theoretical function (9.9) with one varied parameter $C_{1,\text{sat}}$ and with λ_1 equal to the average $\langle \lambda_1 \rangle$. The results of that fitting are shown in the window “Activation_curve” (see Fig. 28).

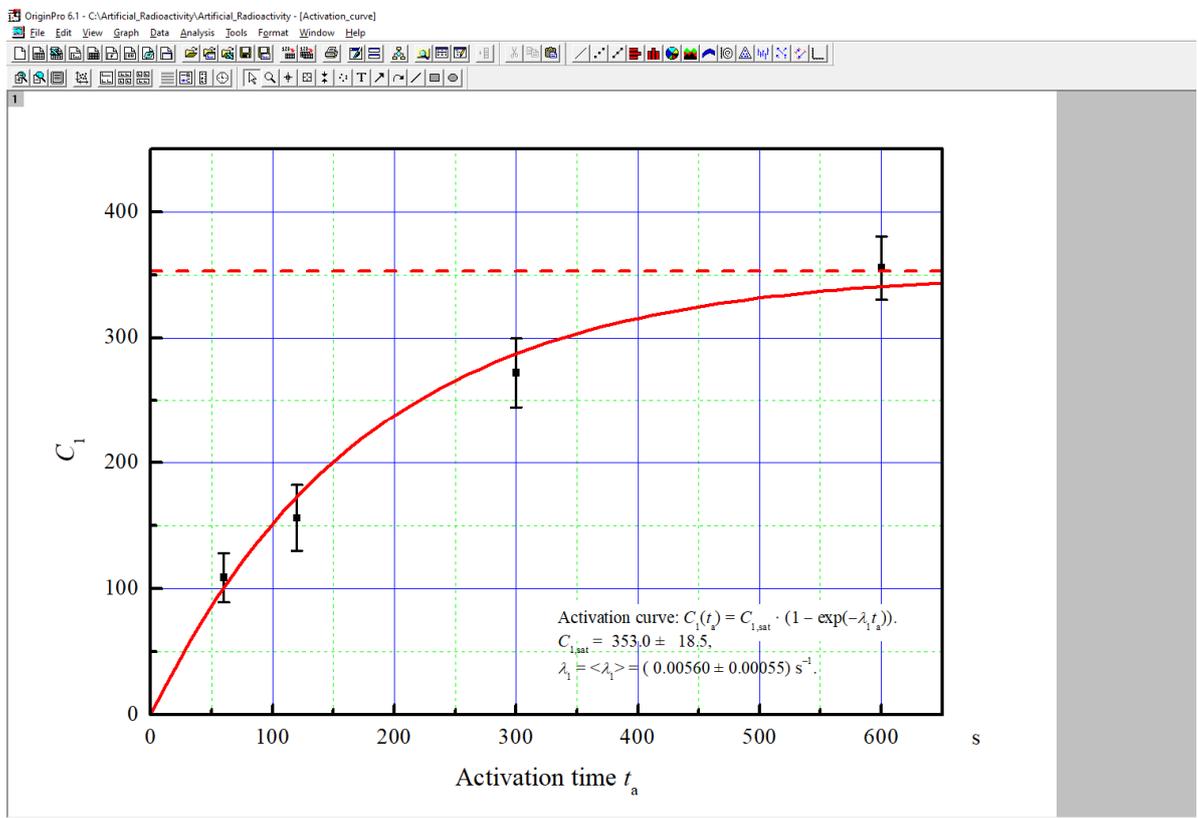


Fig. 28. The activation curve window. The horizontal dashed line indicates the fitted saturation value of C_1

The mentioned average $\langle \lambda_1 \rangle$ is the so-called “weighted average”, with weight factors inversely proportional to squared standard errors:

$$\langle \lambda_1 \rangle = \frac{1}{D_1} \left[\frac{\lambda_1^{(1)}}{(\Delta \lambda_1^{(1)})^2} + \frac{\lambda_1^{(2)}}{(\Delta \lambda_1^{(2)})^2} + \frac{\lambda_1^{(3)}}{(\Delta \lambda_1^{(3)})^2} + \frac{\lambda_1^{(4)}}{(\Delta \lambda_1^{(4)})^2} \right]; \quad (12.1)$$

where $\Delta \lambda_1^{(i)}$ is the standard error of the parameter λ_1 obtained by fitting the slower part of the i -th decay curve ($i = 1, 2, 3, 4$), and the coefficient D_1 is defined as follows:

$$D_1 \equiv \frac{1}{(\Delta \lambda_1^{(1)})^2} + \frac{1}{(\Delta \lambda_1^{(2)})^2} + \frac{1}{(\Delta \lambda_1^{(3)})^2} + \frac{1}{(\Delta \lambda_1^{(4)})^2}. \quad (12.2)$$

The standard error of the weighted average $\langle \lambda_1 \rangle$ is equal to

$$\Delta \langle \lambda_1 \rangle = \frac{1}{\sqrt{D_1}}. \quad (12.3)$$

- Click the button “Subtract the slow component” (see Fig. 28). This script subtracts the slow component from the initial part of the decay curve. Then the decay curve window becomes as shown in Fig. 29.

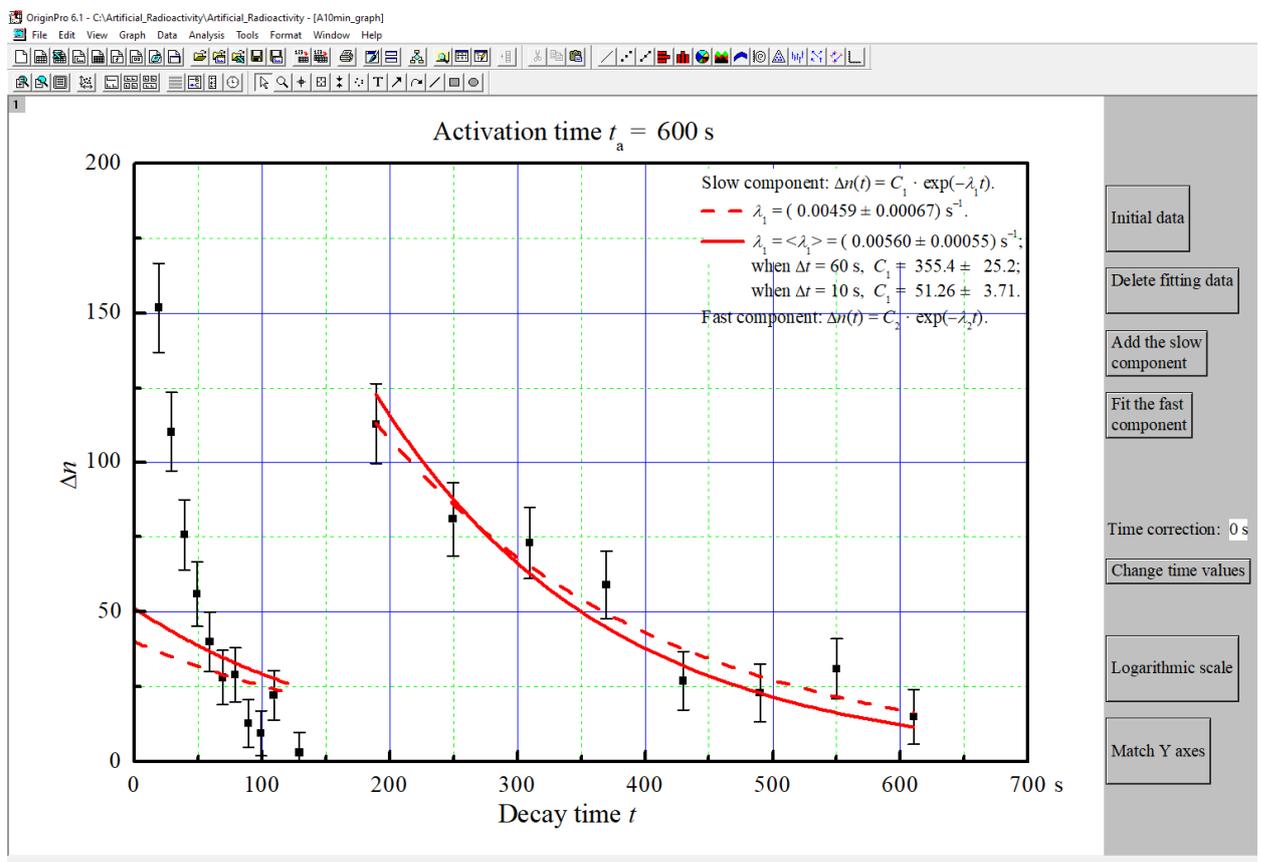


Fig. 29. The decay curve window after subtracting the slow component

- Click the button “Fit the fast component” (see Fig. 29). This script fits the initial part of the decay curve (corresponding to 10 s counts) by the exponential function $C_2 e^{-\lambda_2 t}$. Then a dash-dotted line appears (see Fig. 30). This line corresponds to “optimum” values of C_2 and λ_2 . **Note:** The fitting script excludes the first point ($t \approx 10$ s), because it is inaccurate, as explained in Section 5.
- Perform steps 4 and 5 in the remaining three graph windows with the decay curves.

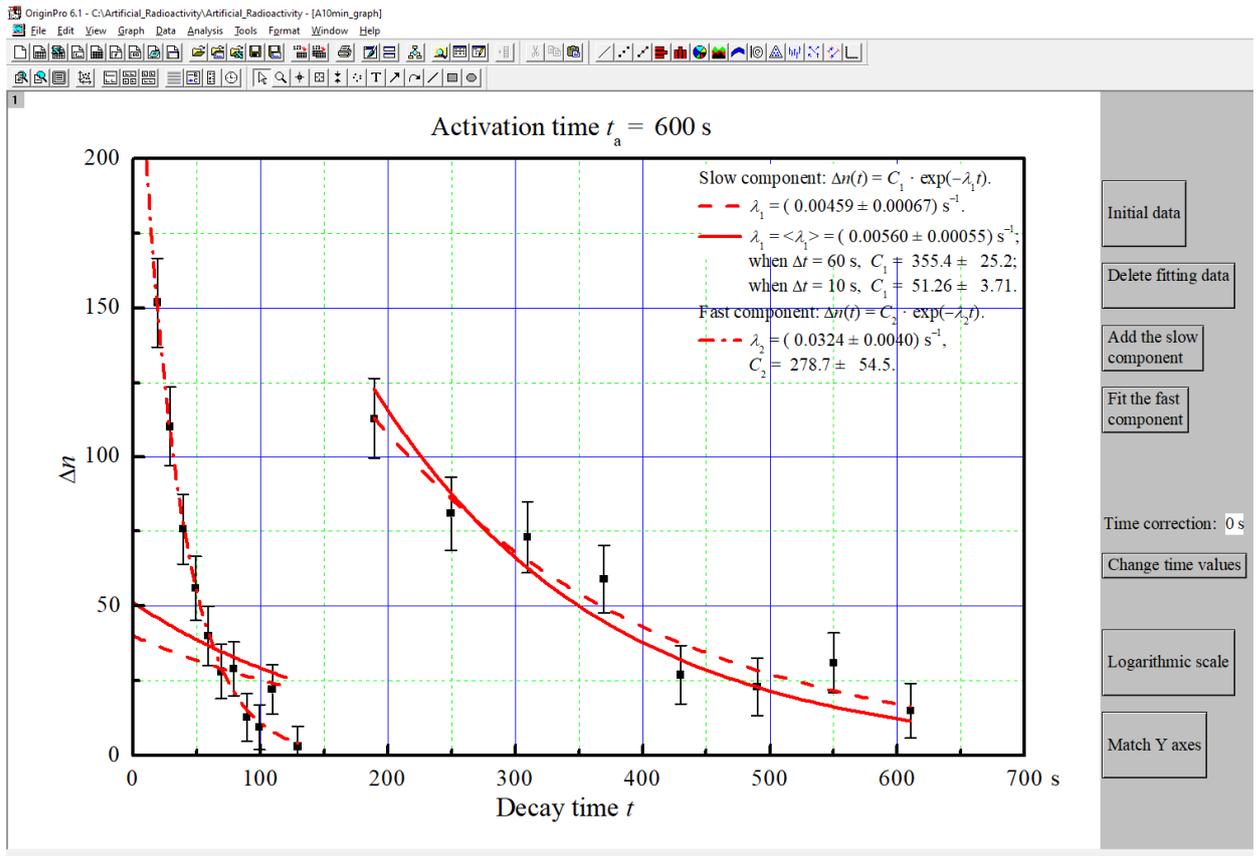


Fig. 30. The decay curve window after fitting the fast component

7. Calculate the average value $\langle \lambda_2 \rangle$ of the decay constant of ^{110}Ag and its standard error $\Delta\langle \lambda_2 \rangle$. There is no script for doing this, therefore those calculations must be done “by hand”. The most accurate result is obtained when the mentioned average is calculated in the same way as the average value of λ_1 , i.e. as the weighted average

$$\langle \lambda_2 \rangle = \frac{1}{D_2} \left[\frac{\lambda_2^{(1)}}{(\Delta\lambda_2^{(1)})^2} + \frac{\lambda_2^{(2)}}{(\Delta\lambda_2^{(2)})^2} + \frac{\lambda_2^{(3)}}{(\Delta\lambda_2^{(3)})^2} + \frac{\lambda_2^{(4)}}{(\Delta\lambda_2^{(4)})^2} \right]; \quad (12.4)$$

where $\Delta\lambda_2^{(i)}$ is the standard error of the parameter λ_2 obtained by fitting the fast initial part of the i -th decay curve ($i = 1, 2, 3, 4$), and the coefficient D_2 is defined as follows:

$$D_2 \equiv \frac{1}{(\Delta\lambda_2^{(1)})^2} + \frac{1}{(\Delta\lambda_2^{(2)})^2} + \frac{1}{(\Delta\lambda_2^{(3)})^2} + \frac{1}{(\Delta\lambda_2^{(4)})^2}. \quad (12.5)$$

The standard error of the weighted average $\langle \lambda_2 \rangle$ is equal to

$$\Delta\langle \lambda_2 \rangle = 1/\sqrt{D_2}. \quad (12.6)$$

8. Calculate the half-lives of ^{108}Ag and ^{110}Ag (denoted T_1 and T_2 , respectively), using Eq. (7.1a). Calculate their standard errors using the formula

$$\Delta T_{1/2} = \frac{\ln 2}{\langle \lambda \rangle^2} \Delta\langle \lambda \rangle, \quad (12.7)$$

where $\Delta T_{1/2} = \Delta T_1$ or ΔT_2 , $\langle \lambda \rangle = \langle \lambda_1 \rangle$ or $\langle \lambda_2 \rangle$, and $\Delta\langle \lambda \rangle = \Delta\langle \lambda_1 \rangle$ or $\Delta\langle \lambda_2 \rangle$.

The final report of the experiment must include the following 5 graphs: 4 decay curves with all fitting results (an example of such a graph is given in Fig. 30) and the activation curve with fitting results (an example is given in Fig. 28). The calculated values of $\langle \lambda_2 \rangle$, half-lives T_1 and T_2 and their standard errors must be presented, too.

The observed regularities must be discussed and compared with theory. In particular, take notice of the general shape of the decay curves, dependence of that shape on activation time, the shape of the activation curve. The measured half-lives must be compared with the true ones (142 s and 24.6 s). The discussion of the observed differences between the true values and the experimental ones must take into

account the fact that the mentioned standard error has the meaning of the half-width of the 68 % confidence interval (see the next section).

13. Using confidence intervals of experimental values

If the experimental value of a particular parameter (for example, half-life or decay constant) is p_{exp} , then its 68 % confidence interval is $[p_{\text{exp}} - \Delta p, p_{\text{exp}} + \Delta p]$, where Δp is the standard error. The term “68 % confidence interval” can be explained as follows. Let us suppose that an experiment was performed 10000 times under identical conditions, and that all observed deviations of experimental values from the true ones (including the deviations of individual points from the fitted curves) are random and described by the Gaussian distribution. Then the deviation of the optimal value p_{exp} from the true value p_{true} would be less than Δp in approximately 6800 cases (in those cases, the true value would be inside the 68 % confidence interval), and it would be greater than Δp in the remaining 3200 cases (in these cases, the true value would be outside the 68 % confidence interval). The 95 % confidence interval is twice wider than the 68 % confidence interval. This means that the 95 % confidence interval is $[p_{\text{exp}} - 2\Delta p, p_{\text{exp}} + 2\Delta p]$. I.e., after performing the experiment 10000 times at identical conditions, the deviation of the experimental value p_{exp} from the true value p_{true} would be less than $2\Delta p$ in approximately 9500 cases, and it would be greater than $2\Delta p$ in the remaining 500 cases. The 99.7 % confidence interval is wider than the 68 % confidence interval by a factor of 3. This means that the 99.7 % confidence interval is $[p_{\text{exp}} - 3\Delta p, p_{\text{exp}} + 3\Delta p]$. I.e., after performing the experiment 10000 times at identical conditions, the deviation of the optimal value p_{exp} from the true value p_{true} would be less than $3\Delta p$ in approximately 9970 cases, and it would be greater than $3\Delta p$ in the remaining 30 cases.

The random deviations mentioned in the previous paragraph are caused by the random nature of radioactive decay. Eq. (9.5) is applicable only to the statistical average, i.e., the average over an infinitely large number of identical experiments (assuming that the times of individual counts were identical in all experiments). In each experiment, individual points would deviate from the theoretical curve (9.5) randomly, and their standard errors would be approximately equal to $\sqrt{\Delta n}$ (this follows from the fact that the radioactive decay is a Poisson process). For example, if the number of particles counted during a particular interval of time is 100, then the 68 % confidence interval of the corresponding *true* value (statistical average) is [90, 110]. In this example, the *relative* standard error is approximately 10 %. In general, the relative standard error of a Poisson process is approximately equal to $\sqrt{\Delta n} / \Delta n = 1 / \sqrt{\Delta n}$. Thus, in order to achieve accuracy of 1 % of an individual count, the number of particles must exceed 10000. In this experiment, the results of individual counts are comparatively small (for example, see Fig. 19), which leads to large relative uncertainties. Those relative uncertainties are further increased by additional mathematical transformations (such as subtraction of the background) described above (the “Origin” scripts, which perform the calculations, automatically recalculate the standard errors after each transformation of the data, according to the usual statistical rules of propagation of errors in arithmetic operations).

In addition to the random errors discussed above, systematic errors (also called “bias”) are possible. The term “bias” is applied to non-random errors that have the same sign and magnitude in each experiment, if the measurement conditions are unchanged. Bias may be caused either by deficiencies of the measurement equipment, or by erroneous measurement procedure, or by inadequacy of the fitting procedure (which is used to extract the values of certain parameters from the “raw” measurement data, as in the present experiment). In order to distinguish bias from random errors, additional information is needed. In this experiment, the true values of the decay half-lives are known, and they may be used as the mentioned additional information. I.e., if the deviations of experimental half-lives from the true ones are too large to be considered purely random, then those deviations are (at least in part) a result of a bias. In order to determine the likelihood that a given deviation from a true value is purely random, the mentioned confidence intervals are used. By convention, the probability of 5 % or smaller to obtain an experimental value that differs from the true one as much as the current experimental result or more in absence of bias is considered so small as to be practically zero. I. e., such an event is considered too improbable to be caused only by random errors. Consequently, if the true value of a particular parameter (such as the half-life) does not belong to the 95 % confidence interval $[p_{\text{exp}} - 2\Delta p, p_{\text{exp}} + 2\Delta p]$, then one concludes that the deviation of the experimental value from the true one is at least partly a result of bias.

In summary, one has to check if the true values of half lives of ^{108}Ag and ^{110}Ag are inside the 68 % and 95 % confidence intervals defined above (the 95 % confidence interval must be checked only if the true value does not belong to the 68 % confidence interval), and to make the appropriate conclusions about the accuracy of this experiment.